bridge**course**

Physics

A Self Study Course after **Class 10** Board Exams

to Bridge the gap between Class X & XI &
to Build Foundation for Engineering/Medical Entrances

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ARIHANT PRAKASHAN (SERIES), MEERUT

PREFACE

Being involved in preparing the students for JEE for the last many years, I personally felt that when students enter in class XIth they feel a lot of difficulty in maintaining their spirit and enthusiasm of High School. The reason being that there is a vast gap between the levels of the two classes. Almost everything taught to them in 11th class is either new to them or has been taught to them in a different manner. Moreover, the vast syllabus and the big fat books frightened them, as a result of which even the very bright students of High school tend to loose their confidence in the initial stage of their intermediate. There are many good books available in the market which serves the purpose for curriculum of classes 10th and 11th but there is no book available in the market which can bridge the gap of these two classes and help the High school students to become successful student in their intermediate also.

The book in your hand is written with the motto-"Maintain the pace and spirit of our young minds for Towards JEE. The idea behind the introduction of this product is of Mr. Deepesh Jain (Director, Arihant Prakashan). With this motto in mind I completely went through the syllabus of class 8th, 9th, 10th and talked to various pioneer teachers of High School and those involving in IIT-foundation program about their curriculum, level of students and teaching methodologies. With this small research I started to develop the content of Towards JEE.

The major problem what I faced is that up to High school Algebra based physics has been taught and in 11th class calculus based, moreover the students came to know the things about calculus in the 2nd year of their intermediate, so to develop the content of this book I can't use calculus and it is also a proven fact that without calculus it is very difficult to explain certain concepts in physics, still I tried to explain those concepts in a different way by taking certain examples from daily life or I skipped those concepts. To understand the content of this book you do not need any previous concepts of physics. The book is complete in all sense. The only thing which I feel, is required, is the basic receptive mind, curiosity to learn, habit of understand the things from core and little mathematical skills.

This book can be useful for students of 8th, 9th and 10th class or for 10th to 11th moving students or by those who are having fear in physics or not having any interest in physics. This book will try to keep you in race for JEE.

I have put in my best sincere efforts to develop the content of this book and tried my level best to present the theory in a very simple and interesting way. Even though I have tried my best to provide the most accurate matter, the mistakes/errors might have gone unnoticed. If you find some, then please bring it to my notice. It would be highly appreciated.

I feel that this book will serve the motto ["In maintaining the pace and spirit of our young minds'] with which it has been written. Still to serve the student community in a better way-feedback and suggestions of students and teachers are invited.

Vikas Jain

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Chapter 10

Gravitation

The First Steps' Learning

- Newton's Law of Universal Gravitation
- Earth's Gravity and Acceleration Due to Gravity
- Planetary Motion and Kepler's Laws
- Motion of Satellites in Circular Orbits
- Geosynchronous Satellites

Throughout our discussion of physics till now in this book, we always frequently encountered the term 'gravity force' which is nothing but the measure of gravitational attraction between the earth and an object on its surface or near to its surface. In present chapter we shall explore this concept of gravitation in somewhat more detail.

We have seen in chapter 5 (Newton's Laws of Motion) that gravitational force is one of the four basic forces that exist in the nature. Let's imagine that you make a jump to kick the ball, and after kicking, the ball doesn't come down and you are flying in air. This incident seems to be a fantasy, but this can be possible if we were to have a gravity-free world ie, if there is no gravity, then you won't come back to earth or in other words we can say, it is the gravitation which binds us to the earth and also holds the other planets and the sun in the solar system. In a broader sense, it is the gravity which holds the universe together, and makes the "Newton's Apples" fall to the ground only. It was Sir Issac Newton, who first analysed and interpreted the gravitation in great detail and propounded a mathematical law related to gravitational phenomenon. Although many scientists were able to understand the phenomenon of gravitation before Newton, but none was able to formulate the things in a very precise terms.

Newton's Law of Universal Gravitation

It has been said that once Sir Issac Newton was sitting in his garden, and observing the falling apple from a tree (which is a very common sight in apple growing regions for ordinary people), but intellectual brain of Newton start asking variety of questions from Newton—why the apple is falling down ? Why it is not going up or remains in air ? If apple is falling down, then why not the moon ? Or moon is also falling down ? If apple is falling down, then some force must act on it according to my second law of motion—who is exerting this force and why ?

All these questions and some more led Newton to generalize the idea that the earth attracts all objects towards its centre. He also realized that same force *ie*, the force of gravitation governs the motion of falling apple and also the motion of moon and this realization of Newton was of great achievement to science. This particular generalization of Newton, that earth attracts every object towards is centre—is termed as the gravity of earth or we can say that it is the gravitational force experienced by objects due to earth's gravity. The force with which the earth attracts objects towards its centre—is termed as the earth's gravity force. It is the earth's gravity force only, which is responsible to bind us with the earth, holds the atmosphere above the earth, falling of rain drops to the earth, motion of moon around the earth and many more illustrations can be added to the list.

But things were not yet over for Newton in the pursuit of gravitation, there was something more and even more revolutionary, was to be deduced by Newton, and this comes in the form of **Newton's universal law** of gravitation, which states.

"Every particle of the matter in the universe attracts every other particle with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them."

Consider two particles A and B of masses m_A and m_B respectively, placed at a separation r as shown in the figure, then from Newton's universal law of gravitation they attract each other with a force F which is characterized by

$$F \propto m_A m_B, \text{ and}$$

$$F \propto \frac{1}{r^2}$$

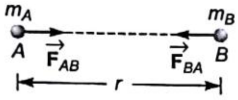


Fig. 10.1

Combining above two equations, we get

$$F \propto \frac{m_A m_B}{r^2}$$

After removing the proportionality sign, we get

$$F = \frac{G m_A m_B}{r^2}$$

where G is a constant known as **universal constant of gravitation**. Its value was measured experimentally by Cavendish, long after Newton's work. The value of G is measured as 6.67×10^{-11} N-m^{2}kg^{-2}. The value of universal constant of gravitation, G does not depend on place and time *ie*, value of G at your table and also at moon would be same (*ie*, independent of place), and the other truth is that the value of G ten years back was same and after ten years also it would be the same. The value of G is also independent of the nature of particles and the separation between them. As value of G is doesn't depend upon any factor, it is said to be universal constant *ie*, a constant whose value remains constant and is independent of position and time. And for the same reasons the Newton's law of gravitation is also called an universal law as it is applicable for all types of particles having mass and is valid at all places at all times.

It is important to keep in mind that Newton's law of gravitation is valid only for point objects and not for extended objects*. Although Newton had done tedious mathematical calculations to show that if two spheres having **uniform mass distribution**** are placed at a certain distance apart then the Newton's law of gravitation gives the force of attraction between them by considering r as is the separation between their centres."

Now, we mention here few of the important points related to universal law of gravitation :

(a) It is a universal law, according to which every particle attracts every other particle of the universe. We shall discuss about this in greater detail towards the end of this section.

(b) The gravitational force between two particles A and B acts along the line joining the particles. If A experiences a force $\vec{F}_{AB}$ due to B, then B experiences a force $\vec{F}_{BA}$ due to A such that $\vec{F}_{BA} = -\vec{F}_{AB}$, *ie*, both particles attract each other with equal and opposite force given by $\frac{G m_A m_B}{r^2}$

ie,

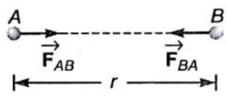


Fig. 10.2

$$|\vec{F}_{AB}| = |\vec{F}_{BA}| = F = \frac{G m_A m_B}{r^2}$$

So we can say that gravitational force on A and B due to each other forms an action-reaction pair.

(c) The gravitational force between two particles is independent of the medium present between them.

(d) Universal law of gravitation is valid even for very small distances of the order of 10^{-10} m as well as for very large distances of the order of 10^{12} m. In other words, we can say that it is valid for atomic distances as well as for astronomical distances.

(e) Universal law of gravitation is valid for subatomic particles as well as for terrestrial objects like planets, stars etc.

*For extended objects to compute gravitational force use of calculus is required.

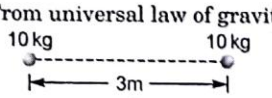
**Uniform mass distribution means mass per unit volume is constant.

C-BIs

Concept Building Illustrations

Illustration | 1 *Two particles of mass 10 kg each are placed at a separation of 3 m. Determine the gravitational force between them.*

Solution From universal law of gravitation



$$F = \frac{Gm_1m_2}{r^2}$$

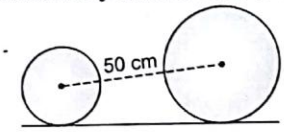
Here, $m_1 = m_2 = 10$ kg and $r = 3$ m

So, $$F = \frac{6.67 \times 10^{-11} \times 10 \times 10}{(3)^2} \text{ N}$$

$$= 7.41 \times 10^{-10} \text{ N}$$

*Just think that how much small this force is.

Illustration | 2 *Two spheres, one of mass 5 kg and radius 10 cm and other of mass 10 kg and radius 20 cm are placed at a distance of 50 cm [Separation between their centres] as shown in figure. Assume the mass distribution of both the spheres to be uniform. Determine the gravitational force with which they attract each other.*



Solution As two uniformly distributed mass spheres could be considered as two point masses placed at their respective centres for application of universal law of gravitation,

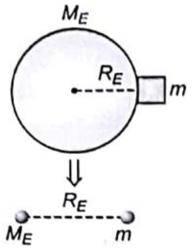
so, $$F = \frac{Gm_1m_2}{r^2}$$

$$\Rightarrow F = \frac{6.67 \times 10^{-11} \times 5 \times 10}{(0.5)^2} = 1.34 \times 10^{-8} \text{ N}$$

Illustration | 3 *A block of mass 5 kg is placed on the surface of earth. Determine the gravitational force experienced by the block due to earth.*

[Take mass of the earth, $M_E = 6 \times 10^{24}$ kg, radius of the earth, $R_E = 6.4 \times 10^3$ km].

Solution Earth can be considered as a sphere of uniform mass distribution, and we know a uniformly distributed mass sphere can be replaced by a point mass (whose mass is equal to mass of sphere) placed at the centre of sphere. The equivalent arrangement is shown in the figure.



The force experienced by the block due to earth's gravitation is

$$F = \frac{GM_Em}{R_E^2} = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 5}{(6.4 \times 10^6)^2} \text{ N}$$

$$= 48.85 \text{ N}$$

Illustration | 4 *The mass of the earth is 6×10^{24} kg and that of moon is 7.4×10^{22} kg. The distance between the centre of the moon and earth is 3.84×10^5 km. Assume both the earth and moon to be uniform spheres. Determine the gravitational force experienced by the earth on moon.*

Solution Using concepts of illustration 2,

$$F = \frac{GM_EM_M}{(R_{EM})^2}$$

where M_E is mass of earth equal to 6×10^{24} kg, M_M is mass of moon equal to 7.4×10^{22} kg.

R_{EM} is separation between centre of moon and earth equal to 3.84×10^5 km.

$$F = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 7.4 \times 10^{22}}{(3.84 \times 10^8)^2}$$

$$= 2 \times 10^{20} \text{ N}$$

Last four illustrations give you the insight that every object in the universe attracts every other object with certain force that depends on their masses and the separation between them. This force can be very small as in illustrations 1 and 2, can be having moderate values as in illustration 3 and can also be very high as in illustration 4. Now you may ask a question, if every object attracts every other object in universe, then why different objects on earth (like human beings, cars, plants, mountains etc) are not colliding to make a single object? The reason is that gravitational force is very small for objects present on the earth, which is unable to overcome other forces acting on objects. For example, let us consider you are standing on the platform (railway) and a train is stationary on the rails in front of you. If your mass is say 40 kg and train mass is say 10^5 kg (let us make an assumption for simplification purpose consider train to be a point mass) and separation between you and train to be 1 m. Then the gravitational force experienced by you due to train is

$$F = \frac{GM_T \times m}{r^2}$$

$$= \frac{6.67 \times 10^{-11} \times 10^5 \times 40}{1^2} = 2.67 \times 10^{-5} \text{ N}$$

If we take coefficient of friction between your shoes and platform as $\mu = 0.2$, then the limiting friction force between your shoes and platform is $f_L = \mu mg = 0.2 \times 40 \times 9.8 = 78.4$ N.

Now it means, minimum 78.4 N force must act on you to cause your motion, but the gravitational force experienced by you due to a very big object (train) is negligible as compared to minimum force required to cause your motion so, it can't cause your motion. But it doesn't mean that gravitational force can never cause the motion, in illustrations 3 and 4 you can easily see that gravitational force is large enough to cause the motion of body. The concept is that if at least one of the bodies is large (like sun or earth or any other heavenly body) then gravitational force becomes comparable to the force required for motion and hence may cause the motion. From illustration 4 you can infer that moon is experiencing a very large force ($= 2 \times 10^{20}$ N) due to gravitational attraction of earth, and it is this large force which makes the moon revolve around the earth. So, we can say that moon is bounded to earth only due to gravitational force of earth.

Earth's Gravity and Acceleration Due to Gravity

If you throw a ball upwards then it comes back, motion of moon around the earth, motion of satellites around the earth, working of hydroelectric power plants etc can be only made possible because of earth's gravitation.

Earth attracts all the bodies which are in the region of its gravitational influence, say a ball is dropped from a high tower, then it will fall towards the surface of earth. If you drop a piece of paper from some height, then it will also fall towards the surface of earth but slowly as compared to ball. So, from this observation can we conclude that under the earth's gravitational influence alone, the lighter bodies will fall slowly as compared to heavier bodies? No, because here, air resistance is also acting on the objects as they are falling down.

Air Resistance

The force exerted by air friction is termed as air resistance. Its value depends on the mass of the object, its speed and its surface area.

It has been known earlier (before Galileo) that, when a feather and a stone are dropped from same height, then the stone will reach the ground first and from this people have concluded that lighter objects are falling slowly as compared to heavier objects in the earth's gravity. Galileo suggested that all objects are falling with the same acceleration under earth's gravity and he explained the observation on the basis that because of air friction the feather is falling slowly. Later on, Robert Boyle also experimentally explained the fact that both the

feather and the stone are falling with same acceleration due to earth's gravity. He dropped a feather and a coin from same height in a closed tall glass tube fitted with a vacuum pump (Being used to pump out air and thus creating vacuum in the glass tube) and observed that both the feather and the coin sink to the bottom of tube at the same time, thus showing that both are falling at same rate under earth's gravity. In other words, we can say that acceleration of all falling bodies near the surface of earth is same and is independent of the mass of the body provided air friction is neglected.

The acceleration of the body falling under the earth's gravitation alone is termed as acceleration due to gravity. Here, falling doesn't mean that the body is moving towards the earth's surface, it can be moving up or it can be in projectile motion. Let us consider, a body of mass m moving near the surface of earth under the earth's gravitational influence. It will experience a force due to earth's gravity given by

$$F = \frac{GM_E m}{R_E^2} \text{ towards the centre of earth}$$

where M_E is the mass of earth and R_E the radius of earth.

The acceleration of the body is, $a = \frac{F}{m} = \frac{GM_E}{R_E^2}$, towards the centre of earth. It is clear from above expression that acceleration of a body is independent of its mass.

If gravity is the only force acting on an object near the surface of earth, then its acceleration is independent of mass and is termed as acceleration due to gravity (g), which is given by

$$g = \frac{GM_E}{R_E^2} \text{ directed towards the centre of}$$

earth.

$$= \frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{(6.4 \times 10^6)^2} = 9.8 \text{ ms}^{-2}$$

This value of g is valid at the surface of earth or for the points near to the surface of earth *ie*, whose height from the surface of earth is negligible as compared to the radius of earth. The acceleration of the particle moving under earth's gravity remains the same (both magnitude and direction), whether the particle is falling down, moving up or moving at some angle with the horizontal, and in all cases, the particle is said to move freely under gravity.

The acceleration due to gravity is not a universal constant, its value is different at different positions, and it also depends on various other factors.

Let us consider a body falling freely under earth's gravity, and at an instant it is at a height h from the earth's surface as shown in the figure. Here h is not negligible as compared to the radius of earth, R_E.

The force experienced by the body due to the earth's gravity is,

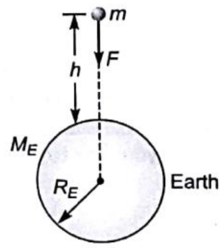


Fig. 10.3 A ball is at a height h above the earth's surface.

$$F = \frac{GM_E m}{(R_E + h)^2}$$

So, acceleration due to gravity at a height h from the surface of earth is

$$g = \frac{F}{m} = \frac{GM_E}{(R_E + h)^2}$$

variation in acceleration *due to gravity with height.*

If $h << R_E$ or $h = 0$, then $g = \frac{GM_E}{R_E^2}$ $= 9.8 \text{ ms}^{-2}$ which we have already seen.

From above expression, we can conclude that as one moves away from the surface of earth, the value of g decreases. Not only this, value of g also changes as we go deep inside the mine or if one goes from equatorial region to polar region. It has also to be kept in mind that acceleration due to gravity is defined separately for moon, sun, mars etc.

C-BIs

Concept Building Illustrations

Illustration | 5 *Determine the acceleration due to gravity at a height of $h = \frac{R_E}{2}$ from earth's surface. At earth's surface, $g = 9.8\ ms^{-2}$.*

Solution Acceleration due to gravity at a height h from the earth's surface is given by

$$g = \frac{GM_E}{(R_E + h)^2}$$

$$= \frac{GM_E}{(R_E + R_E/2)^2} = \frac{4}{9} \times \frac{GM_E}{R_E^2}$$

$= \frac{4}{9} \times$ Acceleration due to gravity at Earth's surface

$$= \frac{4}{9} \times 9.8$$

$$= 4.36\ \text{ms}^{-2}$$

Weight

In chapter 5 we discussed about the weight of an object, here we are discussing something more about weight. Generally, weight is defined as the force with which earth attracts it, but more accurately weight of an object is defined as "the force with which a heavenly body (earth, moon, sun, mars etc.) attracts an object."

If we consider, an heavenly body (which can be considered as a uniformly distributed mass sphere) of mass M and radius R, then it will attract an object of mass m, on its surface with a force given by

$$F = \frac{GMm}{R^2}$$

This itself is the weight of object on this heavenly body *ie*, weight,

$$w = \frac{GMm}{R^2}$$

$$= mg'$$

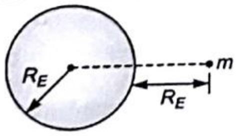
where $g' = \frac{GM}{R^2}$ is the acceleration to gravity at the surface of this heavenly body. If we want the weight of an object at the moon's surface, then M and R would be the mass and radius of moon respectively. As value of g is different at different places, weight of an object is also different at different places.

C-BIs

Concept Building Illustrations

Illustration | 6 *The weight of an object at the earth's surface is 400 N. Determine its weight at a height of $h = R_E$, from the earth's surface.*

Solution The force experienced by the object at $h = R_E$, due to earth is



$$F = \frac{GM_E \times m}{(R_E + h)^2} = \frac{GM_E \times m}{4R_E^2}$$

It is given that weight at the surface of earth is 400 N, so

$$\frac{GM_E m}{R_E^2} = 400\ \text{N}$$

So, the required weight,

$$w = F = \frac{1}{4} \times \frac{GM_E m}{R_E^2} = \frac{1}{4} \times 400\ \text{N} = 100\ \text{N}$$

Illustration | 7 *The weight of an object at the earth's surface is 600 N. Determine its weight on moon's surface.* $M_E = 6 \times 10^{24}$ *kg,* $R_E = 6.4 \times 10^3$ *km,* $M_M = 7.4 \times 10^{22}$ *kg,* $R_M = 1740$ *km.*

Solution The weight of the object at moon's surface is given by,

$$w' = \frac{GM_M m}{R_M^2}$$ where m is the mass of object.

It is also given that, weight of the object at the earth's surface is 600 N.

$$w = 600\text{ N} = \frac{GM_E m}{R_E^2}$$

Dividing above two equations, we get

$$\frac{w'}{w} = \frac{G(M_M m)}{R_M^2} \times \frac{R_E^2}{GM_E \times m}$$

$$= \frac{M_M}{M_E} \times \left(\frac{R_E}{R_M}\right)^2$$

$$= \frac{7.4 \times 10^{22}}{6 \times 10^{24}} \times \left(\frac{6.4 \times 10^6}{1740 \times 10^3}\right)^2 \simeq \frac{1}{6}$$

$$w' = \frac{w}{6} = 100\text{ N}$$

From above relation it can be generalized that weight of an object at moon's surface is approximately $\frac{1}{6}$th of the weight of object at the earth's surface. Or in other words, we can say that acceleration due to gravity at the earth's surface is 6 times of the acceleration due to gravity at the moon's surface.

Illustration | 8 *An object is thrown in vertical upward direction from earth's surface in such a way that it reaches to a height of* 10 m *before reversing its direction of motion. If the same object is thrown with same speed on the moon's surface in vertical upward direction, then by how much distance it will move before reversing its direction of motion ?*

Solution Let v_0 be the initial projected velocity of object and g be the acceleration due to gravity at earth's surface. Then from previous illustration, acceleration due to gravity at moon's surface is, $g' = \frac{g}{6}$.

When the object is thrown from earth's surface : The speed of the object at its highest point of motion is zero, so from the equations of motion,

$$0 = v_0^2 - 2gh \quad \text{where } h = 10\text{ m (given)}$$

$$\Rightarrow \quad h = \frac{v_0^2}{2g}$$

When the object is thrown from the moon's surface :

Let h' be the distance travelled by the object before it reverses its direction of motion, then

$$0 = v_0^2 - 2g'h'$$

$$\Rightarrow \quad h' = \frac{v_0^2}{2g'} = \frac{v_0^2}{2g/6}$$

$$= 6 \times \frac{v_0^2}{2g}$$

$$= 6 \times 10 = 60\text{ m}$$

Planetary Motion and Kepler's Laws

You all may be familiar about our solar system, in which the planets (heavenly bodies like Earth, Mars, Jupiter etc.) around the star (Sun). These planets revolve around the Sun in elliptical orbits, under the gravitational influence of Sun. Our solar system is shown adjacent :

Planets are those heavenly bodies, which revolve around the star.

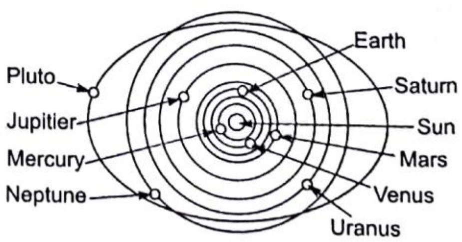


Fig. 10.4 Solar system

A satellite is a heavenly or any other body which revolves around the planet.

In 16th century, a great astronomer named Johannes Kepler deduced three laws which govern the motion of all planets, the same laws govern the motion of satellites too. Kepler deduced these laws after the rigorous and tiring observation of motion of planets for about 20 years. He also took the help of the recorded data (related to motion of planets) of Tycho Brahe of whom Kepler was once an assistant. He corelated the data recorded by Brahe and his observations and tried to predict the regularities or irregularities in the motion of planets and after analysing the varied data he went on to attempt to put his conclusions in mathematical forms or in form of laws. Here, we are presenting these laws of Kepler which were prevalent long before the Newton's law of universal gravitation. The surprising thing is that these laws now can be derived from Newton's law of universal gravitation. In other words, we can say that Kepler found how the planets are moving, and later Newton explained why they are moving in such a way?

Kepler's First Law

For basic knowledge of ellipse, you can refer the Mathematical Appendix at the end of the book.

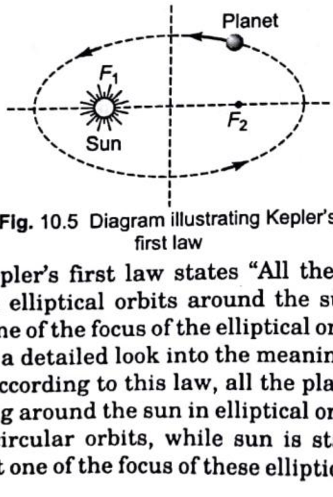


Fig. 10.5 Diagram illustrating Kepler's first law

Kepler's first law states "All the planets move in elliptical orbits around the sun, with sun at one of the focus of the elliptical orbit." Let us have a detailed look into the meaning of this law— According to this law, all the planets are revolving around the sun in elliptical orbits and not in circular orbits, while sun is stationary and is at one of the focus of these elliptical orbit. In the figure we have shown how a planet is revolving around the sun in an elliptical orbit.

Kepler's Second Law

Kepler's second law states "Each planet revolves around the sun in such a way, that the line joining the planet and the Sun sweeps out equal area in equal interval of time."

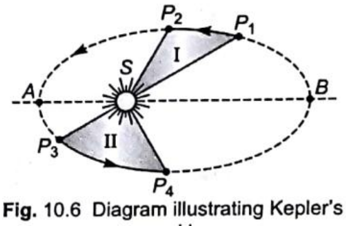


Fig. 10.6 Diagram illustrating Kepler's second law.

Consider a planet, which is moving around the Sun in elliptical orbit as shown in the figure. If the planet moves from P_1 to P_2 in time t, then in same time t it moves from P_3 to P_4, such that area of region I is equal to area of region II. Now, it is clear from the figure that P_1P_2 is less than P_3P_4, so we can conclude that the speed with which the planet is revolving around the sun is not constant, and the speed of planet is more when it is nearer to Sun. Also, the planet moves slowly when it is farther from the Sun. During its motion the planet moves with maximum speed when it is nearest to the Sun, in the figure this position is shown by A, and planet moves with minimum speed when it is farthest from the Sun, this point is marked as B in the diagram.

Kepler's Third Law

Kepler's third law states "The cube of a semi-major axis of elliptical orbit of a planet is directly proportional to the square of

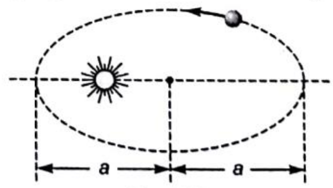


Fig. 10.7

time-period of the planet." Mathematically, $T^2 \propto a^3$ where T is the time period of planet and a is semi-major axis of elliptical orbit. Consider a planet which is revolving around the sun in an elliptical orbit as shown in figure. If T is the time period of planetary motion *ie*, T is the time taken by planet to complete one revolution around the sun, and a be the semi-major axis of elliptical orbit, then according to Kepler's third law, $T^2 \propto a^3$.

The Kepler's three laws are totally based on the observation of planetary motion, and he was not able to explain the theory related to the motion of planets. But later on Newton revealed the cause of motion of planets, and on the basis of his theory of gravitation and the dynamics laws (Newton's laws of motion), he explains that gravitational force between a planet and the Sun is responsible for the motion of the planet and also confirms the authenticity of Kepler's laws.

Motion of Satellites in Circular Orbits

In this section we are going to discuss the motion of satellites in circular orbits using Newton's theory and from which you would be able to judge how precisely Kepler had observed the solar system for 20 years. Before coming to the details of satellite motion, let us first get familiar about the satellites. The artificial or natural bodies which revolve around the planets are termed as satellites. There are two types of satellites—natural satellites and artificial satellites.

The natural bodies which revolve around the planet are termed as its natural satellies. For example, moon is a natural satellite of the earth. And the artificial bodies which revolve around the planet are termed as artificial satellites. For example, INSAT-IA is an artificial satellite of earth.

Consider a satellite of mass m, which is revolving around the earth in a circular orbit of radius R as shown in the figure. Let the satellite be orbiting with speed v_o, around the earth. This

Fig. 10.8 Satellite of mass m revolving around the earth

speed of the satellite is termed as orbital speed of satellite. As the satellite is performing circular motion, some force must act on it which will provide the necessary centripetal force required for circular motion. Here, the gravitational force experienced by the satellite due to the earth serves this purpose.

So, $$\frac{GM_E m}{r^2} = \frac{mv_o^2}{r}$$

where M_E is the mass of earth and r the orbital radius of satellite.

$$\Rightarrow \qquad v_o = \sqrt{\frac{GM_E}{r}}$$

It has to be kept in mind that for successful orbiting of a satellite, the plane of satellite must contain the centre of earth.

From the above expression it is clear that orbital speed of a satellite is independent of its mass and is depending only on the mass of planet and the orbital radius of earth. It means that different satellites having different masses, revolving in the same orbit are having same orbital speeds. The satellites are moving with constant speed as there is no tangential force acting on them and hence their motion is uniform circular motion. The time period of a satellite is the time period of its circular motion, and is given by,

$$T = \frac{2\pi r}{v_o}$$

$$T = \frac{2\pi r}{\sqrt{\frac{GM_E}{r}}} = \frac{2\pi r^{3/2}}{\sqrt{GM_E}}$$

Squaring the above equation, we get

$$T^2 = \frac{4\pi^2 r^3}{GM_E} \quad ie, \quad T^2 \propto r^3$$

which is in agreement with the Kepler's third law.

As in circular orbit, the speed of a satellite is constant and hence the line joining the satellite and the planet sweeps out equal areas in equal intervals of time, thus confirming the applicability of Kepler's laws for satellite motion too. Here, we have restricted our discussion to circular orbits only, but we can develop theoretical frame works for elliptical orbits on the same reasoning.

Geosynchronous Satellites

The satellite which revolves around the earth in such a way that to any person on the earth it seems to be stationary is termed as a *geosynchronous satellite* or *geostationary satellite*. The concept behind analysis of motion of geostationary satellites is that earth is revolving about its own axis from east to west with time period of 24 h, if the satellite also moves with same time period in same direction, then to any observer on the earth this satellite seems to be always stationary.

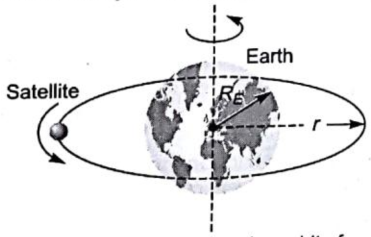


Fig. 10.9 Diagram showing orbit of geostationary satellite.

For a satellite to be geostationary, following requirements must be fulfilled :

I. It must have time period of 24 h.
II. It must revolve from east to west.
III. It must be in equatorial plane.

Let us compute the orbital radius of geostationary satellite. As we know,

$$T^2 = \frac{4\pi^2 r^3}{GM_E},$$

and for geostationary satellite $T = 24$ h.

$$\text{So, } (24 \times 60 \times 60)^2 = \frac{4\pi^2 r^3}{6.67 \times 10^{-11} \times 6 \times 10^{24}}$$

$$\Rightarrow r^3 = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times (24 \times 60 \times 60)^2}{4\pi^2}$$

$$\Rightarrow \quad r = 4.2 \times 10^4 \text{ km} = 42{,}000 \text{ km}$$

ie, $(42000 - 6400)$ km $= 35{,}600$ km from the earth's surface.

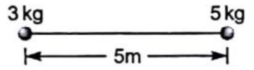
Geostationary satellites are placed at high altitudes allowing them to inspect the entire earth's surface area except for small regions at the south and north geographic poles, which significantly helps us in meteorological studies. Use of highly directional dish antennas can reduce signal interventions from earth-based sources and from other satellites too. The orbital sector is a really thin loop in the plane of the equator; hence a very small number of satellites can be maintained within this sector without mutual conflicts and collisions. The precise hovering location of a geostationary satellite fluctuates a little over each 24-hour period loop. This fluctuation happens due to the gravitational interference among the satellite, the earth, the sun, the moon, and other planets. Radio signals take roughly 1/4th of a second for a two-way trip to the satellite, resulting in a small but major signal wait. This wait raises the trouble of interactive communication like telephonic conversation. Geostationary satellites have modernized and transformed worldwide communications, television broadcasting, meteorological and weather forecasting. It also has a number of significant and intelligence applications.

Proficiency in Concepts (PIC)

Problems

Problem | 1 *Two particles of masses 3 kg and 5 kg respectively are placed at a separation of 5 m in air. Determine the gravitational force of interaction between them.*

Solution From Newton's law of universal gravitation,

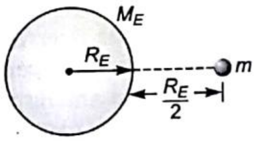


$$F = \frac{Gm_1m_2}{r^2} = \frac{6.67 \times 10^{-11} \times 3 \times 5}{(5)^2}$$

$$= 4.002 \times 10^{-11}\ \text{N}$$

Problem | 2 *A particle of mass 100 kg is at a distance of 3200 km from the earth's surface. Determine the gravitational force experienced by the earth due to this particle. [Take, $M_E = 6 \times 10^{24}$ kg, $R_E = 6400$ km, g at surface of earth $= 9.8\ ms^{-2}$]*

Solution As gravitational force is an action-reaction pair, the force experienced by the particle due to the earth and force experienced by the earth due to particle are equal and opposite. So, the force experienced by the earth due to the particle is,



$$F = \frac{GM_E \times m}{\left(R_E + \frac{R_E}{2}\right)^2}$$

$$= \frac{4}{9} \times \frac{GM_E}{R_E^2} \times m$$

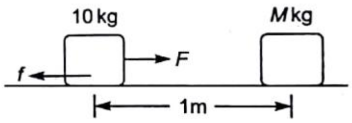
$$= \frac{4}{9} \times g \times m$$

$$= \frac{4}{9} \times 9.8 \times 100$$

$= 435.56$ N towards the particle.

Problem | 3 *A block of mass 10 kg is placed at a distance of 1 m from another block of mass M kg on a rough horizontal surface. The coefficient of friction between the 10 kg block and the surface is 0.05. Determine the minimum value of M, so that the motion of 10 kg block takes place.*
[Take $g = 10\ ms^{-2}$]

Solution For the motion of a block of mass 10 kg to take place, the gravitational force experienced by it due to M kg block must be able to overcome the frictional force.



The gravitational force of interaction between the blocks is,

$$F = \frac{GM \times m}{r^2}$$

$$F = \frac{GM \times 10}{1} = 6.67 \times 10^{-10} \times M \text{ newton}$$

The limiting friction force between the 10 kg block and the surface is, $f_L = \mu \times 10\,g = 5$ N

For required condition,

$$F > f_L$$

$$\Rightarrow \quad 6.67 \times 10^{-10}\, M > 5$$

$$M > 7.5 \times 10^9 \text{ kg}$$

So, minimum value of M is 7.5×10^9 kg.

Problem | 4 *Determine the value of acceleration due to gravity at a height of $h = 3R_E$ from the surface of earth. [Take the value of g at the earth's surface as 10 ms^{-2}].*

Solution At a height h from the earth's surface, value of g is given by

$$g' = \frac{GM_E}{(R_E + h)^2}$$

So, at $h = 3R_E$

$$g' = \frac{GM_E}{(R_E + 3R_E)^2}$$

$$= \frac{GM_E}{16R_E^2}$$

$$= \frac{1}{16} \times \frac{GM_E}{R_E^2}$$

$= \frac{1}{16} \times$ Acceleration due to gravity at the earth's surface

$$= \frac{10}{16}\ \text{ms}^{-2} = 0.625\ \text{ms}^{-2}$$

Problem | 5 *If an object of mass 5 kg is at a height of $h = 3R_E$ from the earth's surface, then determine its weight.*
[Take $g = 11\ ms^{-2}$]

Solution From Problem 4, $g' = 0.625\ \text{ms}^{-2}$

So, weight of object at $h = 3R_E$ is, $w = mg'$

$$w = 5 \times 0.625\ \text{N} = 3.125\ \text{N}$$

Problem | 6 *If the moon is revolving around the earth in a circular orbit of radius 3.84×10^5 km, then determine the orbital speed of moon. [Take $M_E = 6 \times 10^{24}$ kg, and $R_E = 6.4 \times 10^3$ km]*

Solution From the expression $v_o = \sqrt{\frac{GM_E}{r}}$ we can compute the required orbital speed.

$$v_o = \sqrt{\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{(3.84 \times 10^8)}}\ \text{ms}^{-1}$$

$$= 1.02\ \text{kms}^{-1}.$$

Problem | 7 *The time period of a satellite A whose orbital radius is r_o is T_o, then determine the time period of a satellite B whose orbital radius is $4r_o$. Assume the circular orbits.*

Solution From Kepler's third law, $T^2 \propto r^3$.

or $$T^2 = \frac{4\pi^2 r^3}{GM}$$

So, $$\frac{T_A^2}{T_B^2} = \frac{r_A^3}{r_B^3}$$

$$\Rightarrow \quad \frac{T_A}{T_B} = \left(\frac{r_A}{r_B}\right)^{3/2}$$

$$\Rightarrow \quad \frac{T_o}{T_B} = \left(\frac{r_o}{4r_o}\right)^{3/2}$$

$$\Rightarrow \quad T_B = T_o \times 4^{3/2} = 8\,T_o$$

Towards Proficiency Problems
Exercise 1

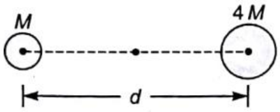
A. Subjective Discussions

1. Is Newton's law of universal gravitation is valid only for particles ? If yes, then how can we determine the gravitational force of interaction between the two extended bodies ? Between two spheres ?
2. Is it possible that two particles under their mutual gravitational interaction are in equilibrium ? Any one of them ? If instead of two particles we take three particles, then what would be your answer ?
3. Gravitational force is an action-reaction pair, then if an object is released near to the earth's surface it starts moving, then why is not the earth moving ? Or is it moving ?
4. Kepler's laws are based on certain observations. Just ponder how Kepler concluded these laws ?
5. Newton's law of gravitation says that each and every object attracts one another, then why they all are not moving towards each other and eventually collide ?
6. Which force provides the necessary contripetal force for the moon to revolve around the earth in an almost circular orbit ?
7. Can a body have mass but no weight ? Give your arguments.
8. Kepler's laws have been derived from Newton's law of gravitation and classical mechanics. Comment on this statement.
9. Earth is attracting you in vertical downward direction and the gravitational force is an action-reaction pair. Does it mean that you are also attracting the earth ? If yes, in which direction ? If you are attracting the earth, then why is it not moving ?
10. To measure the mass of a planet with the same radius as the earth, an astronaut drops an object from rest (relative to the planet) from an altitude of one radius above the surface. When the object hits the surface of planet with a speed which is 4 times, then what it would be if the same experiment is carried out at the earth. Determine the mass of planet in terms of the earth's mass.

B. Numerical Answer Types

1. Two particles of masses 1 kg and 3 kg are placed at a separation of 1 m. Determine the gravitational interaction between these two mases.
2. Two electrons are kept at a separation of 10 fm ($1 \text{fm} = 10^{-15}$ m). Determine the gravitational force of interaction between the two electrons. (Mass of an electron $= 9.1 \times 10^{-31}$ kg.)
3. Determine the gravitational force of interaction between two spheres of masses 20 kg and 50 kg and radii 3 m and 2 m, respectively. The separation between their centres is 10 m.
4. Determine the gravitational force of interaction between the Sun and the earth. ($M_E = 6 \times 10^{24}$ kg, $M_{\text{sun}} = 2 \times 10^{30}$ kg, separation between the sun and the earth is 1.5×10^{11} m.)

5. In Q. 1, if two particles are free to move, then what would be the acceleration of these two particles ? Are they same or different ?
6. Two spherical bodies of masses M and $4M$ are kept at a separation of d as shown in the figure. Where should we place a third particle of mass m so that it will experience a zero net force ? Is there any restriction on the value of m to satisfy above condition ? Also determine the net force experienced by M and $4M$.



7. In PIC Problem 4, determine the acceleration of moon.
8. Determine the acceleration due to gravity at a height of $h = 2\ R_E$ from the earth's surface ? Take acceleration due to gravity at the earth's surface to be 10 ms^{-2}.
9. Determine the weight of a 5 kg body at the earth's surface ? At $h = 2\ R_E$ from earth's surface ? Take g at the earth's surface to be 10 ms^{-2}.
10. A planet is having a mass twice to that of earth's mass and its radius as 4 times that of the earth's radius. Determine the acceleration due to gravity at the surface of this planet. Acceleration due to gravity at the earth's surface is 10 ms^{-2}.
11. A body of mass 5 kg is at a height twice the radius of planet from its surface. For the planet mentioned is Q. 10, determine the weight of body.
12. A ball of mass 1 kg is dropped from a height of 10 m above the surface of moon. Determine the speed of ball just before striking the moon's surface and also find out the time of flight of ball.
13. Two particles of masses m_1 and m_2 are kept at a separation of r. When a third particle is kept at a distance of $d/3$ from m_1, then it won't experience any net force. Determine the ratio $\frac{m_1}{m_2}$.
14. The ratio of acceleration due to gravity at the earth's surface and at a height of h from the earth's surface is 4. Determine the value of h in terms of the radius of earth.
15. The weight of an object reduces by a factor of 4 as it is taken from the earth's surface to a height of 6400 km. Determine the earth's radius.
16. The orbital speed of a satellite moving in a circular orbit around the earth is v_o. If another satellite having half the mass of the first satellite has to revolve in some orbit, then determine its orbital speed.
17. A satellite is revolving round the earth at a height of 600 km from the surface. Find the speed of satellite, magnitude of its acceleration and time period of its motion around the earth. [*Take* $M_E = 6 \times 10^{24}$ *kg and* $R_E = 6400$ *km*].
18. Three particles each of mass m are situated at the vertices of an equilateral triangle of side a. Determine the magnitude of net force experienced by any one of them.
19. Four particles each of mass m are situated at the corners of a square of edge length a. Determine the magnitude of the net force experienced by any one of them.
20. Find the height over the earth's surface at which the weight of a body becomes half of its value at the surface.
21. A body stretches a spring by a particular length at the earth's surface when suspended. At what height above the earth's surface the elongation in the spring is half of its initial value.
22. The time taken by a planet to revolve around the sun is 1.5 earth years. Find out the ratio of semi- major axis of planet's orbit to semi-major axis of the earth's orbit around the sun.

23. A satellite is revolving around a planet in a circular period with time period T. The orbital radius of satellite is r and radius of planet is R. Determine the acceleration due to gravity at its surface, and also the mass of the planet.

24. A satellite is in a circular orbit around an unknown planet. The satellite has a speed of 1.70×10^4 ms^{-1}, and radius of the orbit is 5.25×10^6 m. A second satellite also has a circular orbit around the same planet. The orbital radius of second satellite is 8.6×10^6 m. Determine the orbital speed of the second satellite.

C. Fill in the Blanks

1. The value of universal gravitational constant G at moon is ....
2. The weight of a person on the earth is 600 N, the weight and mass of the same person on the moon are .... and .... respectively. [$g = 10$ ms^{-2}]
3. The gravitational force acting on an object due to the earth is $\vec{F}_o$ and due to an object on earth is $\vec{F}_e$. Then the relation between $\vec{F}_o$ and $\vec{F}_e$ is ....
4. Let M and R denote the mass and radius of earth, respectively. The ratio of g/G at the earth's surface is ....
5. Mars has about 1/10th the mass of earth and about 1/2 the diameter of earth. The acceleration due to gravity at the surface of Mars is about ....
6. An object at the surface of earth weights 90 N. Its weight at a distance of $3R$ (where R is radius of earth) from the centre of earth is ....
7. A planet is in a circular orbit around the sun. Its distance from the sun is four times the semi-major axis of earth's orbit. The period of the planet is .... earth years.

D. True/False

1. The value of G as well as of g are same at both the earth's and moon's surface.
2. If the separation between two point masses doubles, then the gravity force between them decreases by a factor of 4.
3. Mass is an intrinsic property of a body and it is same at all places at all times.
4. Newton's law of universal gravitation is valid only on the earth.
5. It is necessary that the plane of the orbit of a satellite must contain the centre of the earth.
6. An object is raised from the surface of earth to a height of two earth radii above it. Then its mass and weight both remain constant.
7. An astronaut finishes some work on the outside of his satellite, which is in a circular orbit around the earth. By mistake, his wrench gets released from his hand, the wrench will continue to move in the same orbit.

High Skill Questions

Exercise 2

A. Only One Option Correct

1. The universal law of gravitation is valid for
   (a) spherical bodies
   (b) elliptical bodies
   (c) any arbitrary-shaped bodies
   (d) rectangular objects

2. In order that the force of attraction between two bodies may become noticeable and cause motion, one of the bodies must have
   (a) an extremely large size
   (b) an extremely large mass
   (c) an extremely large density
   (d) None of the above

3. The value of g at the earth's surface is $10\ ms^{-2}$, the value of g at a height of $3R_E$ from the surface of earth is
   (a) $\frac{10}{9}\ ms^{-2}$ (b) $\frac{5}{8}\ ms^{-2}$
   (c) $\frac{3}{8}\ ms^{-2}$ (d) $\frac{10}{8}\ ms^{-2}$

4. Two particles of masses m and $2m$ are placed at a separation of d m. A particle of mass $5m$ is to be placed on the line joining two point masses so that it doesn't experience any net force. The particle of mass $5m$ should be placed at a distance of
   (a) $\frac{d}{1+\sqrt{2}}$ m from m
   (b) $\frac{d}{\sqrt{2}-1}$ m from m
   (c) $\frac{\sqrt{2}\,d}{\sqrt{2}+1}$ m from m
   (d) $\frac{\sqrt{2}\,d}{\sqrt{2}-1}$ m from m

5. A satellite is moving in a circular orbit of radius $5R_E$. Its acceleration would be $\left(\frac{GM_E}{R_E^2} = 10\ ms^{-2}\right)$
   (a) $0.2\ ms^{-2}$ (b) $0.4\ ms^{-2}$
   (c) $0.8\ ms^{-2}$ (d) $1.8\ ms^{-2}$
   Symbols have their usual meanings

6. A planet (having spherical shape) of uniform mass density is having double the mass of earth and radius some as that of earth. The acceleration due to gravity at the surface of this planet is [Acceleration due to gravity at the earth's surface is $10\ ms^{-2}$]
   (a) $10\ ms^{-2}$ (b) $5\ ms^{-2}$
   (c) $20\ ms^{-2}$ (d) $40\ ms^{-2}$

7. The gravitational force between two objects placed at some separation on earth is F. If the same objects are placed at the same separation on moon, then the gravitational force between these two objects would be
   (a) F
   (b) $\frac{F}{6}$
   (c) $6F$
   (d) Information insufficient

8. The time period of an earth's satellite in circular orbits is independent of
   (a) the mass of satellite
   (b) radius of the orbit
   (c) None of them
   (d) Both of them

9. The magnitude of the acceleration of a planet in an orbit around the Sun is proportional to
   (a) the mass of the planet
   (b) the mass of the Sun
   (c) the reciprocal of the distance between the planet and the Sun
   (d) the product of mass of the planet and mass of the Sun

10. Earth exerts a gravitational force on the moon, keeping it in its orbit. The reaction to this force in the sense of Newton's third law is
   (a) the centripetal force on the moon
   (b) the gravitational force on earth by the moon
   (c) the tides due to the moon
   (d) None of the above
11. Of the following where would the weight of an object would be the least ?
   (a) 2000 miles above earth's surface
   (b) At north pole
   (c) At equator
   (d) At centre of earth
12. An artificial satellite releases a bomb. Neglecting air resistance, the bomb will
   (a) strike the earth under the satellite at the instant of release
   (b) strike the earth under the satellite at the instant of impact
   (c) strike the earth ahead of the satellite at the instant of impact
   (d) Never strike earth
13. Two planets are orbiting a star in a distant galaxy. The first has a semi-major axis of 150×10^6 km and time period of 1 earth year. The second has a semi-major axis of 250×10^6 km. The time period of the second one is
   (a) 0.46 earth years
   (b) 2.2 earth years
   (c) 1.24 earth years
   (d) 2.8 earth years
14. A satellite is in a circular orbit around an unknown planet. The satellite has a speed of 1.7×10^4 ms^{-1} and the radius of the orbit is 5.25×10^6 m. A second satellite also has a circular orbit around the same planet. The orbital radius of this second satellite is 8.6×10^6 m. The orbital speed of the second satellite is
   (a) 1.7×10^4 ms^{-1} (b) 3.4×10^4 ms^{-1}
   (c) 1.86×10^4 ms^{-1} (d) 1.33×10^4 ms^{-1}
15. A satellite circles the earth in an orbit whose radius is twice the earth's radius. The mass of earth is 6×10^{24} kg and its radius is 6.4×10^6 m. The time period of the satellite is approximately
   (a) 14346 s (b) 2 yr
   (c) 84 h (d) 1 h
16. Two of Jupiter's moons have orbit radii which differ by a factor of 2. Their periods
   (a) differ by a factor of $2\sqrt{2}$
   (b) differ by a factor of $\sqrt{2}$
   (c) differ by a factor of 4
   (d) are the same
17. The height above the earth's surface where g is having one-ninth of its value at the surface, is
   (a) three times the radius of earth
   (b) two times the radius of earth
   (c) four times the radius of earth
   (d) None of the above

B. More Than One Options Correct

1. Mark out the correct statement(s) related to Newton's law of universal gravitation.
   (a) It is valid only for particles and spherically symmetric bodies.
   (b) It is an action-reaction pair.
   (c) It is valid for interatomic distances also.
   (d) It is a very strong force.
2. Two particles A and B of masses 5 kg and 10 kg are kept at a separation of 1 m. Both the particles are free to move. If the two particles are released from rest, then mark out the correct statement(s).
   (a) Both the particles experience an equal and opposite force.
   (b) Magnitudes of acceleration of both the particles are the same.
   (c) Magnitude of acceleration of both the particles are different.
   (d) Magnitude of acceleration of A is double of that of B.
3. Mark out the incorrect statement(s) *wrt* mass
   (a) It is a vector.
   (b) It is same at different locations.
   (c) It is different at different locations.
   (d) It is same for all objects of same size and shape.

4. A astronaut on the moon simultaneously drops a feather and a hammer. The fact that they land together shows that
   (a) no gravity forces acts on a body in vacuum
   (b) the acceleration due to gravity on the moon is less than that on earth
   (c) in the absence of air friction all bodies at a given location fall with the same acceleration
   (d) $g = 0$ on the moon
5. The value of g depends upon the
   (a) mass of the body which is falling
   (b) mass of the planet
   (c) radius of the planet
   (d) height from surface of the planet
6. Weight of an object depends upon the
   (a) mass of the object
   (b) location of the object
   (c) mass of the planet
   (d) radius of the planet
7. Geostationary satellites must
   (a) be in an equatorial plane
   (b) have time-periods of 24 h
   (c) be revolving from east to west
   (d) be revolving from west to east

C. Assertion & Reason

Directions (Q. Nos. 1 to 7) *Some questions (Assertion-Reason type) are given below. Each question contains **Statement I** (**Assertion**) and **Statement II** (**Reason**). Each question has 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct. So, select the correct choice.*

Choices are

(a) **Statement I** is True, **Statement II** is True; **Statement II** is a correct explanation for **Statement I**
(b) **Statement I** is True, **Statement II** is True; **Statement II** is **NOT** a correct explanation for **Statement I**
(c) **Statement I** is True, **Statement II** is False
(d) **Statement I** is False, **Statement II** is True

1. **Statement I** Earth moves in a stable orbit around the sun and hence is in equilibrium.
   Statement II Earth experiences a net non-zero gravitational force due to Sun.
2. **Statement I** Orbital speed of a satellite in a circular orbit is independent of the mass of satellite.
   Statement II The gravitational force between the earth and satellite in circular orbit provides the necessary centripetal force.
3. **Statement I** The gravitational force between two masses in water is same as that in vacuum under same situation.
   Statement II Gravitational force between two masses is independent of the medium present between the two masses.
4. **Statement I** The weight of a person in a satellite is the same as that on the earth's surface.
   Statement II The weight of an object is equal to the gravitational force between the object and the planet.
5. **Statement I** The acceleration due to gravity is independent of the mass of the body which is considered.
   Statement II All the bodies at a particular location have same the acceleration due to gravity.
6. **Statement I** Mass of a body is independent of its location, and also irrespective of time.
   Statement II Mass is an intrinsic property of any substance.
7. **Statement I** Acceleration due to gravity is a universal constant.
   Statement II Universal gravitational constant is a universal constant.

D. Comprehend the Passage Questions

Passage I

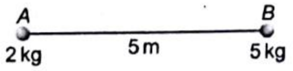
A satellite has a mass of 5850 kg and is in a circular orbit 4.1×10^5 m above the surface of a planet. The time period of the satellite is 2 h. The radius of the planet is 4.15×10^6 m.

Based on above information answer the following questions :

1. The orbital radius of satellite is
   (a) 4.15×10^6 m
   (b) 4.1×10^5 m
   (c) 4.56×10^6 m
   (d) None of the above
2. The weight of the satellite when it is in its orbit is
   (a) 2.03×10^4 N
   (b) 2000 N
   (c) 4.68×10^4 N
   (d) None of the above
3. The weight of the satellite when it is at rest on the planet's surface is
   (a) 2.03×10^4 N (b) 2.45×10^4 N
   (c) 2.4×10^2 N (d) None of these

Passage II

Two point masses of 2 kg and 5 kg are placed at a distance of 5 m as shown in the figure. A point mass of 1 kg has to be placed on the line joining them.



Based on above information answer the following questions :

4. If 1 kg mass is placed at the mid-point of AB, then it will experience a net force of
   (a) $\frac{12\,G}{25}$ N towards A
   (b) $\frac{12\,G}{25}$ N towards B
   (c) $\frac{28\,G}{25}$ N towards A
   (d) $\frac{28\,G}{25}$ N towards B
5. The distance of the point from A where the 1 kg mass should be placed so that it doesn't experience any net force, is
   (a) 1.94 m
   (b) 2.68 m
   (c) 1.52 m
   (d) 1.25 m
6. If the 1 kg mass is placed at a distance of 5 m to the left of A, then if will experience a net force of
   (a) $\frac{10\,G}{33}$ N towards A
   (b) $\frac{110\,G}{33}$ N towards A
   (c) $\frac{13\,G}{100}$ N towards A
   (d) None of the above

E. Match the Columns

1. Match the following.

	Column I		Column II
(A)	Mass	(P)	Same value at all places
(B)	Weight	(Q)	Different values at different places
(C)	Gravitational constant G	(R)	Depends upon the mass of planet, say earth
(D)	Acceleration due to gravity	(S)	Doesn't depend upon mass of planet, say earth

2. Match the entries of Column I with the entries of Column II.

	Column I		Column II
(A)	Mass	(P)	Scalar
(B)	Weight	(Q)	Vector
(C)	Gravitational constant, G	(R)	Universal constant
(D)	Acceleration due to gravity	(S)	Not an universal constant

3. In Column I some physical quantities are given related to a satellite revolving in a circular orbit around the earth while in Column II are given some factors, on which these physical quantities may depend. Match the entries of Column I with the entries of Column II.

	Column I		Column II
(A)	Orbital speed	(P)	Mass of satellite
(B)	Time period	(Q)	Mass of earth
(C)	KE of satellite	(R)	Orbital radius of satellite
(D)	Linear momentum of satellite	(S)	Universal constant, G

Answers

Towards Proficiency Problems
Exercise 1

B. Numerical Answer Types

1. 2×10^{-10} N **2.** 5.5×10^{-43} N **3.** 6.67×10^{-10} N **4.** 3.55×10^{22} N

5. 2×10^{-10} ms^{-2} for 1 kg particle, $\frac{2}{3}\times10^{-10}$ ms^{-2} for 3 kg particle

6. $\frac{d}{3}$ from centre of M, No, $F_m=\frac{GM}{d^2}[4M+9m]$, $F_{4M}=\frac{GM}{d^2}[4M+9m]$ **7.** 0.0027 ms^{-2}

8. $\frac{10}{9}$ ms^{-2} **9.** $\frac{50}{9}$ N **10.** 1.25 ms^{-2} **11.** 0.7 N **12.** $\frac{10}{\sqrt{3}}$ ms^{-1}

13. $\frac{1}{4}$ **14.** $h=R$ **15.** 6400 km **16.** v_o

17. 7.56 kms^{-1}, 8.17 ms^{-2}, 5817 s **18.** $\frac{\sqrt{3}\,Gm^2}{a^2}$ **19.** $\frac{Gm^2}{a^2}\left[\frac{1}{2}+\sqrt{2}\right]$ **20.** $R_E(\sqrt{2}-1)$

21. $(\sqrt{2}-1)R_E$ **22.** $(1.5)^{2/3}$ **23.** $g=\frac{4\pi^2r^3}{R^2T^2}$, $M=\frac{4\pi^2r^3}{GT^2}$

24. 1.33×10^4 ms^{-1}

C. Fill in the Blanks

1. 6.67×10^{-11} N·m^2 kg^{-2} **2.** 100 N, 60 kg **3.** $\vec{F}_e+\vec{F}_o=0$ **4.** M/R^2

5. 3.9 ms^{-2} **6.** 10 N **7.** 8

D. True/False

1. F **2.** T **3.** T **4.** F **5.** T **6.** F **7.** T

High Skill Questions
Exercise 2

A. Only One Option Correct

1. (a) **2.** (b) **3.** (b) **4.** (a) **5.** (b) **6.** (c) **7.** (a) **8.** (a) **9.** (b) **10.** (b)
11. (d) **12.** (d) **13.** (b) **14.** (d) **15.** (a) **16.** (a) **17.** (b)

B. More Than One Options Correct

1. (a, b, c) **2.** (a, c, d) **3.** (a, c, d) **4.** (c) **5.** (b, c, d)
6. (a, b, c, d) **7.** (a, b, d)

C. Assertion & Reason

1. (d) **2.** (b) **3.** (a) **4.** (d) **5.** (b) **6.** (a) **7.** (d)

D. Comprehend the Passage Questions

1. (c) **2.** (a) **3.** (b) **4.** (b) **5.** (a) **6.** (c)

E. Match the Columns

1. A → P, S; B → Q, R; C → P, S; D → Q, R
2. A → P, S; B → Q, S; C → P, R; D → Q, S
3. A → Q, R, S; B → Q, R, S; C → P, Q, R, S; D → P, Q, R, S

Explanations

Towards Proficiency Problems
Exercise 1

Numerical Answer Types

1. $F = \dfrac{Gm_1m_2}{r^2}$

$= \dfrac{6.67 \times 10^{-11} \times 1 \times 3}{1^2}$

$= 2 \times 10^{-10}$ N

2. $F = \dfrac{6.67 \times 10^{-11} \times (9.1 \times 10^{-31})^2}{(10 \times 10^{-15})^2}$

$= 5.5 \times 10^{-43}$ N

3. For spheres also we can apply Newton's law of gravitation after considering that point masses having masses same as that of spheres are placed at their respective centres.

$$F = \frac{6.67 \times 10^{-11} \times 20 \times 50}{10^2}$$

$$= 6.67 \times 10^{-10} \text{ N}$$

4. $F = \dfrac{GM_EM_S}{r^2}$

$= \dfrac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 2 \times 10^{30}}{(1.5 \times 10^{11})^2}$

$= 3.55 \times 10^{22}$ N

5. Both the particles experience the same force, but are having different masses, so they experience different acceleration.

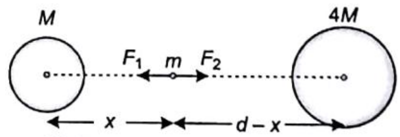
Acceleration of 1 kg particle,

$$a_1 = \frac{F}{1} = \frac{2 \times 10^{-10}}{1} \text{ ms}^{-2}$$

Acceleration of 3 kg particle,

$$a_2 = \frac{F}{3} = \frac{2 \times 10^{-10}}{3} \text{ ms}^{-2}$$

6. Suppose we place the particle at a distance x from M.



$F_1 = \dfrac{GMm}{x^2} \rightarrow$ *(The force experienced by m due to M.)*

$F_2 = \dfrac{G \times 4Mm}{(d-x)^2} \rightarrow$ *(The force experienced by m due to 4 M.)*

$$F_1 = F_2 \Rightarrow x = \frac{d}{3}.$$

8. $g' = \dfrac{g}{\left(1 + \dfrac{h}{R_E}\right)^2} = \dfrac{10}{(1+2)^2} = \dfrac{10}{9} \text{ ms}^{-2}$

9. At the earth's surface, $W_1 = 5\,g = 50$ N

At $h = 2\,R_E$,

$$W_2 = 5g' = \frac{5g}{\left(1 + \dfrac{h}{R_E}\right)^2}$$

$$= \frac{50}{9} \text{ N}$$

10. $M_P = 2M_E$ and $R_P = 4R_E$

$$g_P = \frac{GM_P}{R_P^2} = \frac{G \times 2\,M_E}{(4\,R_E)^2}$$
$$= \frac{1}{8} \times g_E = 1.25\ \text{ms}^{-2}$$

11. $w = 5g'_P$

$$g_P' = \frac{g_P}{\left(1 + \frac{h}{R_P}\right)^2} = \frac{g_P}{9}$$

12. The acceleration due to gravity at the moon's surface is $\frac{1}{6}$th of that at the earth's surface.

$$\frac{mv^2}{2} = \frac{mgh}{6}$$
$$\Rightarrow \quad v = \sqrt{\frac{gh}{3}} = \frac{10}{\sqrt{3}}\ \text{ms}^{-1}$$

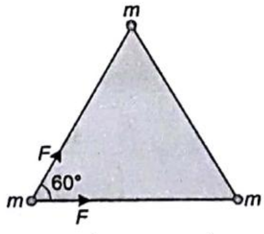
14. $$\frac{g'}{g} = \frac{1}{4} = \frac{1}{\left(1 + \frac{h}{R_E}\right)^2}$$
$$\Rightarrow \quad h = R_E$$

16. Orbital speed of a satellite is independent of its mass.

17. $$v_o = \sqrt{\frac{GM_E}{r}}$$
$$= \sqrt{\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{7000 \times 10^3}}\ \text{ms}^{-1}$$
$$= 7.56\ \text{kms}^{-1}$$
$$\text{Acceleration} = \frac{GM_E}{r^2} = \frac{v_o^2}{r} = 8.17\ \text{ms}^{-2}$$
$$T = \frac{2\pi r}{v_o} = 5817\ \text{s}$$

18. Each particle experiences a force due to the other two.

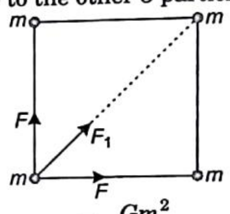
$$F = \frac{Gm^2}{a^2}$$



The resultant force experienced by any particle is

$$\sqrt{F^2 + F^2 + 2F^2 \cos 60^\circ} = \sqrt{3}F$$
$$= \frac{\sqrt{3}\,Gm^2}{a^2}$$

19. The situation is shown clearly in the figure. Each particle experiences the gravitational force due to the other 3 particles.



$$F = \frac{Gm^2}{a^2},$$
and $$F_1 = \frac{Gm^2}{(\sqrt{2}\,a)^2}$$

The net force experienced by any particle is,

$$F_{\text{res}} = \sqrt{2}\,F + F_1$$
$$= \frac{Gm^2}{a^2}\left[\sqrt{2} + \frac{1}{2}\right]$$

20. Let at height h above the earth's surface,

$$w = \frac{w_{\text{surface}}}{2}$$
$$w = mg' = \frac{mg}{2}$$
$$\Rightarrow \quad \frac{g'}{g} = \frac{1}{2} = \frac{1}{\left(1 + \frac{h}{R_E}\right)^2}$$
$$\Rightarrow \quad h = (\sqrt{2} - 1)\,R_E$$

22. $T^2 \propto a^3$

So, $$\frac{T_1}{T_2} = \left(\frac{a_1}{a_2}\right)^{3/2}$$
$$\Rightarrow \quad \frac{a_1}{a_2} = \left(\frac{T_1}{T_2}\right)^{2/3} = (1.5)^{2/3}$$

23. $T^2 = \dfrac{4\pi^2 r^3}{GM}$ and $g = \dfrac{GM}{r^2}$

24. $$v_o = \sqrt{\frac{GM}{r}}$$

So, $$\frac{v_1}{v_2} = \sqrt{\frac{r_2}{r_1}}$$
$$\Rightarrow \quad v_2 = v_1\sqrt{\frac{r_1}{r_2}}$$
$$= 1.7 \times 10^4\sqrt{\frac{5.25 \times 10^6}{8.6 \times 10^6}}$$
$$= 1.33 \times 10^4\ \text{ms}^{-1}$$

Chapter 11

The Wave Motion

The First Steps' Learning

- What is a Wave?
- What Do You Mean by a Medium?
- The Molecular Picture of a Wave
- Classification of Waves
- Transverse Waves
- Longitudinal Waves
- Frequency and Period of a Wave
- Energy Concept of a Wave
- Speed of a Wave
- Sound Waves
- Sound as a Pressure Wave
- Pitch and Frequency of Sound Waves
- The Speed of Sound
- The Human Ear

Waves are everywhere, whether we recognize them or not, we encounter waves on a daily basis. Sound waves, visible light waves, radio waves, micro waves, water waves, earthquake waves, waves on a string and slinky waves are just a few of the examples of our daily encounters with waves. In addition to waves, there are a variety of phenomena in our physical world which resemble waves so closely that we can describe these phenomena as being wavelike. The motion of a pendulum, the motion of a mass suspended by a spring, the motion of a child on a swing could be thought of as wave-like phenomenon. Thus we can say waves (and wave-like phenomena) are everywhere!

We study the physics of waves because it provides us with a deep insight into the physical world which we seek to explore and describe as students of physics. Before coming to the core part of the subject of waves, it is often useful to discuss the various general descriptions about waves.

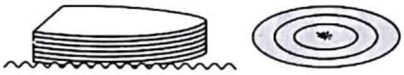


Fig. 11.1 A stone tossed into the water will create a circular disturbance (whirls) which travel outward in all directions

For many people, as the first idea about waves comes as a picture of a wave moving across the surface of an ocean, lake, pond or any other body of water. The waves are created by some form of a disturbance, such as a stone being thrown into the water, a duck shaking its tail in the water or a boat moving through the water.

Another example of waves you are aware of is the movement of a slinky or similar set of coils. If a slinky is stretched out from end to end, a wave can be introduced into the slinky by either vibrating the first coil up and down vertically or back and forth horizontally as shown in Fig. 11.1. *A wave will subsequently be seen travelling from one end of the slinky to the other. As the wave moves along the slinky, each individual coil is seen to move out of place and then returns to its original position. The coils always move in the same direction that the first coil was vibrated. A continued vibration of the first coil results in a continued back and forth motion of the other coils. If looked at closely, one notices that the wave does not stop when it reaches the end of the slinky, rather it seems to bounce off the end and to head back from where it started. Not only these two, there are a large number of physical situations in which wave motion is involved, but here it is not possible to mention them all.*

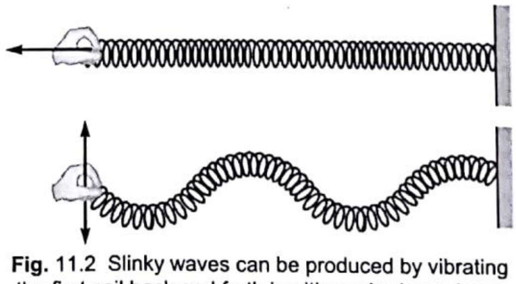


Fig. 11.2 Slinky waves can be produced by vibrating the first coil back and forth in either a horizontal or a vertical direction

Waves are everywhere in nature. Our understanding of the physical world is not complete until we understand the nature, properties and behaviour of waves. The goal of this chapter is to develop a clear mental picture of waves and their related phenomena in you, and ultimately apply your understanding to study the most common type of waves—the sound waves.

What is a Wave ?

So, waves are everywhere. But what makes a wave *a wave* ? What characteristics, properties, or behaviors are shared by all the phenomena which we typically characterize as being a wave ? How can waves be described in a manner that allows us to understand their basic nature and qualities ? What actually a wave is ?

A wave can be described as a disturbance that travels through a medium from one location to another location, without any net transportation of the medium. Consider a slinky wave as an example of a wave. When the slinky is stretched from end to end, and is held at rest, it assumes a natural position known as the **equilibrium or rest position.** The coils of the slinky naturally assume this position, spaced equally far apart. To introduce a wave into the slinky, the first particle is displaced or moved from its equilibrium or rest position. The particle might be moved upwards or downwards, forwards or backwards; but once moved, it returns to its original equilibrium or rest position. The act of moving the first coil of the slinky in a given direction and then returning it to its equilibrium position creates a disturbance in the slinky. We can then observe this disturbance moving through the slinky from one end to the other. If the first coil of the slinky is given a single back-and-forth vibration, then we call the observed motion of the disturbance through the slinky a *slinky pulse*. A pulse is a single disturbance moving through a medium from one location to another location. However, if the first coil of the slinky is continuously and periodically vibrated in a back-and-forth manner, we would observe a repeating disturbance moving within the slinky which endures over some prolonged period of time. The repeating and periodic disturbance which moves through a medium from one location to another is referred to as a wave.

When a slinky is stretched, the individual coils assume an equilibrium or rest position

(a)

When the first coil of the slinky is repeatedly vibrated back and forth, a disturbance is created which travels through the slinky from one end to the other

(b)

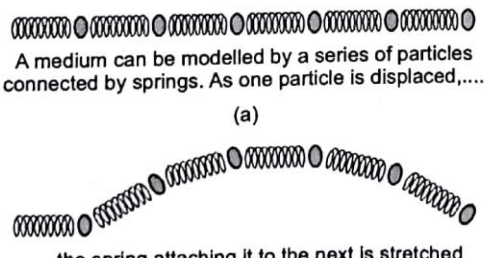
Fig. 11.3

What Do You Mean By a Medium ?

So now we know any disturbance caused in a medium is termed as wave, but what is meant by the word *medium* ? A **medium** is a substance or material which carries the wave. A wave medium is the substance which carries a wave (or disturbance) from one location to another. The wave medium is not the wave and it doesn't make the wave; it merely carries or transports the wave from one location to another. In the case of our slinky wave, the medium through which the wave travels is the slinky coils. In the case of a water wave in the ocean, the medium through which the wave travels is the ocean water. In the case of a sound wave the medium through which the sound wave travels is the air in the room.

The Molecular Picture of a Wave

To fully understand the nature of a wave, it is important to consider the medium as a collection of interacting *particles*. In other words, the medium is composed of parts which are capable of interacting with each other. The interactions of one particle of the medium with the next adjacent particle allows the disturbance to travel through the medium. In the case of the slinky wave, the *particles* or interacting parts of the medium are the individual coils of the slinky. In the case of a sound wave in air, the *particles* or interacting parts of the medium are the individual molecules of air. Consider the presence of a wave in a slinky. The first coil becomes disturbed and begins to push or pull on the second coil; this push or pull on the second coil will displace the second coil from its equilibrium position. As the second coil becomes displaced, it begins to push or pull on the third coil; the push or pull on the third coil displaces it from its equilibrium position. As the third coil becomes displaced, it begins to push or pull on the fourth coil. This process continues in consecutive fashion, with each individual *particle* acting to displace the adjacent particle. Subsequently, the disturbance travels through the medium. The medium can be pictured as a series of particles connected by springs. As one particle moves, the spring connecting it to the next particle begins to stretch and apply a force to its adjacent neighbour. As this neighbour begins to move, the spring attaching this neighbour to its neighbour begins to stretch and applies a force on its adjacent neighbour.



A medium can be modelled by a series of particles connected by springs. As one particle is displaced,....

(a)

.... the spring attaching it to the next is stretched out and begins to exert a force on its neighbour, thus displacing the neighbour from its rest position.

(b)

Fig. 11.4

A Wave Transports Energy—Not the Matter

When a wave is present in a medium (that is, when there is a disturbance moving through a medium), the individual particles of the medium are only temporarily displaced from their rest positions. There is always a force acting upon the particles which restores them to their original position. In a slinky wave, each coil of the slinky ultimately returns to its original position. In a water wave, each molecule of the water ultimately returns to its original position. It is for this reason, that is said to involve the movement of a disturbance without the movement of matter. The particles of medium (water molecules, slinky coils) simply vibrate about a fixed position as the pattern of the disturbance moves from one location to another location.

As a disturbance moves through a medium from one particle to its adjacent particle, energy is being transported from one end of the medium to the other. In a slinky wave, a person imparts energy to the first coil by doing work upon it. The first coil receives a certain amount of energy which it subsequently transfers to the second coil. When the first coil returns to its original position, it possesses the same amount of energy as it had before it was displaced. The first coil transfers its energy to the second coil. The second coil then has the energy which it subsequently transfers to the third coil. When the second coil returns to its original position, it possesses the same amount of energy as it had before it was displaced. The third coil has received the energy of the second coil. This process of energy transfer continues

as each coil interacts with its neighbour. In this manner, energy is transported from one end of the slinky to the other, from its source to another location. Thus we can say that in a wave the energy is transported from one place to another but there is no net transportation of matter. Each individual particle of the medium is temporarily displaced and then returns to its original equilibrium position.

Always remember that there are two motions associated with a wave—one is the wave motion *ie*, the motion of disturbance and other is the oscillating motion of the medium particles.

Classification of Waves

(A) Depending on the number of dimensions in which a wave can propagate, the waves are classified as

1. **One dimensional Waves** The waves in which disturbance is propagated along one dimension only. For example, slinky waves.
2. **Two dimensional Waves** The waves in which disturbance is propagated along two dimensions. For example, water waves or ripples.
3. **Three dimensional Waves** The waves in which disturbance is propagated along three dimensions. For example, sound waves.

(B) Depending on the basis of whether wave requires a medium or not for its propagation

1. **Non-mechanical Wave** A non-mechanical wave is a wave which is capable of transmitting its energy through a vacuum (*ie*, empty space) *ie*, these waves can propagate in vacuum also and don't require any medium for their propagation. Examples of non-mechanical waves are light waves and electromagnetic waves.
2. **Mechanical Wave** A mechanical wave is a wave which is not capable of transmitting its energy through vacuum. Mechanical waves require a medium in order to transport their energy from one place to another. A sound wave is an example of a mechanical wave. Sound waves are incapable of travelling through a vacuum. Slinky waves, water waves, are other examples of mechanical waves. Each requires some medium in order to exist. The constituent particles of the medium undergo oscillatory motion about their mean position in mechanical waves but there is no net motion of the medium particles.

In this chapter we shall consider only mechanical waves.

(C) Depending on the direction in which the medium particles oscillate *wrt* direction of wave propagation, mechanical waves are classified as

1. **Transverse Waves** A transverse wave is a wave in which particles of the medium oscillate in a direction perpendicular to the direction in which the wave propagates. For example, slinky waves, string waves etc. Suppose that a slinky is stretched out in a horizontal direction across the classroom and that a pulse is introduced into the slinky on the left end by vibrating the first coil up and down. Energy will begin to be transported through the slinky from left to right. As the energy is transported from left to right, the individual coils of the medium will be displaced upwards and downwards. In this case, the particles of the medium move perpendicular to the direction in which the pulse moves. This

type of wave is a transverse wave. Transverse waves are always characterized by particle motion being perpendicular to the wave motion. Transverse waves travel in the form of crest and troughs.

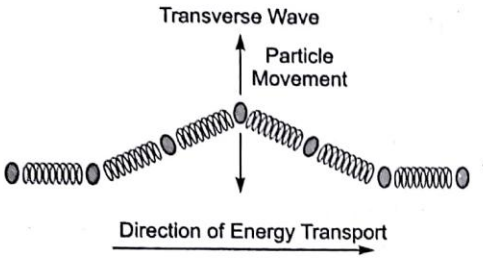


Fig. 11.5

2. **Longitudinal Waves** A longitudinal wave is a wave in which particles of the medium move in a direction parallel to the direction in which the wave propagates. For example, Slinky waves, sound waves etc. Suppose that a slinky is stretched out in a horizontal direction across the classroom and that a pulse is introduced into the slinky on the left end by vibrating the first coil left and right. Energy will begin to be transported through the slinky from left to right. As the energy is transported from left to right, the individual coils of the medium will be displaced leftwards and rightwards. In this case, the particles of the medium move parallel to the direction in which the pulse moves. This type of wave is a longitudinal wave. Longitudinal waves travels in the form of a series of compressions and rarefactions.

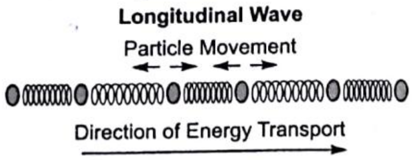


Fig. 11.6

Waves travelling through a solid medium can be either transverse waves or longitudinal waves. Yet waves travelling through the bulk of a fluid (such as a liquid or a gas) are always longitudinal waves. Transverse waves require a relatively rigid medium in order to transmit their energy. As one particle begins to move it must be able to exert a pull on its nearest neighbour. If the medium is not rigid as is the case with fluids, the particles will slide past each other. This sliding action which is a characteristic of liquids and gases prevents one particle from displacing its neighbour in a direction perpendicular to the energy transport. It is for this reason that only longitudinal waves are observed moving through the bulk of liquids such as our oceans.

Thus we can conclude that to create a mechanical wave we must have a system and a source. The source must be there to create the disturbance in the system and the medium particles carry this disturbance from one place to another. At the location where the wave is introduced into the medium, the particles which are displaced from their equilibrium position always moves in the same direction as the source of the vibration. So, if you wish to create a transverse wave in a slinky, then the first coil of the slinky must be displaced in a direction perpendicular to the entire slinky.

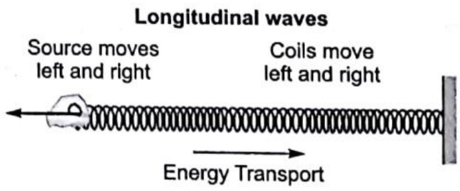


Fig. 11.7 (a)

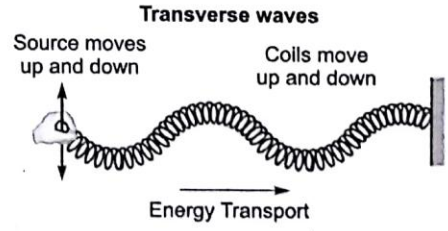


Fig. 11.7 The subsequent direction of motion of individual particles of a medium is the same as the direction of vibration of the disturbance.

Transverse Waves

A transverse wave is a wave in which the particles of the medium are displaced in a direction perpendicular to the direction of energy transport. A transverse wave can be created in a string, if the string is stretched out horizontally and the end is vibrated back-and-forth in a vertical direction. If a snapshot of such a transverse wave could be taken, then it would look like the following diagram :

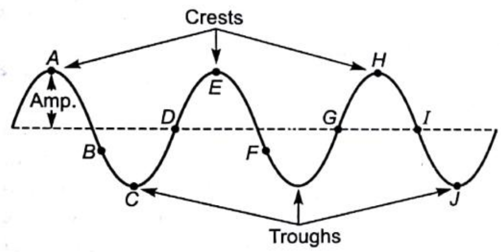


Fig. 11.8

The dashed line drawn through the centre of the diagram represents the equilibrium or rest position of the string. This is the position that the string would assume if there were no disturbance moving through it. Once a disturbance is introduced into the string, the particles of the string begin to vibrate upwards and downwards. At any given moment in time, a particle on the medium could be above or below the rest position. Points A, E and H on the diagram represent the crests of this wave. The **crest** of a wave is the point on the medium which exhibits the maximum amount of positive or upward displacement from the rest position. Points C and J on the diagram represent the troughs of this wave. The **trough** of a wave is the point on the medium which exhibits the maximum amount of negative or downward displacement from the rest position.

Properties of a Wave

Amplitude The **amplitude** of a wave refers to the maximum amount of displacement of a particle on the medium from its rest position. In a sense, the amplitude is the distance *from rest to crest.*

Similarly, the amplitude can be measured from the rest position to the trough position. In the diagram above, the amplitude could be measured as the distance of a line segment which is perpendicular to the rest position and extends vertically upward from the rest position to point A.

Wavelength The wavelength is another property of a wave which has been portrayed in the diagram above. The **wavelength** of a wave is simply the length of one complete wave cycle. If you were to trace your finger across the wave in the diagram above, you would notice that your finger repeats its path. A wave is a repeating pattern. It repeats itself in a periodic and regular fashion over both time and space. And the length of one such spatial repetition (known as a *wave cycle*) is the wavelength. The wavelength can be measured as the distance from crest to crest or from trough to trough. In fact, the wavelength of a wave can be measured as the distance from a point on a wave to the corresponding point on the next cycle of the wave. In the diagram above, the wavelength is the horizontal distance from A to E, or the horizontal distance from B to F, or the horizontal distance from D to G, or the horizontal distance from E to H. Any one of these distance measurements would suffice in determining the wavelength of this wave.

Longitudinal Waves

A longitudinal wave is a wave in which the particles of the medium are displaced in a direction parallel to the direction of energy transport. A longitudinal wave can be created in a slinky if the slinky is stretched out horizontally and the end coil is vibrated back-and-forth in a horizontal direction. If a snapshot of such a longitudinal wave could be taken, then it would look like the following diagram :

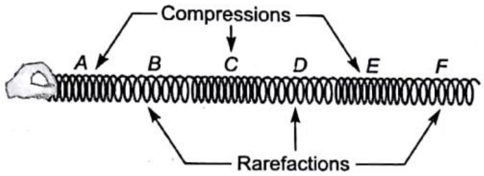


Fig. 11.9

Because the coils of the slinky are vibrating longitudinally, there are regions where they become pressed together and other regions where they are spread apart. A region where the coils are pressed together in a small amount of space is known as a compression. A **compression** is a point on a medium through which a longitudinal wave is travelling which has the maximum density. A region where the coils are spread apart, thus maximizing the distance between coils, is known as a rarefaction. A **rarefaction** is a point on a medium through which a longitudinal wave is travelling which has the minimum density. Points A, C and E on the diagram above represent compressions and points B, D and F represent rarefactions. While a transverse wave has an alternating pattern of crests and troughs, a longitudinal wave has an alternating pattern of compressions and rarefactions.

As discussed above, the wavelength of a wave is the length of one complete cycle of a wave. For a transverse wave, the wavelength is determined by measuring from crest to crest. A longitudinal wave doesn't have a crest; so how can its wavelength be determined ? The wavelength can always be determined by measuring the distance between any two corresponding points on adjacent waves. In the case of a longitudinal wave, a wavelength measurement is made by measuring the distance from a compression to the next compression or from a rarefaction to the next rarefaction. On the diagram above, the distance from point A to point C or from point B to point D would be representative of the wavelength.

Frequency and Period of a Wave

We have discussed earlier that a wave is created in a slinky by the periodic and repeating vibration of the first coil of the slinky. This vibration creates a disturbance which moves through the slinky and transports energy from the first coil to the last coil. A single back-and-forth vibration of the first coil of a slinky introduces a pulse into the slinky. But the act of continually vibrating the first coil with a back-and-forth motion in periodic fashion introduces a wave into the slinky.

Suppose that a hand holding the first coil of a slinky is moved back-and-forth performing two complete cycles in one second. The rate of the hand's motion would be 2 cycles/s. The first coil, being attached to the hand, in turn would vibrate at a rate of 2 cycles/s. The second coil, being attached to the first coil, would vibrate at a rate of 2 cycles/s. The third coil, being attached to the second coil, would vibrate at a rate of 2 cycles/s. In fact, every coil of the slinky would vibrate at this rate of 2 cycles/s. This rate of 2 cycles/s is referred to as the frequency of the wave.

The **frequency** of a wave refers to how often the particles of the medium vibrate when

a wave passes through the medium. Frequency is a part of our common, everyday language. In mathematical terms, the frequency is the number of complete vibrational cycles of a medium for a given amount of time. Given this definition, it is reasonable that the quantity *frequency* would have units of cycles/second, waves/second, vibrations/second, or something/second. Another unit for frequency is the **Hertz** (abbreviated Hz) where 1 Hz is equivalent to 1 cycle/s. If a coil of slinky makes 2 vibrational cycles in one second, then the frequency is 2 Hz. If a coil of the slinky makes 3 vibrational cycles in one second, then the frequency is 3 Hz. And if a coil makes 8 vibrational cycles in 4 s, then the frequency is 2 Hz (8 cycles/4s = 2 cycles/s).

The quantity frequency is often confused with the quantity period. Period refers to the time which it takes to complete one cycle. When an event occurs repeatedly, then we say that the event is **periodic** and refer to the time for the event to repeat itself as the period. The **period** of a wave is the time for a particle on a medium to make one complete vibrational cycle. *Period*, being a time, is measured in units of time such as seconds, hours, days or years. The period of orbit for the earth around the sun is approximately 365 days; it takes 365 days for the earth to complete a cycle.

Frequency and period are distinctly different, yet related, quantities. *Frequency* refers to how often something happens. *Period* refers to the time it takes something to happen. Frequency is a rate quantity. *Period* is a time quantity. Frequency is the cycles/second. Period is the seconds/cycle. Mathematically, the period is the reciprocal of the frequency and *vice-versa*. In equation form, this is expressed as follows.

$$\text{Period} = \frac{1}{\text{frequency}}, \text{ and frequency} = \frac{1}{\text{period}}$$

Since the symbol f is used for frequency and the symbol T is used for period, these equations are also expressed as :

$$T = \frac{1}{f}$$

$$f = \frac{1}{T}$$

C-BIs

Concept Building Illustrations

Illustration | 1 *A wave is repeating its pattern after every* 0.2 s. *Determine frequency of the wave.*

Solution From $f = \dfrac{1}{T}$

Here, $T = 0.2\text{ s}$

So, $f = \dfrac{1}{0.2}\text{ Hz} = 5\text{ Hz}$

Energy Concept of a Wave

When we create a disturbance in a system from its equilibrium position then we are doing some work on the system in creating this disturbance. This work done by us is the energy associated with the wave and this energy wave transports from one place to another. Let us consider that a pulse or a wave is introduced into a slinky when a person holds the first coil and

gives it a back-and-forth motion. The energy is imparted to the medium by the person as he/she does work upon the first coil to provide it a kinetic energy. This energy is transferred from coil to coil until it arrives at the end of the slinky. If you were holding the opposite end of the slinky, then you would feel the energy as it reaches your end. The energy associated with the wave is proportional to the square of the product of amplitude and frequency of waves.

C-BIs

Concept Building Illustrations

Illustration | 2 *A wave is moving with amplitude A and frequency f. If the amplitude of wave gets half while frequency remaining the same, then how the energy associated with the wave changes ?*

Solution Initially energy, $E_1 = K \times A^2 f^2$

Finally energy, $E_2 = K\left(\frac{A}{2}\right)^2 f^2$

$E_2 = \frac{E_1}{4}$, *ie*, energy decreases by a factor of 4.

Speed of a Wave

A wave is a disturbance which moves along a medium from one place to another. If one watches an ocean wave moving along the medium (the ocean water), one can observe that the crest of the wave is moving from one location to another over a given interval of time. The crest is observed to *cover* distance. In the case of a wave, the speed is the distance travelled by a given point on the wave (such as a crest) in a given interval of time.

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

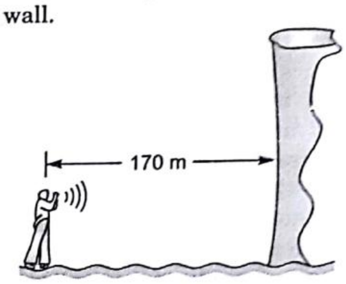
Sometimes a wave encounters the end of a medium and the presence of a different medium. For example, a wave introduced by a person into one end of a slinky will travel through the slinky and eventually reach the end of the slinky. One behaviour which waves undergo at the end of a medium is reflection. The wave will reflect or bounce off the person's hand. When a wave undergoes reflection, it remains within the medium and merely reverses its direction of travel. In the case of a slinky wave, the disturbance can be seen travelling back to the original end. A slinky wave which travels to the end of a slinky and back has *doubled its distance*. That is, by reflecting back to the original location, the wave has travelled a distance which is equal to twice the length of the slinky.

C-BIs

Concept Building Illustrations

Illustration | 3 *Shayam stands 170 m away from a steep wall. He shouts and hears the echo of his voice 1 s later. What is the speed of the wave ?*

Solution In this situation, the sound wave travels 340 ms^{-1} in 1 s, so the speed of the wave is 300 ms^{-1}. Remember, when there is a reflection, the wave *doubles its distance.* In other words, the distance travelled by the sound wave in 1 s is equivalent to the 170 m down to the wall plus(+) the 170 m back from the wall.



Relation between Wave-Speed, Wavelength and the Frequency of Wave

As discussed earlier, a wave is produced when a vibrating source periodically disturbs the system from its equilibrium position. This creates a wave pattern which begins to travel along the medium from particle to particle. The frequency at which each individual particle vibrates is equal to the frequency at which the source vibrates. In one period, the source is able to displace the first particle upwards from rest, back to rest, downwards from rest, back to rest, downwards from rest, and finally back to rest. This complete back-and-forth movement constitutes one complete wave cycle.

We know that in a time period, the wave has moved a distance of one wavelength. Combining this information with the equation for speed (speed = distance/time), it can be said that the speed of a wave is also the wavelength/period.

$$\text{Speed} = \frac{\text{wavelength}}{\text{period}}$$

Since the period is the reciprocal of the frequency, the expression $1/f$ can be substituted into the above equation for period. Rearranging the equation yields a new equation of the form :

$$\text{Speed} = \text{Wavelength} \times \text{frequency}$$

The above equation is known as the wave equation. It states the mathematical relationship between the speed (v) of a wave and its wavelength (λ) and frequency (f). Using the symbols v, λ and f, the equation can be rewritten as

$$v = f \times \lambda$$

C-BIs

Concept Building Illustrations

Illustration | 4 *The separation between consecutive crests is 35 cm and the disturbance is moving with a speed of 15 ms^{-1}. Determine the frequency of wave.*

Solution Here $v = 15\ ms^{-1}$, and $\lambda = 0.35$ m.

So, $$f = \frac{v}{\lambda} = \frac{15}{0.35}\ s^{-1}$$
$$= 42.9 \text{ Hz.}$$

Sound Waves

Sound and music are parts of our everyday sensory experience. Just as humans have eyes for the detection of light and colour, so we are equipped with ears for the detection of sound. We seldom take the time to ponder the characteristics and behaviours of sound and the mechanisms by which sounds are produced, propagated, and detected. Sound is a wave which is created by vibrating objects and propagated through a medium from one location to another.

A sound wave travelling through air is a very common example of a longitudinal wave. As a sound wave moves from the speaker to the ear of a listener, the particles of air vibrate back and forth in the same direction as that of direction of energy transport or the direction in which the disturbance propagates. Each individual particle pushes on its neighbouring particle so as to push it forward. The *collision* of particle 1 with its neighbour serves to restore particle 1 to its original position and displace particle 2 in a forwards direction. This back and forth motion of particles in the direction of energy transport creates regions within the medium where the particles are pressed together and other regions where the particles are spread apart. Longitudinal waves can always be quickly identified by the presence of such regions. This process continues along the *chain* of particles until the sound wave reaches the ear of listener.

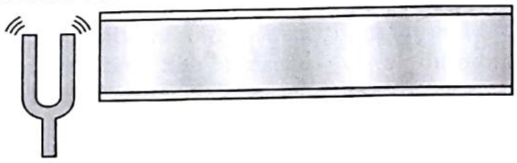


Fig. 11.10 A vibrating tuning fork will force air within a pipe to begin vibrating back and forth in a direction parallel to the energy transport; sound is a longitudinal wave

Since a sound wave is a disturbance which is transported through a medium *via* the mechanism of particle-to-particle interaction, a sound wave is characterized as a mechanical wave.

The creation and propagation of sound waves are often demonstrated in the class through the use of a tuning fork. A tuning fork is a metal object consisting of two tines capable of vibrating if struck by a rubber hammer. As the tines of the tuning forks vibrate back and forth, they begin to disturb the surrounding air molecules. These disturbances are passed on to the adjacent air molecules by the mechanism of particle interaction. The motion of the disturbance, originating at the tines of the tuning fork and travelling through the medium (in this case, air) is what is referred to as a sound wave.

In many Physics demonstrations, tuning forks are mounted on a sound box. In such a situation, the vibrating tuning fork, being *connected* to the sound box, sets the sound box into vibrational motion. In turn, the soundbox, being *connected* to the air inside it, sets the air inside of the sound box into a vibrational motion. As the tines of the tuning fork, the structure of the sound box, and the air inside of the sound box begin vibrating at the same frequency, a louder sound is produced. In fact, the more particles which can be made to vibrate, the louder or more amplified is the sound. This concept is often demonstrated by the placement of a vibrating tuning fork against the glass panel of an overhead projector or on the wooden door of a cabinet. The vibrating tuning fork sets the glass panel or wood door into vibrational motion and results in an amplified sound.

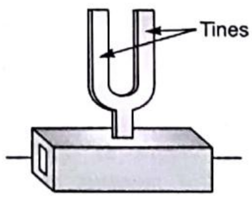


Fig. 11.11 A tuning fork mounted on a sound box

Sound as a Pressure Wave

Sound is a mechanical wave which results from the back and forth vibration of the particles of the medium through which the sound wave is moving. If a sound wave is moving from left to right through air, then the particles of air will be displaced both rightward and leftward as the energy of the sound wave passes through it. The motion of the particles are parallel (and anti-parallel) to the direction of the energy transport. This is what characterizes sound waves in air as longitudinal waves.

A vibrating tuning fork is capable of creating such a longitudinal wave. As the tines of the fork vibrate back and forth, they push on to the neighbouring air particles. The forward motion of a tine pushes air molecules horizontally to the right and the backward retraction of the tine creates a low pressure area allowing the air particles to move back to the left.

Because of the longitudinal motion of the air particles, there are regions in the air where the air particles are compressed together and other regions where the air particles are spread apart. These regions are known as **compressions** and **rarefactions**, respectively. The compressions are regions of high air pressure while the rarefactions are regions of low air pressure. The diagram below depicts a sound wave created by a tuning fork and propagated through the air in an open tube. The compressions and rarefactions have been neatly labelled.

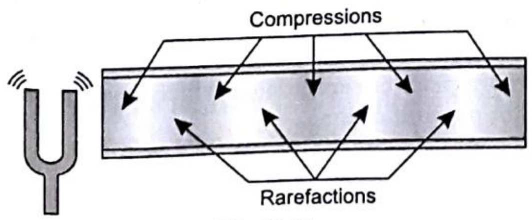


Fig. 11.12

Since a sound wave consists of a repeating pattern of high pressure and low pressure regions moving through a medium, it is sometimes referred to as a **pressure wave.** If a detector, whether it be the human ear or a man-made instrument, is used to detect a sound wave, it would detect fluctuations in pressure as the sound wave impinges upon the detecting device. At one instant in time, the detector would detect a high pressure; this would correspond to the arrival of a compression at the detector site. At the next instant of time, the detector might detect a normal pressure. And then finally a low pressure would be detected, corresponding to the arrival of a rarefaction at the detector site. The fluctuations in pressure as detected by the detector occurs at periodic and regular time-intervals. In fact, a plot of pressure *versus* time would appear as a sine curve. The peak points of the sine curve correspond to compressions; the low points correspond to rarefactions; and the "zero points" correspond to the pressure which the air would have if there were no disturbance moving through it. The diagram below depicts the correspondence between the longitudinal nature of a sound wave in air and the pressure-time fluctuations which it creates at a fixed detector location.

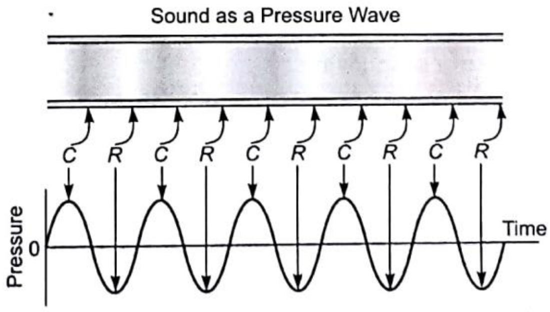


Fig. 11.13 Note ε stands for compression and *R* stands for rarefaction

The above diagram can be somewhat misleading if you are not careful. The representation of sound by a sine wave is merely an attempt to illustrate the sinusoidal nature of the pressure-time fluctuations. Do not conclude that sound is a transverse wave which has crests and troughs. Sound waves travelling through air are indeed longitudinal waves with compressions and rarefactions. As sound passes through air (or any fluid medium), the particles of air don't vibrate in a transverse manner.

Pitch and Frequency of Sound Waves

A sound wave, like any other wave, is introduced into a medium by a vibrating object. The vibrating object is the source of the disturbance which moves through the medium. The vibrating object which creates the disturbance could be the vocal chords of a person, the vibrating string and sound board of a guitar or violin, the vibrating tines of a tuning fork, or the vibrating diaphragm of a radio speaker. Regardless of what vibrating object is creating the sound wave, the particles of the medium through which the sound moves are vibrating in a back and forth motion at a given frequency. The frequency of a wave refers to how often the particles of the medium vibrate when a wave passes through the medium. The frequency of a wave is measured as the number of complete back-and-forth vibrations of a particle of the medium per unit time. The sensation of frequencies is commonly referred to as the pitch of a sound. A high pitch sound corresponds to a high frequency sound wave and a low pitch sound corresponds to a low frequency sound wave.

As a sound wave moves through the medium, each particle of the medium vibrates at the same frequency. This is sensible since each particle vibrates due to the motion of its nearest neighbour. The first particle of the medium begins vibrating, say at 500 Hz, and begins to set the second particle into vibrational motion at the same frequency of 500 Hz. The second particle begins vibrating at 500 Hz and thus sets the third particle of the medium into vibrational motion at 500 Hz. The process continues throughout the medium, each particle vibrating at the same frequency. And of course the frequency at which each particle vibrates is the same as the frequency of the original source of the sound wave. The back-and-forth vibrational motion of the particles of the medium would not be the only observable phenomenon occurring at a given frequency. Since a sound wave is a pressure wave, a detector could be used to detect oscillations in pressure from a high pressure to a low pressure and back to a high pressure. As the compressions (high pressure) and rarefactions (low pressure) move through the medium, they would reach the detector at a given frequency. For example, a compression would reach the detector 500 times per second if the frequency of the wave were 500 Hz. Similarly, a rarefaction would reach the detector 500 times per second if the frequency of the wave were 500 Hz. The frequency of a sound wave not only refers to the number of back-and-forth vibrations of the particles per unit time, but also refers to the number of compressions or rarefactions which pass a given point per unit of time.

The Speed of Sound

A sound wave is a pressure disturbance which travels through a medium by means of particle-to-particle interaction. As one particle becomes disturbed, it exerts a force on the adjacent particle, thus disturbing that particle from rest and transporting the energy through the medium. Like any other wave, the speed of a sound wave refers to how fast the disturbance is passed from particle to particle. While frequency refers to the number of vibrations which an individual particle makes per unit time, speed refers to the distance which the disturbance travels per unit time. Always be cautious to distinguish between the two often confused quantities of speed (*how fast...*) and frequency (*how often...*).

The speed of any wave depends upon the properties of the medium through which the wave is travelling. Typically there are two essential types of properties which affect the wave speed—inertial properties and elastic properties.

The speed of a sound wave in air depends upon the properties of the air, namely the temperature and the pressure. The pressure of air (like any gas) will affect the mass density of the air (an inertial property) and the temperature will affect the strength of the particle interactions (an elastic property). At normal atmospheric pressure, the temperature dependence of the speed of a sound wave through air is approximated by the following equation :

$$v = (331 + 0.6T)\ \text{ms}^{-1}$$

where T is the temperature of the air in degrees Celsius.

C-BIs

Concept Building Illustrations

Illustration | 5 *A fire whistle emits a tone of 170 Hz. Take speed of sound in air to be 340 ms^{-1}. Determine the wavelength of this sound wave.*

Solution Using $v = f\lambda$

$$\Rightarrow \quad 340 = 170 \times \lambda \quad \Rightarrow \quad \lambda = 2\ \text{m}$$

Illustration | 6 *Determine the speed of sound in air at 5°C.*

Solution From $v = (331 + 0.6\ T)\ \text{ms}^{-1}$

where T is temperature of air in Celsius.

$$v = (331 + 0.6 \times 5)\ \text{ms}^{-1}$$
$$= 334\ \text{ms}^{-1}.$$

The Human Ear

Here we shall explain to you the working of human ear. It is very difficult to understand the how humans hear. But we are trying to explain the working and construction of a human ear in a simple manner.

The ear consists of three basic parts—the outer, the middle, and the inner. Each part of the ear serves a specific purpose in the task of detecting and interpreting a sound. The outer ear serves to collect and send sound waves to the middle ear. The middle ear serves to transform the energy of a sound wave into the internal vibrations of the bone structure of the middle ear and ultimately transform these

vibrations into a compressional wave in the inner ear. The inner ear serves to transform the energy of a compressional wave within the inner ear fluid into nerve impulses which can be transmitted to the brain. The three parts of the ear are as shown below :

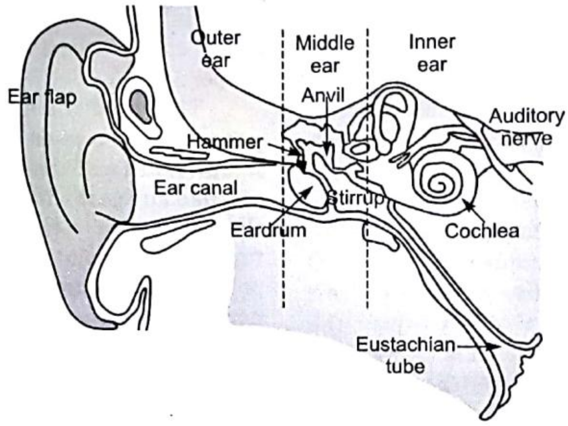


Fig. 11.14

The outer ear consists of an ear flap and an approximately 2-cm long ear canal. The ear flap provides protection for the middle ear in order to prevent damage to the eardrum. The outer ear also sends sound waves which reach the ear through the ear canal to the eardrum of the middle ear. Because of the length of the ear canal, it is capable of amplifying sounds with frequencies of approximately 3000 Hz. As sound travels through the outer ear, the sound is still in the form of a pressure wave, with an alternating pattern of high and low pressure regions. It is not until the sound reaches the eardrum at the interface of the outer and the middle ear that the energy of the mechanical wave becomes converted into vibrations of the inner bone structure of the ear.

The middle ear is an air-filled cavity which consists of an eardrum and three tiny, interconnected bones—the hammer, anvil, and stirr-up. The eardrum is a very durable any tightly stretched membrane which vibrates as the incoming pressure waves reach it.

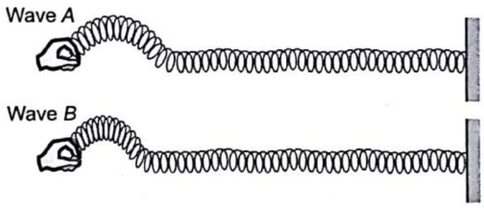
Being connected to the hammer, the movements of the eardrum will set the hammer, anvil, and stirr-up into motion at the same frequency of the sound wave. The stirr-up is connected to the inner ear; and thus the vibrations of the stirr-up are transmitted to the fluid of the inner ear and create a compression wave within the fluid. The three tiny bones of the middle ear act as levers to amplify the vibrations of the sound wave. Due to a mechanical advantage, the displacements of the stirrup are greater than that of the hammer. Further more, since the pressure wave striking the large area of the eardrum is concentrated into the smaller area of the stirr-up, the force of the vibrating stirr-up is nearly 15 times larger than that of the eardrum. This feature enhances our ability to hear the faintest of sounds. The middle ear is an air-filled cavity which is connected by the Eustachian tube to the mouth. This connection allows for the equalization of pressure within the air-filled cavities of the ear. When this tube becomes clogged during a cold, the ear cavity is unable to equalize its pressure, this will often lead to ear-aches and other pains.

Towards Proficiency Problems

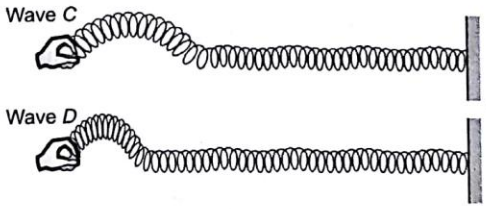
Exercise 1

A. Subjective Discussions

1. Describe how the fans in a stadium must move in order to produce a longitudinal stadium wave?
2. A science fiction film depicts inhabitants of one spaceship (in outer space) hearing the sound of a nearby spaceship as it zooms past at high speeds. Criticise the physics of this film.
3. Minute after minute, hour after hour, day after day, ocean waves continue to splash onto the shores. Explain why the beach is not completely submerged and why the middle of the ocean has not yet been depleted of its water supply?
4. A teacher attaches a slinky (a coiled spring) to the wall and begins introducing pulses with different amplitudes. Which of the two pulses (A or B) below will travel from the hand to the wall in the least amount of time? Justify your answer.

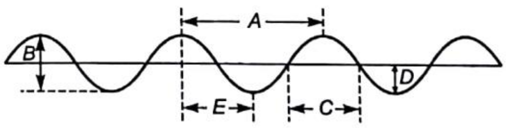


5. In Q. No. 4, the teacher then begins introducing pulses with different wavelengths. Which of the two pulses (C or D) will travel from the hand to the wall in the least amount of time? Justify your answer.



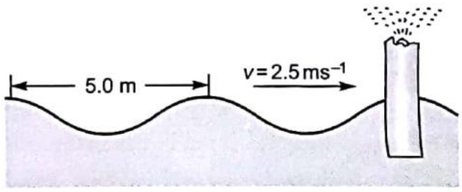
B. Numerical Answer Types

Consider the diagram below in order to answer Q. Nos. 1-2.

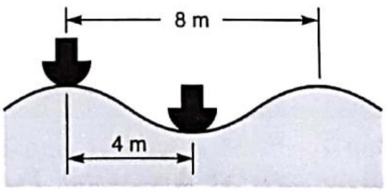


1. The wavelength of the wave in the diagram above is given by letter ....
2. The amplitude of the wave in the diagram shown is given by letter ....

3. A wave is introduced into a thin wire held tight at each end. It has an amplitude of 3.8 cm, a frequency of 51.2 Hz and a distance from a crest to the neighbouring trough of 12.8 cm. Determine the period of such a wave.
4. Ram and Shyam stand 8 m apart and demonstrate the motion of a transverse wave on a string. The wave can be described as having a vertical distance of 32 cm from a trough to a crest, a frequency of 2.4 Hz, and a horizontal distance of 48 cm from a crest to the nearest trough. Determine the amplitude, period and wavelength of such a wave.
5. An automatic focus camera is able to focus on objects by use of an ultrasonic sound wave. The camera sends out sound waves which reflect off distant objects and return to the camera. A sensor detects the time it takes for the waves to return and then determines the distance an object is from the camera. The camera lens then focuses at that distance. If a sound wave (speed $= 340\ ms^{-1}$) returns to the camera 0.150 s after leaving the camera, then how far away is the object ?
6. While hiking through a mountain, a climber lets out a scream. An echo (reflection of the scream off a nearby mountain) is heard 0.82 s after the scream. The speed of the sound wave in air is $342\ ms^{-1}$. Calculate the distance of the climber from the nearby mountain.
7. The water waves shown below are travelling along the surface of an ocean at a speed of $2.5\ ms^{-1}$, and splashing up periodically against a pole. Each adjacent crest is 5 m apart. The crests splash upon reaching feet of pole. How much time passes between each successive splashing ?



8. Ocean waves are observed to travel along the water surface during a developing storm. A Coast Guard weather station observes that there is a vertical distance from high point to low point of 4.6 m and a horizontal distance of 8.6 m between adjacent crests. The waves splash into the station once every 6.2 s. Determine the frequency and the speed of these waves.
9. Two boats are anchored 4 m apart. They bob up and down, returning to the same up position every 3 s. When one is up the other is down. There are never any wave crests between the boats. Calculate the speed of the waves.



10. On a hot summer day, a little mosquito produced its warning sound near your ear. The sound is produced by the beating of its wings at a rate of about 600 wing beats/s.
    (a) What is the frequency in Hertz of the sound wave ?
    (b) Assuming the sound wave moves with a velocity of $350\ ms^{-1}$, what is the wavelength of the wave ?
11. Playing middle C on a piano keyboard produces a sound with a frequency of 256 Hz. Assuming the speed of sound in air is $345\ ms^{-1}$, determine the wavelength of the sound corresponding to the note of middle C.

12. Most people can detect frequencies as high as 20000 Hz. Assuming the speed of sound in air to be 345 ms^{-1}, determine the wavelength of the sound corresponding to this upper range of audible hearing.

13. An elephant produces a 10 Hz sound wave. Assuming the speed of sound in air to be 345 ms^{-1}, determine the wavelength of this infrasonic sound wave.

Information for Q. No. 14 to 16 Ram and Shyam are conducting a slinky experiment. They are studying the possible effect of several variables upon the speed of a wave in a slinky. Their data table is shown below. Fill in the blanks in the table, analyze the data, and answer the following questions :

Medium	Wavelength	Frequency	Speed
Zinc, 1-in. diameter coils	1.75 m	2.0 Hz	A
Zinc, 1-in. diameter coils	0.90 m	3.9 Hz	B
Copper, 1-in. diameter coils	1.19 m	2.1 Hz	C
Copper, 1-in. diameter coils	0.60 m	4.2 Hz	D
Zinc, 3-in. diameter coils	0.95 m	2.2 Hz	E
Zinc, 3-in. diameter coils	1.82 m	1.2 Hz	F

14. As the wavelength of a wave in a uniform medium increases, then what will happen with its speed ?

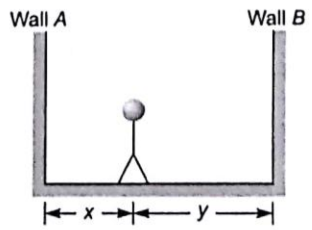
15. As the wavelength of a wave in a uniform medium increases, then what will happen with its frequency ?

16. The speed of a wave depends upon (*ie*, is causally affected by) [Mark out the correct choice(s)]
(a) the properties of the medium through which the wave travels.
(b) the wavelength of the wave.
(c) the frequency of the wave.
(d) both the wavelength and the frequency of the wave.

17. Suppose that you hear a clap of thunder 16.2 s after seeing the associated lightining stroke. The speed of sound waves in air is 343 ms^{-1} and the speed of light in air is 3×10^8 ms^{-1}. How far you are from the lightning stroke ?

18. A rescue plane flies horizontally at a constant speed searching for a disabled boat. When the plane is directly above the boat, the boat's crew blows a loud horn. By the time the plane's sound detector receives the horn's sound, the plane has travelled a distance equal to one half of its altitude above the ocean. If it takes sound 2 s to reach the plane, then determine (a) speed of plane, and (b) its altitude. Speed of sound = 340 ms^{-1}.

19. A person standing in between two walls as shown in figure shouts out and hears three echos after 1s, 2s and 3s, respectively. Determine the values of x and y. Take the speed of sound to be 330 ms^{-1}.



C. Fill in the Blanks

1. A fly flaps its wings back and forth 121 times each second. The period of the wing flapping is .... sec.
2. A common physics lab involves the study of the oscillations of a pendulum. If a pendulum makes 33 complete back-and-forth cycles of vibrations in 11 s, then its period will be ....
3. The period of the sound wave produced by a 440 Hz tuning fork is ....
4. Two waves are travelling through the same container of nitrogen gas. Wave *A* has a wavelength of 1.5 m. Wave *B* has a wavelength of 4.5 m. The speed of wave *B* must be .... the speed of wave *A*.
5. Two sound waves are travelling through a container of unknown gas. Wave *A* has a wavelength of 1.2 m. Wave *B* has a wavelength of 3.6 m. The velocity of wave *B* must be .... the velocity of wave *A*.

D. True/False

1. In order for Ram to hear Shyam, air molecules must move from the lips of Shyam to the ears of Ram.
2. Doubling the frequency of a wave source doubles the speed of the waves.

High Skill Questions

Exercise 2

A. Only One Option Correct

1. A transverse wave is transporting energy from east to west. The particles of the medium will move
   (a) east to west only
   (b) both eastward and westward
   (c) north to south only
   (d) both northward and southward

2. A wave is transporting energy from left to right. The particles of the medium are moving back and forth in a leftward and rightward direction. This type of wave is known as
   (a) mechanical
   (b) electromagnetic
   (c) transverse
   (d) longitudinal mechanical

3. A sound wave is a mechanical wave, not an electromagnetic wave. This means that
   (a) particles of the medium move perpendicular to the direction of energy transport
   (b) a sound wave transports its energy through a vacuum
   (c) particles of the medium regularly and repeatedly oscillate about their rest positions
   (d) a medium is required in order for sound waves to transport energy

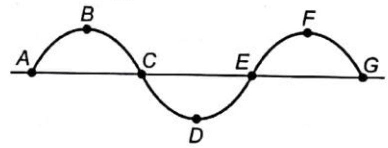
4. If you strike a horizontal rod vertically from above, what can be said about the waves created in the rod ?
   (a) The particles vibrate horizontally along the direction of the rod.
   (b) The particles vibrate vertically, perpendicular to the direction of the rod.
   (c) The particles vibrate in circles, perpendicular to the direction of the rod.
   (d) The particles travel along the rod from the point of impact to its end.

5. Which of the following is not a characteristic of mechanical waves ?
   (a) They consists of disturbances or oscillations of a medium.
   (b) They transport energy.
   (c) They travel in a direction which is at right angles to the direction of the particles of the medium.
   (d) They are created by a vibrating source.

6. A medium is able to transport a wave from one location to another because the particles of the medium are
   (a) frictionless
   (b) isolated from one another
   (c) able to interact
   (d) very light

7. A tennis coach paces back and forth along the sideline 10 times in 2 m. The frequency of her pacing is .... Hz.
   (a) 5.0 (b) 0.20
   (c) 0.12 (d) 0.083

8. Indicate the interval which represents one full wavelength.



   (a) A to C (b) B to D
   (c) A to G (d) C to G

9. Non-digital clocks (which are becoming more rare) have a seconds' hand which rotates around in a regular and repeating fashion. The frequency of rotation of a seconds' hand on a clock is .... Hz.
   (a) 1/60 (b) 1/12
   (c) 1/2 (d) 1

10. Sita accompanies her father to the park for fun. While there, she hops on the swing and begins a motion characterized by a complete back-and-forth cycle every 2 s. The frequency of swing is
(a) 0.5 Hz (b) 1 Hz
(c) 2 Hz (d) None of these

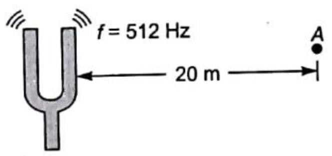
11. In problem 10, the period of swing is
(a) 0.5 s (b) 1 s
(c) 2 s (d) None of these

12. As the frequency of a wave increases, the period of the wave ....
(a) decreases (b) increases
(c) remains the same (d) None of these

13. An ocean wave has an amplitude of 2.5 m. Weather conditions suddenly change such that the wave has an amplitude of 5.0 m. The amount of energy transported by the wave is ....
(a) halved
(b) doubled
(c) quadrupled
(d) Remains the same.

14. Two waves are travelling through a container of an inert gas. Wave *A* has an amplitude of 1 cm. Wave *B* has an amplitude of 2 cm. The energy transported by wave *B* must be .... the energy transported by wave *A*.
(a) one-fourth
(b) one-half
(c) two times larger than
(d) four times larger than

15. The time required for the sound wave ($v = 340\ ms^{-1}$) to travel from the tuning fork to point *A* is



(a) 0.020 s (b) 0.059 s
(c) 0.59 s (d) 2.9 s

16. Ram and Shyam have stretched a slinky between them and begin experimenting with waves. As the frequency of the waves is doubled,
(a) the wavelength is halved and the speed remains constant
(b) the wavelength remains constant and the speed is doubled
(c) both the wavelength and the speed are halved
(d) both the wavelength and the speed remain constant

17. A sound wave is different than a light wave in a way that a sound wave is
(a) produced by an oscillating object while a light wave is not
(b) not capable of travelling through a vacuum
(c) not capable of diffracting while a light wave is
(d) capable of existing with a variety of frequencies while a light wave has a single frequency

18. A sound wave is a pressure wave; regions of high (compressions) and low pressure (rarefactions) are established as the result of the vibrations of the sound source. These compressions and rarefactions result because sound
(a) is more dense than air and thus has more inertia, causing the bunching up of sound
(b) waves have a speed which is dependent only upon the properties of the medium
(c) is like all waves, it is able to bend into the regions of space behind obstacles
(d) is able to reflect off fixed ends and interfere with incident waves
(e) vibrates longitudinally, the longitudinal movement of air produces pressure fluctuations

19. The speed of sound in air is affected by changes in
(a) wavelength (b) frequency
(c) temperature (d) amplitude

20. Mechanical waves are observed to have a wavelength of 300 m and a frequency of 0.7 Hz. The velocity of these waves is
(a) $0.00021\ ms^{-1}$ (b) $21\ ms^{-1}$
(c) $210\ ms^{-1}$ (d) None of these

21. Sinusoidal water waves are generated in a large ripple tank. The waves travel at $20\ cm\text{-}s^{-1}$ and the adjacent crests are 5 cm apart. The time required for each new whole cycle to be generated is

(a) 100 s (b) 4 s
(c) 0.25 s (d) 0.5 s

22. A source of frequency f sends waves of wavelength λ travelling with wave speed v in a medium. If the frequency is changed from f to $2f$, then the new wavelength and new wave speed are respectively
(a) $2\lambda, v$ (b) $\frac{\lambda}{2}, v$
(c) $\lambda, 2v$ (d) $\lambda, \frac{v}{2}$

23. The speed of sound wave is determined by
(a) its amplitude
(b) its intensity
(c) its pitch
(d) the transmitting medium

24. The sound wave has a wavelength of 3 m. The distance from a compression centre to the adjacent rarefaction centre is
(a) 0.75 m (b) 1.5 m
(c) 3.0 m (d) None of these

25. During a time-interval of exactly one period of vibration of a tuning fork, the emitted sound travels a distance
(a) equal to the length of tuning fork
(b) of about 330 m
(c) which decreases with time
(d) of one wavelength in air

Answers

Towards Proficiency Problems
Exercise 1

B. Numerical Answer Types

1. A **2.** D **3.** 0.02 s **4.** 16 cm, 0.42 s, 96 cm
5. 25.5 m **6.** 140.22 m **7.** 2 s **8.** 0.161 Hz, 1.39 ms^{-1}
9. $\frac{8}{3}$ ms^{-1} **10.** (a) 600 Hz, (b) 0.584 m **11.** 1.35 m
12. 1.725 cm **13.** 34.5 m
16. (a, b) **17.** 5.556 km **18.** (a) 152 ms^{-1}, (b) 608 m
19. x = 115 m, y = 330 m

C. Fill in the Blanks

1. $\frac{1}{121}$ **2.** $\frac{1}{3}$ s **3.** $\frac{1}{440}$ s **4.** 3 times **5.** 3 times

D. True/False

1. F **2.** F

High Skill Questions
Exercise 2

A. Only One Option Correct

1. (b) **2.** (d) **3.** (d) **4.** (a) **5.** (d) **6.** (c) **7.** (a) **8.** (d) **9.** (a) **10.** (a)
11. (c) **12.** (a) **13.** (c) **14.** (d) **15.** (b) **16.** (a) **17.** (b) **18.** (e) **19.** (c) **20.** (c)
21. (c) **22.** (b) **23.** (d) **24.** (b) **25.** (d)

Explanations

Towards Proficiency Problems
Exercise 1

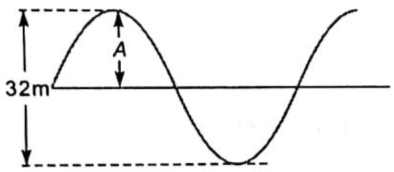
Numerical Answer Types

3. Crest to trough distance $= \dfrac{\lambda}{2} = 12.8$ cm

$$\lambda = 25.6 \text{ cm}, \quad f = 51.2 \text{ Hz}$$
$$T = \frac{1}{f} = \frac{1}{51.2} \approx 0.02 \text{ s}$$

4. $2A = 32$ cm

$$A = 16 \text{ cm}$$



$$f = 2.4 \text{ Hz}, \; T = \frac{1}{f} = \frac{1}{2.4} = 0.42 \text{ s}$$
$$\frac{\lambda}{2} = 48 \text{ cm} \;\Rightarrow\; \lambda = 96 \text{ cm}$$

5. Let d is the required distance, then

$$\frac{2d}{v} = 0.15 \;\Rightarrow\; d = \frac{340 \times 0.15}{2} = 25.5 \text{ cm}$$

6. $2d = 342 \times 0.82$

$$\Rightarrow \qquad d = 140.22 \text{ m}$$

7. $5 = vt \;\Rightarrow\; t = \dfrac{5}{2.5} = 2$ s

8. $T = 6.2 \text{ s} \;\Rightarrow\; f = \dfrac{1}{T} = \dfrac{1}{6.2} \text{ Hz} = 0.161 \text{ Hz}$

$$\lambda = 8.6 \text{ m}$$
$$v = f\lambda = 0.161 \times 8.6 \text{ ms}^{-1} = 1.39 \text{ ms}^{-1}$$

9. $T = 3$ s, $\lambda = 8$ m

$$v = \frac{\lambda}{T} = \frac{8}{3} \text{ ms}^{-1}$$

11. $\lambda = \dfrac{v}{f} = \dfrac{345}{256} \text{ m} = 1.35 \text{ m}$

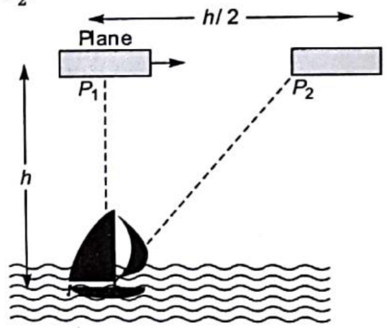
14. $v = f\lambda$

As λ increases which can be due to increase in v or decrease in f or due to combination of both.
So wave speed may increase or remain constant.

15. From above question frequency may decrease or remain constant.

17. $s = 343 \times 16.2 \text{ m} = 5.556 \text{ m}$

18. The time taken by sound wave to reach P_2 is same as time taken by plane to reach from P_1 to P_2.



$$vt = \frac{h}{2}, \text{ where } v \text{ is plane speed}$$
$$\sqrt{h^2 + \frac{h^2}{4}} = 340 \times t$$

It is given $t = 2$ s

So, $\qquad h = 608.2$ m

$$v = \frac{h}{2t} = 152 \text{ ms}^{-1}$$

19. 1st echo he hears due to reflected sound from wall A.
2nd echo he hears due to reflected sound from wall B.
3rd echo he hears due to reflected sound from wall A and then from wall B.

$$1 \text{ s} = \frac{2x}{330}$$
$$2 \text{ s} = \frac{2y}{330}$$
$$\frac{2x + 2y}{330} = 3 \text{ s}$$

$\Rightarrow \qquad x = 115$ m

and $\qquad y = 330$ m

Chapter 12

Thermal Physics

The First Steps' Learning

- Temperature
- Thermometry
- Thermal Equilibrium and Heat
- Zeroth Law of Thermodynamics
- Diathermic and Adiabatic Walls/Materials
- Thermal Expansion
- Calorimetry

In this chapter we are going to explore the concepts related to thermal physics. Thermal physics is the branch of physics which deals with effect and behaviour of thermal energy. This part of physics is very important in explaining the bulk properties of matter and also it is helpful in understanding the mechanics of atoms and molecules ie, it helps in understanding the molecular world.

Have you ever thought how refrigerators, air conditioners, heat engines works or why pump becomes warm as you inflates a bicycle tire ? Why tire becomes cool on bursting ? When you pour hot water on a metallic cap of a sealed glass bottle, then it can be easily opened. The rail track loses their straightness in summer etc. The laws of thermodynamics and other concepts related to thermal physics enable us to answer these questions.

Temperature

Quiet often we associate the concept of temperature with how hot or cold an object feels when we touch it. Thus, we can say that the property by virtue of which we sense the relative hotness or coldness of a body is the measure of its temperature. However, our senses are unreliable and often misleading. For example, if we remove a metal ice tray and a package of butter from the freezer, the ice tray feels colder to the hand than the butter even though both are at the same temperature. This is because of the difference in the rate at which transfer of thermal energy (heat) from the hand to the object takes place and not because of temperature. Thus the above definition of temperature is not an exact one but still works.

From kinetic theory of gases, the temperature is directly related to the kinetic energy of the molecules of gases.

Temperature is one of the seven fundamental quantities. It is a scalar quantity and its SI unit is kelvin (K).

Thermometry

The branch of thermal physics which deals with the measurement of temperature is termed as thermometry and the instruments which are used to measure the temperature is termed as a thermometer. To measure the temperature of an object we keep it in thermal contact (thermal contact means the thermal energy transfer *ie*, heat can take place between two bodies just like electrical contact where charge transfer takes place) with a thermometer for some time and then thermometer gives the temperature of object.

All the thermometers are operating between two fixed points which are steam point (the temperature where steam and water co-exist), and the ice point (the temperature where ice and water co-exist). There are various types of thermometers like constant volume gas thermometer, mercury thermometer etc. Once we fix the two end points (steam point and ice point) for a thermometer we need to define a temperature scale. The three most common temperature scales are

(a) Celsius or Centigrade scale

(b) Fahrenheit scale

(c) Kelvin scale

The following table tells us about the temperature corresponding to steam and ice points on these three scales.

S. No.	Temperature Scale	Steam Point	Ice Point
1.	Celsius scale	100°C	0°C
2.	Fahrenheit scale	212°F	32°F
3.	Kelvin scale	373.15 K	273.15 K

The relation which converts the temperature reading between these three scales is given by

$$\frac{T_C - \text{Ice point of Celsius scale}}{(\text{Steam point} - \text{Ice point})_{\text{Celsius scale}}} = \frac{T_F - \text{Ice point on Fahrenheit scale}}{(\text{Steam point} - \text{Ice point})_{\text{Fahrenheit scale}}} = \frac{T_K - \text{Ice point on Kelvin scale}}{(\text{Steam point} - \text{Ice point})_{\text{Kelvin scale}}}$$

where T_C, T_F and T_K are the temperatures of same object as read by Celsius, Fahrenheit and Kelvin scale respectively.

$$\Rightarrow \quad \frac{T_C - 0}{100} = \frac{T_F - 32}{212 - 32} = \frac{T_K - 273.15}{100}$$

$$\Rightarrow \quad T_F = \frac{9}{5} T_C + 32$$

$$T_K = T_C + 273.15$$

C-BIs

Concept Building Illustrations

Illustration | 1 *The temperature of an object as read by fahrenheit scale is 50°, then find the temperature of same object as read by celsius and kelvin scales ?*

Solution From $T_F = \frac{9}{5} T_C + 32$

It is given $T_F = 50$

$$\Rightarrow \quad 50 = \frac{9}{5} T_C + 32$$

$$\Rightarrow \quad T_C = \frac{(50 - 32)5}{9} = 10°\text{C}$$

$$T_K = T_C + 273.15 = 283.15 \text{ K}$$

Thermal Equilibrium and Heat

If you mix a mug of boiling water to two mugs of water which is somewhat cool, then resulting water would be somewhat like warm. If you keep a hot iron piece in contact with ice then ice melts and the iron piece becomes somewhat cooler. If you keep a hot object in open atmosphere then after a sufficient amount of time the body acquires the atmospheric temperature. There are a number of situations of similar nature. In all these situations initially the two bodies were at different temperatures and finally they acquire the same temperature.

The two or more bodies when kept in thermal contact (two bodies are said to be in thermal contact when thermal energy could be transferred from one to the other) transfer of energy from one to another take place due to their temperature difference and this energy transfer continues till both the bodies acquire the same temperature. Once the bodies acquire the same temperature, no further transfer of energy between bodies takes place and the bodies are said to be in thermal equilibrium with each other. Thus, we can say in thermal equilibrium the two bodies would be at same temperature and no transfer of energy takes place between the two bodies.

Heat

If we ask a question from you—which is having more heat, sun or moon ? Then may be most of you answer in one voice—the Sun. But this is not so, the correct thing is that this question is irrevelent *ie*, question itself doesn't make any sense. Generally, we misuse the word heat in our daily conversation, but in physics heat is defined very precisely. Every object or

body is having some **internal energy*** which is due to the motion of molecules constituting the matter of body and due to the molecular configuration of constituent molecules of the body. According to molecular interpretation of temperature, the temperature of a body is due to the part of the internal energy associated with motion of molecules. When two bodies having different temperatures are kept in thermal contact, then a part of internal energy transfers from the body having higher temperature to the body having lower temperature and this transfer of energy takes place till both acquire the same temperature *ie*, acquire a thermal equilibrium.

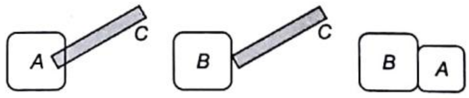
This transference of energy between two bodies due to temperature difference is termed as **heat**, *ie*, heat is the energy in transit and is not the energy contained within the body.

Thus, the question which we posed is meaningless in the way that no doubt the Sun is having more energy than the moon, but heat is not contained within the body but it is the part of thermal energy which transfers from one body to another due to temperature difference.

To summarize, when two bodies having different temperatures are kept in thermal contact, then a part of thermal energy from a body having higher temperature transfers to the body having lower temperature to acquire thermal equilibrium. The energy which is transferred from one body to another due to temperature difference is termed as heat.

Zeroth Law of Thermodynamics

According to the zeroth law of thermodynamics if two bodies A and B are separately in thermal equilibrium with a third body C, then both A and B are also in thermal equilibrium with each other.



A and *C* are kept in thermal contact, and are in thermal equilibrium *ie*, $T_A = T_C$

B and *C* are kept in thermal contact, and are in thermal equilibrium *ie*, $T_B = T_C$

As $T_A = T_C$ and $T_B = T_C$, thus $T_A = T_B$ *ie*, the two bodies *A* and *B* are in thermal equilibrium when placed in thermal contact

(a) (b) (c)

Fig. 12.1

Diathermic and Adiabatic Walls/ Materials

You may know that in summer the people use to keep the water bottle inside a jute cloth so that the water remains cool for a longer time. You may have also noticed that if ice is kept in a styrofoam box (thermocol box), then it won't melt for 7-8 hrs even in hot summer while if ice is kept in open in a hot summer, then it will melt within half an hour. Have you ever wondered, what is the reason behind all these ? Due to this reason only, materials are classified into two classes according as how the energy transfer (slower/fast) takes place across them due to temperature difference.

***In mechanics we never bother about internal energy. Here also we are not discussing internal energy in details.**

The material through which heat flows very fast are termed as diathermic materials. The two bodies kept in contact with the help of diathermic wall will acquire same temperature very soon, *ie*, two bodies are said to be in thermal contact if some diathermic material is placed between them.

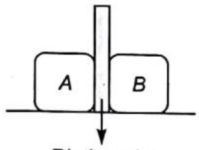


Fig. 12.2 Two bodies *A* and *B* initially having different temperatures when kept in contact *via* diathermic wall will reach thermal equilibrium very fast.

The material through which heat can't flow or heat flow is very slow are termed as adiabatic walls or thermal insulators. If two bodies are kept in contact with a adiabatic material between them, then no energy transfer takes place between them even if they have different temperatures. For examples, : Thermos flask containing hot beverages—the hot drinks inside thermos flask remains hot for a long time even if the thermos flask is kept in a chilling atmosphere, this is because of the adiabatic nature of the walls of the thermos flask.

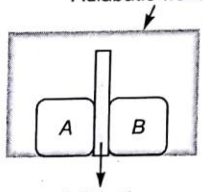


Fig. 12.3 The temperature of two bodies *A* and *B* having different temperatures initially remains constant (*ie*, does not change with time) as they are separated by adiabatic walls.

Thermal Expansion

The working of mercury thermometers which doctors generally use, is based on the concept of thermal expansion. The thermal expansion is one of the most common concepts, whose effect generally we see everyday in our life. The phenomenon, thermal expansion plays a very important role in a large number of other applications. The expansion of joints in bridges, concrete highways, buildings etc, the working of mercury thermometers, bimetallic strips etc, all are based on the concept of thermal expansion.

Thermal expansion means, as the temperature of an object increases its dimensions increase. The overall thermal expansion of an object is a consequence of the change in average separation between its constituent atoms or molecules. The detailed microscopic picture of thermal expansion is not required at this level.

As we increase the temperature of an object, its all three dimensions increase, and hence its linear dimensions, its area as well as volume all increase. For simplicity, thermal expansion is classified into three categories :

(a) Linear expansion
(b) Superficial or areal expansion
(c) Volume or cubical expansion

Linear Expansion

As linear dimensions are defined only for solids, linear expansion is valid only for solids. For liquids and gases, length does not make any sense. If L_0 is the length of an object at any temperature T_0, then as temperature changes by ΔT so that final temperature becomes T, then the increase in length of the object due to increase in temperature is given by $\Delta L = L_0 \alpha \Delta T$ where α is the coefficient of linear expansion of the material of object.

If L be the final length of object, then

$$L - L_0 = L_0 \alpha \, \Delta T$$

$$L = L_0[1 + \alpha \, \Delta T] \qquad \text{where } \Delta T = T - T_0$$

The coefficient of linear expansion is a characteristic property of the materials, and its unit is /°C.

There are various applications of linear expansion like losing or gaining of time by a pendulum clock, bimetallic strips, stress developed in wires etc.

C-BIs

Concept Building Illustrations

Illustration | 2 *The length of a steel rod at 10°C is 20 cm. Determine the length of this steel rod at 50°C. Coefficient of linear expansion for steel is 11×10^{-6} /°C.*

Solution $L = L_0[1 + \alpha \, \Delta T]$

Here, temperature changes from 10°C to 50°C *ie*, temperature increases by 40°C, so $\Delta T = 40°C$

$$\Rightarrow \quad L = (20 \text{ cm})[1 + 11 \times 10^{-6} \times 40]$$

$$= 20.0088 \text{ cm}$$

Illustration | 3 *The maximum permissible change in length of a rod in an experiment is 1 mm. It the length of rod is 20 cm at 60°C, then what would be the range of temperature in which the experiment could be performed? $\alpha_{rod} = 1 \times 10^{-4}$ /°C.*

Solution Let ΔT be the change in temperature which causes an expansion or contraction in the rod of 1 mm. Then,

$$\Delta L = L_0 \alpha \, \Delta T$$

$$1 \times 10^{-3} \text{ m} = (20 \times 10^{-2} \text{ m}) \times 10^{-4} \times \Delta T$$

$$\Rightarrow \quad \Delta T = 50°C$$

It means, within the permissible limits the temperature can be increased or decreased by 50°C from its initial temperature of 60°C. So, the required range of temperature is 10°C to 110°C.

Areal Expansion

As we have seen that linear dimensions of an object changes with temperature, it means its area and volume also change with temperature. The change in area due to change in temperature causes areal expansion and volume change causes volume expansion. For isotropic substances [substances whose properties are same in all directions], the change in area due to change in temperature is given by

$\Delta A = A_0 \beta \, \Delta T$ where $\beta (= 2\alpha)$ is termed as coefficient of superficial expansion

ΔA is change in area.

A_0 is initial area.

ΔT is change in temperature.

A is the final area at temperature T.

$$A - A_0 = A_0 \beta \, \Delta T$$

$$A = A_0[1 + \beta \, \Delta T]$$

As for liquids and gases, area doesn't make any sense, so areal expansion is also valid only for solids.

C-BIs

Concept Building Illustrations

Illustration | 4 *If coefficient of linear expansion for an isotropic material is 25×10^{-6} /°C, then what would be the coefficient of superficial expansion for this material ?*

Solution Using, $\beta = 2\alpha$

$$\beta = 2 \times 25 \times 10^{-6}$$
$$= 5 \times 10^{-5}/°C$$

Illustration | 5 *The surface area of an object at 20°C is 100 cm^2. Determine the area of an object at 50°C. (α for the material of object is 25×10^{-6} /°C.)*

Solution From $A = A_0[1 + \beta\,\Delta T]$

$$A = (100\text{ cm}^2)[1 + 2 \times 25 \times 10^{-6} \times 30]$$
$$= 100.15\text{ cm}^2$$

Volume Expansion

Volume expansion is valid for all the three states of matter—solids, liquids and gases. If V_0 be the volume of an object at temperature T_0, then on changing the temperature of object by ΔT, the volume of the object changes by ΔV given by

$$\Delta V = V_0 \times \gamma \Delta T$$

where $\gamma = 3\alpha$ is the coefficient of cubical or volume expansion.

If V be the final volume, then,

$$V - V_0 = V_0 \gamma\, \Delta T$$

$$\Rightarrow \; V = V_0 + V_0 \gamma\, \Delta T = V_0[1 + \gamma\, \Delta T]$$

As the temperature increases the volume of an object increases but as its mass is independent of temperature *ie*, mass doesn't change as the temperature changes, so from mass = volume × density, we can say that the density of an object decreases as temperature increases.

But it has been observed that behaviour of water is an exceptional one for temperature rise from 0°C to 4°C. It has been found that water contracts (instead of expanding) as its temperature increases from 0°C to 4°C. Beyond 4°C if we increase the temperature of water, the volume of water increases and density decreases while for temperature range from 0°C to 4°C the things are reverse.

The expansion in any matter is due to the increase in average separation between its constituent atoms or molecules which are binded to each other by interatomic or intermolecular forces. As interatomic or intermolecular forces are the strongest in the solids and the weakest in liquids, thermal expansion is maximum in gases, and the least in liquids. Thus $\gamma_{\text{solid}} > \gamma_{\text{liquid}} > \gamma_{\text{gas}}$, this particular fact is used in various applications.

Calorimetry

When you place two objects with different temperatures in a thermal contact, the temperature of the warmer object decreases while the temperature of the cooler object increases. With time, they reach a common equilibrium temperature which is intermediate between their initial temperatures. During this time the energy transfer *ie*, the heat exchange takes place between the two bodies, the science which deals with measurement of heat exchange between various bodies is termed as calorimetry and the vessel which is used to

measure the amount of heat exchanged between various bodies is termed as calorimeter. Before discussing the central idea of calorimetry it is advisable to understand few terms like internal energy, specific heat capacity, latent heats etc.

Internal Energy and Heat

A major distinction must be made between internal energy and heat. Although we discussed about heat earlier also, but after going through this section you will be able to understand the concept of heat more clearly.

Internal energy (U) of a system is the energy associated with the microscopic constituent (atoms or molecules) of the system. It includes kinetic energy of atoms and molecules due to their motion as well as potential energy arising due to the intermolecular interactions.

When two bodies having different temperatures are kept in thermal contact, then a part of internal energy from the body having higher temperature is transferred to the body having lower temperature and thus equalizing the temperature. This mechanism of energy transfer between two bodies due to temperature difference is termed as the heat. Thus sun is having more internal energy as compared to moon is a correct statement while statement like sun is having more heat as compared to moon are meaningless. We use the symbol Q for amount of energy transfer between the system and its surrounding or we can say for heat.

The practical unit of heat is calorie (cal). The calorie is defined as the energy required to raise the temperature of 1 g of water from 14.5°C to 15.5°C.

$$1 \text{ cal} = 4.186 \text{ J}.$$

The above equation is also known as mechanical equivalent of heat.

It means mechanical work and heat are equivalent in terms of energy *ie*, if some work is done on the system, then its internal energy is increased, for example if we stir water in a vessel then it gets warmer. Similarly, if we supply energy (heat) to a system, then also its internal energy increases, for example the water in a beaker kept on a lighted stove gets warmer as water is receiving energy from a burner.

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Concept Building Illustrations

Illustration | 6 *If a person consumes a food which gives 1000 cal of energy to him, of which 80% of food energy is lost in metabolic activities of the body and 20% is available to do some work, then calculate the amount of maximum work done by the person.*

Solution Energy received by the person from food,

$$Q = 1000 \text{ cal}$$

$$= 4186 \text{ J} \qquad [1 \text{ cal} = 4.186 \text{ J}]$$

80% of energy is lost, so the energy loss,

$$Q_1 = 0.8 \times Q = 0.8 \times 4186 \text{ J}$$

So energy available to do work,

$$Q_2 = Q - Q_1 = 0.2 \times 4186 \text{ J}$$

Work done by the person,

$$W = Q_2 = 837.2 \text{ J}.$$

Specific Heat and Latent Heat

If we give some energy to a body then two things can happen to this body—(i) its temperature may increase and/or—(ii) its state may change. For example, we have some water in a beaker at room temperature. Now, if we keep this beaker on a lighted stove, then the burner is supplying some energy to water as a result of which you will observe that water starts heating up *ie*, its temperature increases. Now let's consider the water temperature reaches to 100°C *ie*, boiling point of water, if the energy is supplied to boiling water, then the temperature of water won't increase but the water starts changing to gaseous phase *ie*, steam. Same way, the solid to liquid conversion takes place at the melting point. Then we can say that when we supply energy to a system (body) which is neither at its boiling point nor at its melting point, then the temperature of the body increases. The amount of energy required to increase the temperature of a body of mass m by ΔT is proportional to the product of m and ΔT.

$$Q \propto m\Delta T$$
$$Q = ms\Delta T$$

where s is the constant of proportionality and is termed as the specific heat capacity of substance.

If the temperature of a body is increasing, then it means energy is supplied to it and if the temperature of the body is decreasing, then it means energy is released by the body.

Specific heat capacity is a property of substance and its value depends on various factors such as

- The state of substance *ie*, for same material, value of s is different for its three phases.
- The process *ie*, the way in which energy is supplied to the substance *ie*, by keeping the pressure constant or volume or temperature constant or in some other way.
- It marginally depends on the temperature also but generally we neglect this factor.
- It depends on the nature of the material of substance.

Specific heat capacity is defined as the amount of heat required to raise the temperature of a unit mass of substance by 1°C. Its unit is $Jkg^{-1}\text{-}°C^{-1}$ or cal $g^{-1}\text{-}°C^{-1}$.

To change the phase of a system some energy transition between the system and the surroundings has to take place *ie*, heat transfer takes place. To estimate the heat involved in phase transition we will take help of latent heat (L). Latent heat is defined as amount of energy required to change the state of unit mass of substance. Its SI unit is J kg^{-1}, and the other practical unit is cal-g^{-1}.

The amount of energy required to change the state (phase) of a substance having mass m kg is $Q = mL$.

Generally, we deal with two types of latent heats—latent heat of fusion and latent heat of vaporization.

Latent Heat of Fusion

Latent heat of fusion (L_f) is the latent heat used when the liquid phase is converting to solid phase (freezing) or solid phase is converting to liquid phase (melting). The heat required to change the m kg of solid substance to its liquid phase is given by $Q = mL_f$. Latent heat of fusion is defined as the amount of heat required to change the unit mass of solid to liquid or *vice-versa*.

When solid phase is converted to liquid phase then heat is absorbed by substance and when liquid is converted to solid heat is released by system.

Latent Heat of Vaporization

Latent heat of vaporization (L_V) is used when the phase change occurs during boiling (liquid to gaseous) or condensation (gaseous to liquid). The heat involved in boiling or condensing is $Q = mL_V$. Latent heat of vaporization is defined as the amount of heat required to change unit mass of liquid to gaseous phase or *vice-versa*.

When liquid is converted to gaseous phase heat is absorbed by system and when gaseous phase is converted to liquid then heat is released.

Illustration Consider a ice cube at temperature $-25°C$ (temperature below its melting point), now we are adding (supplying) energy to it so that it will be converted to steam at $120°C$ (temperature above the boiling point of water). Here our aim is to see the full behaviour of ice cube as energy is added to it.

Phase I Initially the ice cube is at $-25°C$ and as we supply energy to it the temperature of ice cube increases to $0°C$. The energy supplied in this process is, $Q_1 = ms_{ice}[0-(-25)]$ $= ms_{ice} \times 25$. After supplying this much energy, the ice cube comes at $0°C$ *ie*, at the melting point.

Phase II Once the melting point has arrived, now whatever energy we give to ice cube it will be used up to melt the ice only and there won't be any increase in temperature and at the end of this phase we have m kg of water at $0°C$. Heat supplied in this phase is $Q_2 = mL_f$.

Phase III Once we have water at $0°C$ and we are giving energy to it, its temperature starts increasing till it reaches the boiling point. The energy supplied in this phase is $Q_3 = ms_{water} \times (100-0)$. After supplying this much energy we have water at $100°C$ (boiling point) at the end of this phase.

Phase IV If we supply further energy to the boiling water, energy will be consumed to change the boiling water to steam. In this process the energy required is $Q_4 = mL_V$. At the end of this phase we have the steam at $100°C$.

Phase V If we supply further energy to the steam at $100°C$, then the temperature of steam increases. The energy supplied in this phase is, $Q_5 = ms_{steam} \times 20$.

Thus, we have seen that as energy is supplied to a body either its temperature increase or its phase changes. When the substance is at its melting or boiling point then the energy supplied will be used only for changing the phase until the phase of entire amount of substance is changed.

Principle of Calorimetry

We have seen that when two or more bodies at different temperatures are kept in thermal contact, then in due course of time all the bodies acquire the same temperature. During this time the heat is transferred from one body to another, in accordance with energy conservation principle. In calorimetry this energy conservation principle is termed as principle of calorimetry. According to calorimetry principle, the total heat gained by a few bodies is equal to the total heat lost by the remaining bodies. Remember that principle of calorimetry is valid only for an isolated system *ie*, for a system on which no work is done by external agent and no energy is supplied to the system of bodies from the outside.

So according to the principle of calorimetry, Total heat lost = Total heat gained.

Some important points related to calorimetry problems are

1. If T_L and T_H are the lowest and the highest temperature of the bodies, then the final equilibrium temperature T would be lying between $T_L \le T \le T_H$.
2. If temperature of a body is increased, then it is absorbing (gaining) heat and if its temperature is decreasing, then it is releasing (losing) heat. Heat absorbed or released by a body is given by, $Q = ms\,\Delta T$.
3. If a solid is converted to liquid, then the body absorbs heat and releases heat when the liquid is converted to solid. The heat released or absorbed during melting or freezing process is given by, $Q = mL_F$.
4. If a liquid is converted to gaseous phase, then the body is absorbing heat and is releasing heat when gaseous phase is converted to the liquid phase. The heat absorbed or released during boiling or condensation process is given by $Q = mL_V$.
5. In final equilibrium, the mixture may consist of two phases, for example, if some amount of water and ice are mixed, then in final thermal equilibrium situation, the mixture can have ice and water both. In this case, the final equilibrium temperature would be $0°C$ (*melting point of water*).

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Illustration | 7 *A body A of mass 2 kg and specific heat capacity 5000 J kg^{-1}-°C^{-1} is kept in contact with another body B of mass 5 kg and specific heat capacity 7000 J kg^{-1}-°C^{-1}. The initial temperature of A is 120°C while that of B is 30°C. Find the final equilibrium temperature.*

Solution Initially temperature of A is greater than the temperature of B, so heat starts flowing from A to B, as a result, the temperature of A decreases while that of B increases. Hence, the body A loses heat while body B gains heat.

Let the final equilibrium temperature be T, then

Heat lost by A,

$$Q_1 = m_A s_A (120 - T)$$
$$= 2 \times 5000(120 - T)\,\text{J}$$

Heat gained by B,

$$Q_2 = m_B s_B (T - 30)$$
$$= 5 \times 7000(T - 30)\,\text{J}$$

From principle of calorimetry, $Q_1 = Q_2$

$$\Rightarrow \quad 2 \times 5000(120 - T) = 5 \times 7000(T - 30)$$
$$\Rightarrow \quad T = 50°\text{C}$$

Illustration | 8 *The temperature of a 0.05 kg metal piece is raised to 200°C and this metal piece is then dropped into a beaker containing 0.4 kg of water initially at 20°C. If the final equilibrium temperature of the mixed system is 22.4°C, then find out the specific heat of the metal. Neglect any heat absorbed by the beaker.* (s_{water} = 4200 J kg^{-1}-°C^{-1}.)

Solution Here, the metal piece is at higher temperature, so it will lose heat and water will gain heat. Let s be the specific heat capacity of metal, then the heat released by metal piece is, $Q_1 = 0.05 \times s \times (200 - 22.4)$ J, while heat absorbed by water is,

$$Q_2 = 0.4 \times 4200 \times (22.4 - 20)\,\text{J}.$$

From principle of calorimetry, $Q_1 = Q_2$

$$\Rightarrow 0.05 \times s\,(200 - 22.4) = 0.4 \times 4200\,(22.4 - 20)$$
$$\Rightarrow \quad s = 453\ \text{J kg}^{-1}\text{-°C}^{-1}$$

Illustration | 9 *100 g of water at 50°C is mixed with 10 g of ice at 0°C. Determine the final equilibrium temperature.*
Take s_{water} = 1 cal-g^{-}-°C^{-1}, L_f = 80 cal g^{-1}.

Solution Here, the final temperature of the system must be between 0°C to 50°C, *ie*, water releases the energy and this released energy is absorbed by ice to melt and may be to increase the temperature of system from 0°C.

Heat required to melt 10 g of ice is,

$$Q_1 = 10 \times L_f = 10 \times 80 = 800\ \text{cal}$$

Let the final temperature of mixture be T°C, then the heat required to raise the temperature of melted ice (water at 0°C) to T°C is,

$$Q_2 = 10\,s\,(T - 0)$$
$$Q_2 = 10 \times 1 \times (T) = 10\,T$$

The heat released by water at 50°C to come to T°C is,

$$Q_3 = 100 \times s \times (50 - T) = 100(50 - T)$$

From the principle of calorimetry,

$$Q_3 = Q_1 + Q_2$$
$$\Rightarrow \quad 100(50 - T) = 800 + 10\,T$$
$$\Rightarrow \quad 5000 - 100T = 800 + 10\,T$$
$$\Rightarrow \quad 110T = 4200$$
$$\Rightarrow \quad T = 38.18°\text{C}.$$

Towards Proficiency Problems

Exercise 1

A. Subjective Discussions

1. A helium-filled balloon is released into the atmosphere. As the balloon rises it—(a) expands, (b) contracts, (c) remains unchanged. Explain your answer.
2. Why does an ordinary glass vessel break when placed on a lighted burner ?
3. Discuss the term heat.
4. A thermometer is laid out in direct sunlight. Does it measure the temperature of the air, or of the sun or of something else ? Explain.
5. Does it make sense to say that one body is twice as hot as another ? Why or why not ?
6. Would a mercury thermometer break if the temperature went below freezing temperature of mercury ? Why or why not ?
7. What is meant by thermal contact ? Is your pen is in thermal contact with you ?
8. Desert travelers sometimes keep water in a canvas bag. Some water seeps through the bag and evaporates. How does this cool the water inside ?

B. Numerical Answer Types

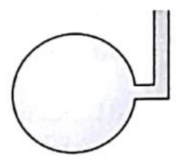
1. A pan of water is heated from 25°C to 80°C. What is the change in its temperature on the kelvin scale, and on the fahrenheit scale ?
2. A steel railroad track has a length of 30.0 m when temperature is 0°C. What is its length on a day when temperature is 40°C ? ($\alpha_{steel} = 11 \times 10^{-6}$/°C.)
3. A hole of cross-sectional area 100 cm^2 is cut into a steel plate at 20°C. What is the area of the hole if the steel is heated from 20°C to 100°C ? ($\alpha_{steel} = 11 \times 10^{-6}$/°C.)
4. For each of the following temperatures, find the equivalent temperature on the indicated temperature scale
   (a) –273.15°C on Fahrenheit scale
   (b) 98.6°F on Celsius scale
   (c) 100 K on Fahrenheit scale
5. The temperature difference between the inside and outside of an automobile engine is 450°C. Express this temperature difference on the (a) Fahrenheit scale, and the (b) Kelvin scale.
6. A copper telephone wire has no sag between poles 35 m apart on a winter day when the temperature is –25°C. Determine the length of wire on a summer day when temperature is 35°C. ($\alpha_{copper} = 17 \times 10^{-6}$/°C.)
7. A pendulum based clock is controlled by a swinging brass pendulum that is 1.3 m long at a temperature of 20°C.
   (a) What is the length of the pendulum rod when temperature drops to 0°C ?
   (b) If the pendulum's time period is given by $T = 2\pi\sqrt{\frac{L}{g}}$, will the clock lose or gain time as temperature changes? ($\alpha_{brass} = 19 \times 10^{-6}$/°C.)
8. A pair of eyeglass frames are made of plastic [$\alpha = 130 \times 10^{-6}$/°C]. At a temperature of 20°C the frames have circular lens holes 2.20 cm in radius. To what temperature must the frames be heated in order to insert lenses 2.21 cm in radius ?

9. A cube of solid aluminium has a volume of 1 m^3 at 25°C. What temperature change is required to produce a 100 cm^3 of increase in volume of the cube ? ($\alpha_{aluminium} = 24 \times 10^{-6}$/°C.)

10. A brass ring of diameter 10 cm at 20°C is heated and slipped over an aluminium rod of diameter 10.01 cm at 20°. (a) To what temperature must this combination by heated or cooled to separate them ? Is this possible ? (b) What if the aluminium rod is 10.02 cm in diameter?
($\alpha_{brass} = 19 \times 10^{-6}$/°C, $\alpha_{aluminium} = 24 \times 10^{-6}$/°C.)

11. An automobile fuel tank is filled to the brim with 45 L of gasoline at 10°C. If the vehicle is parked in open where the temperature is 35°C, then how much gasoline overflows from the vehicle's tank as a result of expansion ? Neglect the expansion of tank. ($\gamma_{gasoline} = 9.6 \times 10^{-4}$/°C.)

12. A liquid having coefficient of volume expansion γ just fills a spherical flask of volume V_0 at temperature T as shown in figure. The flask is made of a material having coefficient of linear expansion α. The flask is attached to a capillary of cross-sectional area A_0. If the temperature of the system is increased by ΔT, then determine the height to which liquid rises in capillary ? Neglect expansion of capillary.



13. How much energy is needed by a body of mass 12 kg to raise its temperature by 15°C ? Specific heat capacity of the material of body is 3000 J kg-°C^{-1}.

14. Determine the amount of energy released when 15 g of ice melts. (L_f = 80 cal g^{-1}.)

15. The temperature of how much water can be raised by 5°C by amount of heat released in above question ? (s_{water} = 1 cal g^{-1}.)

16. 100 g of ice at –20°C is mixed with 500 g of water at 80°C. Determine the final equilibrium temperature. (s_{water} = 1 cal/g-K, s_{ice} = 0.5 cal/g-K, L_f = 80 cal/g)

17. Lead pellets (small pieces) each of mass 1 g are heated to 250°C. How many pellets must be added to 500 g of water that is initially at 20°C to make the equilibrium temperature 25°C ? Neglect any energy transfer to or from the vessel. (s_{lead} = 128 J kg-°C^{-1}.)

18. A 50 g sample of copper is at 25°C. If 1200 J of energy is added to the copper sample by heat, what is its final temperature ? (s_{copper} = 387 J kg-°C^{-1}.)

19. An aluminium rod is 20 cm long at 20°C and has a mass of 350 g. If 10,000 J of energy is added to the rod by heat, then after giving the heat what will be the new length of rod ?
($s_{aluminium}$ = 900 J kg-°C^{-1}, $\alpha_{aluminium} = 24 \times 10^{-6}$/°C.)

20. A person's body converts chemical energy into internal energy at a metabolic rate of 100 kcal-h^{-1}. The person stands neck-deep in an insulated tub containing 0.8 m^3 of water at a temperature of 20°C.
Disregarding energy loss from the tub, determine the change of water temperature in 45 min.
(s_{water} = 1 cal g-°C^{-1}.)

21. A 200 g aluminium cup contains 800 g of water in thermal equilibrium with the cup at 80°C. The combination of cup and water is cooled uniformly so that the temperature decreases by 1.5 °C/min. At what rate is energy being removed ? ($s_{aluminium}$ = 0.215 cal- g^{-1}-°C^{-1}, s_{water} = 1 cal-g^{-1}-°C^{-1}.)

22. An aluminium cup contains 225 g of water and 40 g copper stirrer (spoon) all at 27°C. A 400 g sample of silver at an initial temperature of 87° is placed in the water. The final equilibrium temperature reaches 32°C. Determine the mass of aluminium cup. ($s_{aluminium}$ = 0.215 cal-g^{-1}-°C^{-1}, s_{water} = 1 cal g^{-1}-°C^{-1}, s_{copper} = 0.092 cal g^{-1}-°C^{-1}, s_{silver} = 0.056 cal g^{-1}-°C^{-1}.)

23. If 200 g of water is contained in a 300 g aluminium vessel at 10°C and an additional 100 g of water at 100°C is poured into the container, what is the final equilibrium temperature of the mixture ? ($s_{aluminium}$ = 0.215 cal g^{-1}-°C^{-1}, s_{water} = 1 cal g^{-1}-°C^{-1}.)

24. A 50 g ice cube at 0°C is heated until 45 g has become water at 100°C and 5 g has become steam at 100°C. How much energy was added to accomplish this ? (L_f = 80 cal-g^{-1}; L_V = 540 cal-g^{-1}).

25. A 100 g ice cube at 0°C is dropped into 1 kg of water which is at 80°C. Determine the final temperature of mixture. (L_f = 80 cal-g^{-1}; s_{water} = 1 cal-g^{-1}-°C^{-1}.)
26. How much energy is required to change a 40 g ice cube from ice at –10°C to steam at 110°C ?
(s_{ice} = 0.5 cal/g-°C; s_{water} = 1 cal-g^{-1}-°C^{-1}, L_f = 80 cal-g^{-1}, L_V = 540 cal-g^{-1};
s_{steam} = 0.48 cal-g^{-1}-°C^{-1}.)
27. What mass of steam initially at 130°C is needed to warm 200 g of water in a 100 g glass container from 20°C to 50°C ?
(s_{steam} = 0.48 cal-g^{-1}-°C^{-1}; s_{water} = 1 cal-g^{-1}°C^{-1}, L_V = 540 cal-g^{-1}; s_{glass} = 0.2 cal-g^{-1}-°C^{-1}.)
28. Determine the heat required to increase the temperature of 1 kg water by 20°C.
Take s_{water} = 1 cal/g -°C.
29. Find the change in potential (in cal) of a 10 kg mass after 10 m fall.
30. Find the percentage change in length of 1 m iron rod if its temperature changes by 100°C. α for iron is 2×10^{-5}/°C.
31. An iron block of mass 2 kg, fall from a height 10 m. After colliding with the ground it looses 25% energy to surroundings. Then find the temperature rise of the block. (Take specific heat of Fe = 470 J/kg°C)
32. A bullet of mass 10 g is moving with speed 450 m/s. Find its kinetic energy in calories.
33. Find the rise in temperature if 420 J of energy is supplied to 10 g water.
34. Calculate the heat released by 1 kg steam at 150°C if it converts into 1 kg water at 50°C. Take L_v = 540 cal/g, s_w = 1 cal/g°C, s_{steam} = 0.5 cal/g°C.
35. 1 kg ice at –10°C is mixed with 1 kg water at 100°C. Then find equilibrium temperature and mixture content.
36. If three different liquids of different masses, specific heat and temeprature are mixed with each other then what is the temperature mixture at thermal equilibrium?

C. Fill in the Blanks

1. As temperature increases from 0°C to 4°C, the density of water ....
2. The temperature of a body on celsius scale is 40°C, its reading on fahrenheit scale is ....
3. Two bodies are separated by a diathermic wall, then they would have .... temperature.
4. Linear expansion is possible only for ....
5. Volume expansion is possible in ....
6. Thermal expansion is maximum for .... and the least for ....
7. Value of specific heat capacity are .... for different phases of same substance.

D. True/False

1. Temperature is one of the seven fundamental quantities.
2. Temperature is a microscopic property.
3. The change in temperature on kelvin and celsius scales are the same.
4. Heat is the measure of temperature of a body.
5. In thermal equilibrium the temperature of the bodies may be different.
6. When there is no transfer of thermal energy from one body to another then it means two bodies are at the same temperature.
7. As water melts it releases energy.

High Skill Questions

Exercise 2

A. Only One Option Correct

1. If you were to make a very sensitive glass thermometer, then which of the following working liquids would you choose ?
   (a) Mercury (b) Alcohol
   (c) Gasoline (d) Glycerine

2. Two spheres are made of same material and have the same radius, but one is hollow and the other is solid. The temperature of both the spheres is increased by the same amount, then
   (a) the hollow sphere expands more
   (b) the solid sphere expands more
   (c) Both expand by the same amount
   (d) Information insufficient

3. The specific heat capacity of substance A is greater than that of substance B. These two substances are taken in same amount at same temperature initially and then to both same amount of energy is supplied. Then the final temperature of
   (a) A is greater
   (b) B is greater
   (c) Both would be at the same temperature
   (d) Information insufficient

4. An amount of energy is added to ice, raising its temperature from $-10°C$ to $-5°C$. A larger amount of energy is required by same amount of liquid water to raise its temperature from $15°C$ to $20°C$. From this we can conclude that
   (a) overcoming the latent heat of fusion requires an input of energy
   (b) the latent heat of fusion of ice delivers some energy to the system
   (c) the specific heat of ice is less than that of water
   (d) the specific heat of ice is greater than that of water

5. If two objects are in thermal equilibrium with each other
   (a) they can't be moving
   (b) they can't be undergoing an elastic collision
   (c) they can't have different pressures
   (d) they can't be at different temperatures

6. When two gases separated by a diathermic wall are in thermal equilibrium with each other,
   (a) only their pressures must be the same
   (b) only their volumes must be the same
   (c) only their temperatures must be the same
   (d) they must have same pressures and same volumes

7. Suppose object C is in thermal equilibrium with object A and also with object B. The zeroth law of thermodynamics states
   (a) that C will always be in thermal equilibrium with both A and B
   (b) that C can't transfer energy to anyone
   (c) that A is in thermal equilibrium with B
   (d) that A is in thermal equilibrium with C

8. The zeroth law of thermodynamics allows us to define
   (a) work
   (b) pressure
   (c) temperature
   (d) thermal equilibrium

9. In constructing a thermometer it is necessary to use a substance that
   (a) expands with rising temperature
   (b) expands linearly with rising temperature
   (c) undergoes some change when heated or cooled
   (d) will not boil

10. There is a temperature at which the reading on the kelvin scale is numerically
(a) equal to that on the celsius scale
(b) lower than that on the celsius scale
(c) less than zero
(d) equal to that on the fahrenheit scale

11. Fahrenheit and kelvin scales are in agreement at a reading of
(a) – 40 (b) Zero
(c) 574 (d) 273

12. Mark out the correct statement(s).
(a) Temperature differing by 25°C on the fahrenheit scale must differ by 45° on celsius scale
(b) Temperature which differ by 10° on celsius scale must differ by 18° on fahrenheit scale
(c) 40 K corresponds to –40°C
(d) 0°F corresponds to –32°C

13. A kelvin thermometer and a fahrenheit thermometer both give the same reading for a certain sample. The corresponding celsius temperature is
(a) 574°C (b) 301°C
(c) 276°C (d) 232°C

14. It is more difficult to measure the coefficient of expansion of a liquid than that of a solid because
(a) no relation exists between linear and volume expansion coefficients
(b) a liquid tends to evaporate
(c) a liquid expands too much when heated
(d) the containing vessel also expands

15. Thin strips of iron and zinc are riveted (joined) together to form a bimetallic strip that bends when heated. The iron is on the inside of the bend because
(a) it has lower coefficient of linear expansion
(b) it has higher coefficient of linear expansion
(c) it has high specific heat
(d) it has lower specific heat

16. A surveyor's 30 m steel tape is correct at 68°F. On a hot day the tape has expanded to 30.01 m. On that day, the tape indicates a distance of 15.52 m between two points. The true distance between these points is
(a) 15.50 m (b) 15.51 m
(c) 15.52 m (d) 15.53 m

17. The coefficient of linear expansion of iron is 10^{-5}/°C. The volume of an iron cube, 5 cm on edge will increase by what amount if it is heated from 10°C to 60°C ?
(a) 0.1875 cm^3 (b) 0.00375 cm^3
(c) 0.0225 cm^3 (d) 0.0625 cm^3

18. The mercury column in an ordinary medical thermometer doubles in length when its temperature changes from 95°F to 105°F. Choose the correct statement.
(a) The coefficient of volume expansion of mercury is 0.1/°F.
(b) The coefficient of volume expansion of mercury is 0.3/°F.
(c) The coefficient of volume expansion of mercury is $\frac{0.1}{3}$/°F.
(d) None of the above

19. Metal pipes used to carry water, sometimes burst in the winter because
(a) metal contracts more than water
(b) water expands when it freezes
(c) outside of the pipe expands more than the inside
(d) outside of the pipe contracts more than the inside

20. Heat is
(a) energy transferred by virtue of a temperature difference
(b) energy transferred by microscopic work
(c) energy content of an object
(d) None of the above

21. The specific heat of an object is
(a) the amount of heat energy required to change the state of 1 g of substance
(b) the amount of heat energy required to raise the temperature of unit mass of substance by 1°C
(c) the temperature of the object divided by its mass
(d) None of the above

22. Two different objects have same initial temperature their masses also being the same. When same amount of energy is transferred to them, then final temperature of both of them are different. This may be because of their

(a) same specific heats
(b) different specific heats
(c) same coefficient of expansion
(d) different coefficients of expansion

23. The energy given off as heat by 300 g of an alloy as it cools through 50°C raises the temperature of 300 gm of water from 30°C to 40°C. The specific heat of water is 1 cal-g^{-1}-°C^{-1}. The specific heat of the alloy in cal-g^{-1}-°C^{-1} is

(a) 0.015 (b) 0.10
(c) 0.15 (d) 0.20

24. Object A with heat capacity (product of mass and specific heat) C_A and initially at temperature T_A, is placed in thermal contact with object B, with heat capacity C_B and initially at temperature T_B. If no phase change occurs, then final common temperature is

(a) $\dfrac{C_AT_A + C_BT_B}{C_A + C_B}$ (b) $\dfrac{C_AT_A - C_BT_B}{C_A + C_B}$
(c) $\dfrac{-C_AT_A + C_BT_B}{C_A + C_B}$ (d) None of these

Answers

Towards Proficiency Problems
Exercise 1

B. Numerical Answer Types

1. 55 K and 101°F **2.** 30.0132 m **3.** 100.176 cm^2
4. (a) 0K, (b) 37°C, (c) −279.67°F **5.** (a) 810, (b) 450 **6.** 35.0357 m
7. (a) 1.299506 m, (b) Gains time **8.** 55° C **9.** 1.4°C
10. (a) −179°C, (b) −376.2°C **11.** 1.08 L **12.** $\dfrac{V_0(\gamma - 3\alpha)\Delta T}{A_0(1+2\alpha\Delta T)}$
13. 540 kJ **14.** 1200 cal **15.** 240 g **16.** 51.67°C **17.** 365
18. 87°C **19.** 20.01524 cm **20.** 93.75°C **21.** 1264.5 cal/min
22. 186.6 g **23.** 34.7°C **24.** 11700 cal **25.** 65.45°C
26. 25.792 kcal **27.** 10.42 g

C. Fill in the Blanks

1. Increases **2.** 104°F **3.** Same **4.** Solids
5. All three states of matter **6.** Gases, Solids **7.** Different

D. True/False

1. T **2.** F **3.** T **4.** F **5.** F **6.** T **7.** F

High Skill Questions
Exercise 2

A. Only One Option Correct

1. (c) **2.** (c) **3.** (b) **4.** (c) **5.** (d) **6.** (c) **7.** (c) **8.** (c) **9.** (c) **10.** (d)
11. (c) **12.** (b) **13.** (b) **14.** (d) **15.** (a) **16.** (b) **17.** (a) **18.** (d) **19.** (b) **20.** (a)
21. (b) **22.** (b) **23.** (d) **24.** (a)

Explanations

Towards Proficiency Problems
Exercise 1

Numerical Answer Types

1. $\dfrac{C}{5} = \dfrac{F-32}{9} = \dfrac{K-273.15}{5}$

If, $C = 25$, then $F = 77$ and $K = 298.15$

If, $C = 80$, then $F = 176$ and $K = 353.15$

So, $\Delta F = 101$ and $\Delta K = 55$

2. $l = l_0 [1 + \alpha \Delta t]$

$= 30 [1 + 11 \times 10^{-6} \times 40] = 30.0132$ m

3. $A = A_0 [1 + 2\alpha \Delta T]$

$= (100 \text{ cm}^2)[1 + 2 \times 11 \times 10^{-6} \times 80]$

$= 100.176 \text{ cm}^2$

4. Use the concept and formula of Q. No. 1.

6. $l = l_0 [1 + \alpha \Delta T]$

$= (35 \text{ m})[1 + 17 \times 10^{-6} \times 60]$

$= 35.0357$ m.

7. $l = (1.3 \text{ m})[1 - 19 \times 10^{-6} \times 20]$

$= 1.299506$ m.

As temperature decreases, the time-period of pendulum decreases and hence pendulum-based clock gains time.

8. $2.21 \text{ cm} = (2.20 \text{ cm})[1 + 130 \times 10^{-6} \Delta\theta]$

$\Rightarrow \quad \Delta\theta = 34.96 \cong 35°C$

$\theta - 20 = 35$

$\Rightarrow \quad \theta = 55°C$

9. $\Delta V = V_0 \gamma \Delta\theta$

$\Rightarrow \quad 100 \times 10^{-6} = 1 (3 \times 24 \times 10^{-6}) \Delta\theta$

$\Rightarrow \quad \Delta\theta = 1.4°C$

10. As $\alpha_{\text{aluminium}}$ is greater than α_{brass}, the expansion in aluminium is greater than expansion in brass for a given increase in temperature. As initially diameter of brass ring is less than the diameter of aluminium rod, so the temperature of system has to be decreased to separate them out. Let the final temperature be θ, then at this temperature, the diameter of both the ring and rod would be the same.

$10 \text{ cm} [1 + \alpha_{\text{brass}} (\theta - 20)]$

$= (10.01 \text{ cm}) [1 + \alpha_{\text{aluminium}} (\theta - 20)]$

$\Rightarrow \quad \theta \cong -179°C$

For $\quad 10.02$ cm, $\theta = -376.2°C$

11. $\Delta V = V_0 \gamma \Delta T$

$= (45 \text{ L})(9.6 \times 10^{-4}) \times (35 - 10) = 1.08$ L

12. $\Delta V = V_0 (\gamma - 3\alpha) \Delta T$, where ΔV is amount of liquid that will come into capillary tube.

$\Delta V = A \times h = A_0 [1 + 2\alpha \Delta T] \times h$

$\Rightarrow \quad h = \dfrac{V_0 (\gamma - 3\alpha) \Delta T}{A_0 [1 + 2\alpha \Delta T]}$

13. $\Delta Q = ms\Delta T = 12 \times 3000 \times 15 = 54 \times 10^4$ J

14. $\Delta Q = mL_f = 15 \times 80 \text{ cal} = 1200$ cal

15. $1200 \text{ cal} = m \times (1 \text{ cal/g}) \times 5$

$\Rightarrow \quad m = 240$ g

16. The heat required to bring ice to 0°C is,

$Q_1 = 100 \times 0.5 \times 20 = 1000$ cal.

The heat required to melt ice is,

$Q_2 = 100 \times 80 = 8000$ cal

The heat required to bring water at θ°C is

$Q_3 = 100 \times 1 (\theta)$ cal

$Q = Q_1 + Q_2 + Q_3 = (9000 + 100\theta)$ cal.

The heat released to bring water at 80°C to θ°C is,

$Q' = 500 \times 1 \times (80 - \theta)$.

From principle of calorimetry,

$Q = Q'$

$\Rightarrow \quad 9000 + 1000 = 40000 - 5000$

$\Rightarrow \quad \theta = \dfrac{3100}{600} = 51.67°C$

17. Let n pellets are required

$\dfrac{1}{1000} \times n \times 128 \times 225 = \dfrac{500}{1000} \times 4200 \times 5$

$\Rightarrow \quad n = 364.6$

So, 365 pellets are required.

18. $1200\text{ J} = \frac{50}{1000} \times 387(\theta - 25)$

$\Rightarrow \quad \theta = 87.02°\text{C}$

19. The new temperature of the rod is given by,

$0.350 \times 900(\theta - 20) = 10000$

$\Rightarrow \quad \theta = 51.75°\text{C}$

New length is given by,

$l = 20\text{ cm}[1 + 24 \times 10^{-6} \times 31.75]$

$= 20.01524\text{ cm}$

20. In 45 min, the energy lost by person's body is 100 k cal/h × 3/4 h = 75 kcal. Let temperature of water changes by $\Delta\theta$, then heat taken by water is,

$Q = (0.8 \times 1 \times 1000) \times 1 \times \Delta\theta = 75 \times 10^3$

$\Rightarrow \quad \Delta\theta = 93.75°\text{C}$

21. Rate at which energy is removed,

$\frac{\Delta Q}{\Delta t} = (200 \times 0.215 \times 1.5$

$+ 800 \times 1 \times 1.5)\text{ cal min}^{-1}$

$= 1264.5\text{ cal min}^{-1}$

22. Use the principle of calorimetry.

23. $200 \times 1 \times (\theta - 10) + 300 \times 0.215 \times (\theta - 10)$

$= 100 \times 1 \times (100 - \theta)$

$\Rightarrow \quad \theta = 34.7°\text{C}$

24. Total energy required is,

$Q = (50 \times 80 + 50 \times 1 \times 100 + 5 \times 540)\text{ cal}$

$= 11700\text{ cal}$

25. Let final temperature be θ,

$100 \times 80 + 100 \times 1(\theta) = 1000 \times 1 \times (80 - \theta)$

$\Rightarrow \quad \theta = 65.45°\text{C}$

26. Total energy required to convert 40 g of ice at −10°C to steam at 110°C is,

$Q = Q_1 + Q_2 + Q_3 + Q_4 + Q_5$

where $Q_1 \rightarrow$ the heat required to bring the temperature of ice to 0°C

$Q_2 \rightarrow$ the heat required to melt ice

$Q_3 \rightarrow$ the heat required to bring the temperature of water to 100°C

$Q_4 \rightarrow$ the heat required to vapourize water

$Q_5 \rightarrow$ the heat required to heat the steam

$Q = \left(40 \times \frac{1}{2} \times 10 + 40 \times 80 + 40 \times 1 \times 100\right.$

$\left. + 40 \times 540 + 40 \times 0.48 \times 10\right)\text{ cal}$

$= 25.792\text{ kcal}$

27. Let m gram of steam is needed, then

$m \times 0.48 \times 30 + m \times 540 + m \times 1 \times 50$

$= 200 \times 1 \times 30 + 100 \times 0.2 \times 30$

$\Rightarrow \quad m = 10.42\text{ g}$

28. $Q = ms\Delta\theta$

$= (1\text{ kg}) \times (1\text{ cal/g-°C}) \times 20\,(°\text{C})$

$= (1\text{ kg}) \times (1\text{ kcal/kg-°C}) \times 20°\text{C}$

$= 20\text{ kcal}$

29. Change in potential energy

$\Delta U = mgh = 10 \times 10 \times 10 = 1000\text{ J}$

$= \frac{1000}{4.186}\text{ cal}$

30. Percentage change in length due to temperature change

$\%\, l = \frac{\Delta l}{l} \times 100 = \alpha\,\Delta\theta \times 100$

$= 2 \times 10^{-5} \times 100 \times 100 = 0.2\%$

31. $ms\,\Delta\theta = \frac{1}{4} mgh.$

$\Rightarrow \quad \Delta\theta = \frac{10 \times 10}{4 \times 470} = 0.053°\text{C}$

32. $\Delta k = \frac{1}{2} \times \frac{10}{1000} \times 400 \times 400 = 800$

$\frac{800}{4.2} = 191.11\text{ cal}$

33. $\frac{420 \times 10^{-3}}{4.2} = 10 \times 10^{-3} \times 1 \times \Delta t = 10°\text{C}$

34. $H = 1 \times \frac{1}{2} \times 50 + 1 \times 540 + 1 \times 1 \times 50$

$= 540 + 75 = 615\text{ kcal}$

35. Heat given by 1 kg ice $= 1 \times \frac{1}{2} \times 10 = 5\text{ kcal}$

$5 + 1 \times 80 + 1 \times T = 1 \times (100 - T)$

$85 = 100 - 2T \Rightarrow 2T = 15$

$8 = 152 = 7.5°\text{C}$, water

36. $m_1, s_1, T_1 \rightarrow$ specification for liquid 1

$m_2, s_2, T_2 \rightarrow$ specification for liquid 2

$m_3, s_3, T_3 \rightarrow$ specification for liquid 3

Total heat lost or gained by all substances is zero *ie*, $\Delta Q = 0$

$m_1s_1(T - T_1) + m_2s_2(T - T_2)$

$+ m_3s_3(T - T_3) = 0$

$\therefore \quad T = \frac{m_1s_1T_1 + m_2s_2T_2 + m_3s_3T_3}{m_1s_1 + m_2s_2 + m_3s_3}$

Chapter 13

Electrostatics

The First Steps' Learning

- Concept of Charge
- The Process of Charging
- Conductors and Insulators
- The Meaning of Electrostatics
- Coulomb's Law of Attraction
- The Principle of Superposition of Electric Forces
- Electric Field
- Electric Potential Energy
- Electric Potential
- Behavior of Conductors in Electrostatics

Let us imagine a very difficult life—a life without fan, tube lights, bulb, torch, refrigerators, automobiles without headlights or rather you say no automobiles, computers, television or any other sort of electronic media, radio, geyser, heater, iron etc. Can you imagine your life without these necessary appliances ? Answer would be quiet obvious, no. Now you can think why this type of useless question has been asked ? The reason is that all these things work on some very important principles of physics which are the subject matter if electromagnetism in physics, and if this part of physics plays such an important role in our daily lives, then how can we ignore the importance of all these.

Electricity and magnetism are two broad branches under which electromagnetism is generally studied. The basic source of all electrical and magnetic phenomena is ***charge****, an attribute which is as fundamental as mass.*

Concept of Charge

As we already know that an atom consists of a tiny nucleus (containing protons and neutrons) and electrons, in which electrons revolve around the nucleus. The masses of individual particles of an atom are as follows :

Mass of electron= $m_e = 9.10938 \times 10^{-31}$kg

Mass of proton = $m_p = 1.67262 \times 10^{-27}$ kg

Mass of neutron= $m_n = 1.67493 \times 10^{-27}$ kg

It is clear from above data that m_p and m_n are approximately equal and mass of e^{-1} is approximately $\frac{1}{1840}$ times that of mass of proton and neutron.

As we already know about gravitational force, we expect that when two electrons are placed at a distance of 1 m, they should attract each other with a force of the order of 10^{-71} N. But experimental results were quiet far away from the expected result. From experiments, it has been found that when two electrons are placed at a distance of 1 m they repel each other with a force of the order of 10^{-27} N, *ie*, the experimental and expected results differ in two aspects.

First one the nature of force *ie*, we were expecting attractive force but we got repulsive force, and second is the magnitude of force. Similar type of contradictions have been observed when two protons or one proton and one electron are considered. But it has also been observed that there is no contradiction between expected and experimental results when two neutrons or one neutron and one electron (or proton) have been taken for consideration. And we know that development of science is based on experiments, so after these experiments, physicists start thinking about the reason for this discrepancy and the reason of this discrepancy comes in the form of charge. Charge is one of the most fundamental attributes of the matter just like mass is, just like mass electric charge is an intrinsic property.

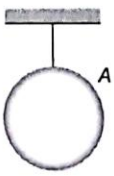
The existence of charge was discovered as early as 600 BC by Greek philosophers. They observed that when amber was rubbed with certain objects, the amber could then attract other objects, however the cause of this attraction was still unexplained. Later on in 16^{th} century, an english physicist William Gilbert studied electrical and magnetic phenomena systematically and explained the cause of Greek's observation more logically. He showed that many substances besides amber acquire an attractive property when being rubbed. He is the one who introduced first the terms like electrical interaction, electric force etc.

Today it has been well established that electron and proton are the basic particles to possess charge while neutron is a neutral

particle. The charge of an electron is -1.6×10^{-19} C while that on a proton is $+1.6 \times 10^{-19}$ C. C stands for coulomb which is the SI unit of charge. In above lines the '+ and –' sign used in mentioning the charge on proton and electron respectively is only signifying that there are two types of charges that exist in nature. It is only way to distinguish between two types of charges, (we can also call them as type A charge and type B charge or any other name if we want), and is not a way to tell that +ve charge is greater than –ve charge or charge of proton is greater than the charge of an electron.

Properties of Charge

1. Charge is of two types *ie*, +ve and –ve as explained above.
2. Charge of an electron is -1.6×10^{-19} C and that of proton is $+1.6 \times 10^{-19}$ C. Coulomb (C) is the SI unit of charge.
3. Like charges repel each other while unlike charges attract each other, *ie*, two electrons or two protons repel each other while an electron and a proton attract each other.
4. The charge on an electron is the smallest amount of free charge that has been discovered. In other words, we can say that charge on any body is always equal to integral multiples of electronic charge (charge on an electron), *ie*, $q = \pm ne$ where q is the charge on any body, n is an integer and e is equal to 1.6×10^{-19} C.
   This statement is also termed as **quantization of charge**.
5. Charge of an isolated system remains constant (conserved). It means if any system is isolated from its surrounding electrically (*ie*, there is no electrical contact between the system and surroundings), then charge of the system will remain conserved although the distribution of charge among various parts of the system can change. This statement is known as **law of conservation of charge**. The law of conservation of charge can also be stated as–"charge can never be created nor be destroyed but can only be transferred from one body to another". For example, consider a body A which possesses the charge q on it, if we suspend it from roof with the help of an insulating thread (say of plastic) then after a very long time also its charge remains same as q. [In this example, we neglected the leakage of charge due to atmosphere present, which is generally the case].

   Regarding insulators we shall discuss later on in the same chapter.
6. A charge produces electric and magnetic fields and also emits electromagnetic radiations (waves). A stationary charge produces electric field alone, while a uniformly moving charge produces both electric and magnetic fields, and an accelerated charge produces electric and magnetic fields, and emits electromagnetic waves as well.
7. Mass can exist without charge but charge can't exist without mass *ie*, charge is the property of a material particle only. For example, if we say that an object is massless *ie*, its mass is zero then it can't have any charge, because if it is having charge then it means that it would have some mass also as both electrons and protons (basic constituent particles which possess charge) have certain a non-zero mass.
8. Two electrically charged objects exert a force on one another, this force is termed as electric force, and is given by Coulomb's law (to be discussed soon). For example, if two balls are suspended by two insulating threads, both having some charge and are allowed to move freely, then due to electrical force, the situation would be as shown in figure.

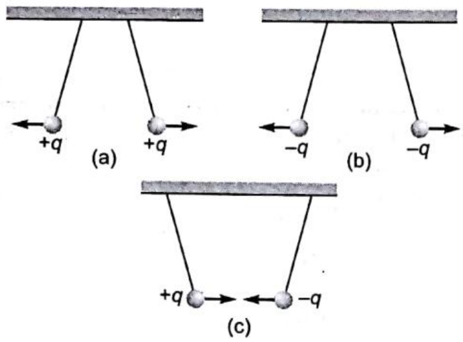


Fig. 13.1

In Fig. 13.1 (a) both the balls are +vely charged, and hence the balls repel each other. In Fig. 13.1 (b) both the balls are −vely charged and hence repel each other while in Fig. 13.1 (c) the two balls attract each other as they are having opposite charges.

9. Charge on a body is always due to excess or deficiency of electrons, as compared to the number of electrons in a neutral object. We know that matter is electrically neutral as number of electrons in the matter is same as that of number of protons. As electrons are free to move, while protons are tightly bounded with neutrons in the nucleus, so either the substance can have excess or deficiency of electron due to addition or removal of electron from neutral matter.
If number of electrons > number of protons, then the object is negatively charged.
If number of electrons = number of protons, then the object is neutral.
If number of electrons < number of protons, then object is positively charged.

10. If we say that a body is having a charge say $+Q$ or $-Q$, then it corresponds to excess charge which the body possesses. The total −ve charge or total +ve charge of the body would be very large as compared to excess charge or simply the charge of body. Thus, we can say $Q = |n_p \times e - n_e \times e|$ where Q is the excess charge of the body, n_p and n_e are the number of protons and electrons respectively in the body and e is the electronic charge. The modulus sign is used to give only the magnitude of the charge.

C-BIs

Concept Building Illustrations

Illustration | 1 *A body is having a charge of 1C, how many less or more electrons will constitute this charge ?*

Solution As body is having +1C charge, so it means the body is having lesser number of electrons. Let n be the number of electrons, then

$$1\text{ C} = n \times 1.6 \times 10^{-19}\text{ C}$$

$$\Rightarrow \quad n = 6.25 \times 10^{18}$$

*From above figure it is clear that 1 C of excess charge on a body is due to very large number of electrons.

Illustration | 2 *A body is having 3×10^{20} excess electrons, then determine the charge on the body.*

Solution From $Q = ne$

$$Q = 3 \times 10^{20} \times 1.6 \times 10^{-19}\text{ C}$$
$$= 48\text{ C}$$

As the electrons are in excess, the charge of the body is −ve and hence the charge of the body is − 48 C.

The Process of Charging

As we know that an atom is electrically neutral (because in an atom the number of electrons are the same as that of number of protons), and all matter is made up of atoms, so matter or any other material object is neutral (electrically) in nature. But, when we add/remove some electrons to/from the substance, then it acquires some net charge which may be +ve/–ve depending on the fact that whether the electron are removed or added. Now the question arises, how to provide a net charge to the body—"The process by which we provide a net charge to a body is termed as charging". There are numerous ways to provide a net charge to a body, but the three most common ways are

1. Frictional electricity
2. Charging by conduction
3. Charging by induction

Frictional Electricity

As we have seen earlier that when we rub two bodies then they get charged, the charging done in this way is termed as frictional electricity or charging by rubbing. Actually what happens, when we rub two bodies against each other is that they get heated up due to friction and internal energy of the bodies increase as a result of which electrons from one body are transferred to the other body, and the bodies get charged.

The body which loses electrons acquires +ve charge due to the removal of electrons from it or we can say due to deficiency of electrons, while the body which gains the electrons acquire –ve charge due to excess of electrons. In this type of charging the two bodies acquire equal and opposite charges as equal number of electrons are removed from one body and given to the other body. During charging by rubbing, masses of the bodies change slightly.

Let us consider two bodies A and B having masses M_1 and M_2 respectively, and when they are rubbed against each other say n electrons get transferred from A to B. Then it means the mass of body A is decreased by mass of n electrons, while mass of body B increases by the same amount.

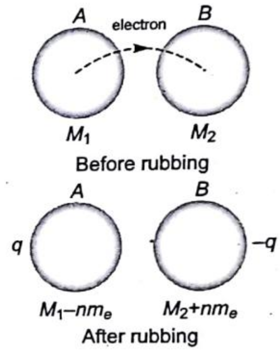


Fig. 13.2 Charging of bodies by rubbing.

So, final mass of $A = M_1 - nm_e$, and final mass of $B = M_2 + nm_e$

where m_e is the mass of an electron.

The charge acquired by A is, $q = ne$ where $e = 1.6 \times 10^{-19}$ C and the charge on B is $-q$.

Charging by Conduction

In this method when a neutral body (which has to be charged) is kept in **electrical contact** with some charged body, then transfer of charge takes place from one body to other till both acquire the same potential (regarding potential we will talk soon). In this way the initially uncharged body acquires a net charge. In this type of charging, the mass of both the bodies change by the same amount.

Electrical contact means the transfer of charge can take place between two bodies.

Let us consider an uncharged body A which is kept in contact with another charged body B having charge $+Q$, then transfer of electrons takes place from A to B, and thus the body A acquires a net +ve charge. As the transfer of electrons takes place from one body to other, the mass of both the bodies changes by

the same amount. In charging by conduction both the bodies acquire the same type of charge.

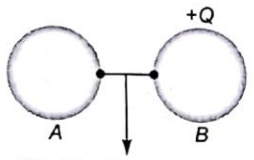


Fig. 13.3 Metallic wire to keep two bodies in electrical contact

Charging by Induction

You may have seen in your daily life that if you pass the comb through your dry hair, and then bring the comb near to certain pieces of paper, then the comb will attract these pieces of paper. Now here we are going to explain you the concept behind this attraction. As we have already seen in frictional electricity, when an object is rubbed against the other both of them get charged. So in above example as the comb is passed through dry hair it gets charged. Now the question arises, why the charged comb attracts neutral paper pieces ? The answer of this question lies in the fact that electric force is inversely proportional to square of distance between the charged particles (will be studied in Coulomb's law), and displacement of charged particles (electrons) within the object when they experience an electric force.

For better understanding of this part it is suggested to go for the next section first.

Let us consider the above mentioned case in greater detail, if the comb acquired +ve charge on rubbing with hair and is brought near to neutral paper pieces, then due to the electric force experienced by +ve and –ve charge constituents of the paper pieces (due to +vely charged comb), they will get displaced as shown in the figure. The surface nearer to the comb acquires some –ve charge and the farther surface acquire +ve charge, although the net charge of paper pieces remains zero as no net charge has been added or removed from it. Thus, we can say that the charge gets redistributed within the paper piece due to presence of a charged comb nearby, this redistribution of charge within the body due to presence of a charged body nearby is termed as **induction**. As the coulomb's force is inversely proportional to square of distance between the charges, the attraction force between comb and paper piece is greater than the repulsive force between them, and hence the paper piece gets attracted towards the comb.

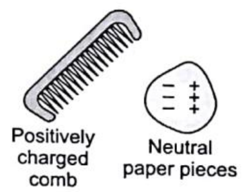


Fig. 13.4

How to give a net charge to a body with the help of induction process, is still an unanswered question. To understand the process of providing a net charge to a body with the help of induction, requires the concept of grounding (earthing) which is somewhat difficult to understand at this stage, so we are skipping this portion here.

Conductors and Insulators

We have talked earlier many times about electrical contact, now you can understand it clearly after going through this section. In any material object, not only electric charge exists, but it can also move/flow through an object. On this particular basis whether charge can flow through an object or not, all the substances have been classified into two categories—namely *conductors* and *insulators*.

Although, the third category is also there namely the semiconductors, but this is not our concern here.

Conductors

Those substances through which electric charges can flow easily are termed as conductors or electrical conductors. All metallic substances like silver, copper, aluminium, iron, gold etc are very good electrical conductors, and thus used in electrical wiring.

Insulators

Those substances through which electric charge can't flow are termed as insulators. Substances like wood, plastics, rubber, wax, glass, nylon etc are examples of good electric insulators.

You all may have seen the wires used in electrical wiring in houses, which are coloured say like red, yellow, green etc which from outside are generally made of vinyl etc which is a very good electrical insulator. If we break it, then you will be able to see the brown coloured copper wires through which the charge flows. You may also have noticed the plug in various electrical appliances in which the pins are made up of metal (conductor) and the casing which you hold is generally of plastic (an insulator).

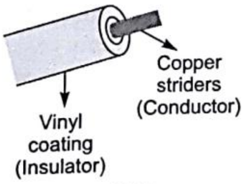


Fig. 13.5

Now the question arises—why different substances behave differentially ? The answer of this question is related to the atomic structure. In an atom, the electrons revolve around the nucleus in different shells (orbits), the electrons in outer orbits experience weaker forces due to positively-charged nucleus as compared to inner orbit electrons, as a result of which the outermost electrons (also termed as valence electrons) can be detached from the atom more easily than the inner ones. In conductors, some valence electrons get detached from the parent atom, and move freely throughout the material, and now don't belong to any particular atom. The ready flow of electrons in a material shows that the substance is a good electrical conductor. In insulators electrons are tightly bound to the nucleus, and are not free to move about.

Now we are going to discuss the meaning of term *electrical contact*. We place two vessels of cross-sectional areas A_1 and A_2 containing liquid upto different heights connected by a tube of negligible cross-sectional area with a stopcock as shown in the figure. Now, we open the stopcock, and see what happens—the water flows from the vessel having higher water level to the vessel having lower water level, and this process continues till both acquire the same water level or in other words we can say that when two vessels initially having different PEs (gravitational) are connected together, then finally both acquire the same gravitational PE although both the vessels have different amounts of water finally.

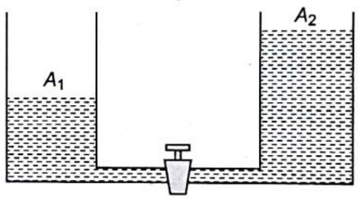


Fig. 13.6

In the same way, let us consider two charged bodies having charges Q_1 and Q_2 as shown in the figure. If we connect them by a conducting wire (metallic wire say of copper or aluminium), then the transfer of electrons (–ve charge) takes place from the one having lower potential to the other having higher potential till both acquire the same potential. In the above described situation in which if transfer of charge can take place from one body to another, then the bodies are said to be in electrical contact. If we connect above two charged spheres by a plastic wire then transfer of electron can't take place, and the bodies are not said to be in electrical contact.

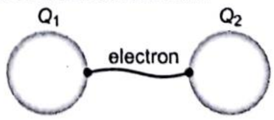


Fig. 13.7

The Meaning of Electrostatics

In this chapter now we are going to deal with the effects of stationary charges only and that is why this branch of physics is known by the name electrostatics. The word "electrostatics" itself tells about its domain, if we break the term electrostatics then it would be *electron+statics*. The word electron is directly a measure of charge and statics means at rest, so it means electrostatics deals with charges at rest or stationary charges. In this chapter, we are first going to look into the electrostatic coulomb's force, and then the effect of stationary charges in the form of electric field, and at last we will find the most important part of this chapter *ie*, the behaviour of conductors in electrostatics.

Coulomb's Law of Attraction

This law gives us the measure of electric force between two point charges. When the size of the bodies are much smaller than the distance separating them, the size of the bodies may be ignored and the charged bodies are called **point charges**.

According to Coulomb's law—**"The electric force of interaction between two point charges is directly proportional to product of their charges and inversely proportional to square of distance between them".**

Consider two point charges q_1 and q_2 placed at a distance r apart as shown in Fig. 13.8. Then according to Coulomb's law, the electric force F between them is,

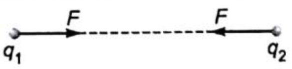


Fig. 13.8

$$F \propto q_1 q_2, \quad \text{and} \quad F \propto \frac{1}{r^2}$$

Combining above two equations, we have

$$F \propto \frac{q_1 q_2}{r^2}$$

By removing the proportionality sign, we would have

$$F = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r^2} = k \times \frac{q_1 q_2}{r^2}$$

where $k = \frac{1}{4\pi\varepsilon_0}$ is the proportionality constant in SI units, and its value is equal to 9×10^9 N-m^{2}C^{-2}. The above proportionality constant is to be used when there is vacuum in between the two charges. The symbol ε_0 pronounced as 'epsilon-naught' is the permittivity of the free space (vacuum). Its value is 8.85×10^{-12} C^{2}N^{-1}-m^{-2}.

If some medium is present in between the charges, say water, glass etc, then ε_0 would be replaced by ε, where ε is the permittivity of medium and is equal to product of relative permittivity of medium with permittivity of free space *ie*, $\varepsilon = \varepsilon_r \times \varepsilon_0$ where ε_r is relative permittivity of the medium.

Now we are discussing some important properties about this force

- Coulomb's force acts along the line joining two charges, *ie*, electric force experienced by one point charge due to other is acting along the line joining the charges.
- Electric force between two charges is an action-reaction pair. The force experienced by a point charge A due to B is equal in magnitude and opposite in direction to the force experienced by B due to A *ie*, $\vec{F}_{AB} = -\vec{F}_{BA}$.

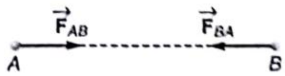


Fig. 13.9

- Electric force is repulsive if the charges have the same sign, and attractive if charges have opposite signs.

- Its value depends on the nature of medium present in between the charges.
- Coulomb's law is valid only for point charges.
- It is an example of action at a distance just like gravitational force. It means two charged particles are exerting a force on each other even when they are kept at certain distance apart.
- Electric force is a conservative force *ie*, work done by this force along a closed path is zero.

The Principle of Superposition of Electric Forces

Just like gravitational force, electric force also obeys the principle of superposition. According to this principle—"If more than two point charges are placed at certain distance apart, then the net force (electric) experienced by any charge is equal to the vector sum of the electric force experienced by it due to remaining individual charges".

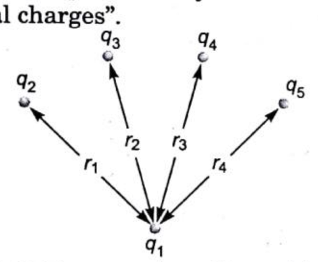


Fig. 13.10 Force experienced by a point charge due to a system of point charges.

For example, if 5 point charges are placed as shown in Fig. 13.10, then the electric force experienced by q_1 is given by

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \vec{F}_{15}$$

where,

$\vec{F}_1$ is net electric force experienced by q_1,

$\vec{F}_{12}$ is electric force experienced by q_1 due to q_2,

$\vec{F}_{13}$ is electric force experienced by q_1 due to q_3,

$\vec{F}_{14}$ is electric force experienced by q_1 due to q_4,

and

$\vec{F}_{15}$ is electric force experienced by q_1 due to q_5

$$\vec{F}_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_1^2}\,\hat{r}_1 + \frac{q_1q_3}{r_2^2}\,\hat{r}_2 + \frac{q_1q_4}{r_3^2}\,\hat{r}_3 + \frac{q_1q_5}{r_4^2}\,\hat{r}_4\right]$$

Here, unit vectors $\hat{r}_1, \hat{r}_2, \hat{r}_3, \hat{r}_4$ represent the directions of $\vec{F}_{12}$, $\vec{F}_{13}$, $\vec{F}_{14}$ and $\vec{F}_{15}$, respectively.

It is to be kept in mind that in vector form the charges have to be substituted with appropriate signs.

In other words, we can state the principle of superposition as—

"The presence of other charges can't effect the electric force of interaction between the two charges". For example, if two charges A and B are kept at some separation, and the electric force experienced by

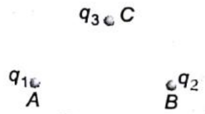


Fig. 13.11

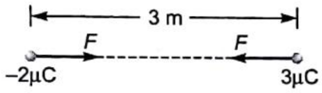
A due to B is $\vec{F}$, and now we bring another charge C nearby to A and B, then also the force experienced by A due to B remains the same as $\vec{F}$, although the net force experienced by A changes.

C-BIs

Concept Building Illustrations

Illustration | 3 *Two point charges* $-2\,\mu C$ *and* $3\,\mu C$ *are placed at a separation of* 3 *m in vacuum. Determine the electrical force between them. If the mass of the particle having charge* $-2\,\mu C$ *is* 2 *kg, then determine the acceleration of this particle at this instant. Can we determine the acceleration of other particle from this information ? If yes, then what would be its acceleration ? Assume no other force except the electric force acting on the charged particles.*

Solution As the charges are of opposite nature, the electric force would be attractive in nature. When the separation between them is 3 m, both of them will experience a force F in the direction shown in the figure. Magnitude of F is given by



$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

$$= \frac{9 \times 10^9 \times (2 \times 10^{-6})(3 \times 10^{-6})}{3^2}$$

$$= 6 \times 10^{-3}\text{ N}$$

So, both the particles will experience a force of 6×10^{-3} N towards each other when they are at a separation of 3 m. As these are free to move (as mentioned in the question) because there are no other forces acting on the charged particles, so both should possess some acceleration.

When the separation between them is 3 m, the free body diagram of the two particles would be like as shown in the figure.

For particle of mass 2 kg,

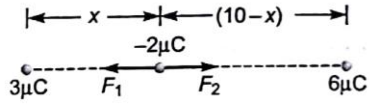
$$\text{acceleration} = \frac{F}{2} = \frac{6 \times 10^{-3}}{2} = 3 \times 10^{-3}\text{ ms}^{-2}$$

As mass of 3 μC charge is not known, so we can't find the acceleration of this charged particle.

It has to be kept in mind that as the particle moves under the action of an electric force, the separation between the particles changes, and thus the force acting on them, and hence the acceleration of particles.

Illustration | 4 *Two point charges 3 μC and 6 μC are kept at a seperation of 10 m. Another point charge of charge −2 μC is to be placed on the line joining two charges in such a way that −2 μC charge doesn't experience any net electric force. Determine the location of the −2 μC charge wrt 3 μC charge.*

Solution Let us place the −2 μC charge is placed at a distance of x m from 3 μC charge, so its distance from 6 μC charge is $(10 - x)$ m.



It experiences an electrical force F_1 due to 3 μC and F_2 due to 6 μC as shown in figure.

$$F_1 = \frac{1}{4\pi\varepsilon_0} \times \frac{(3\,\mu\text{C}) \times (2\,\mu\text{C})}{x^2}$$

$$F_2 = \frac{1}{4\pi\varepsilon_0} \times \frac{(6\,\mu\text{C}) \times (2\,\mu\text{C})}{(10 - x)^2}$$

For given situation, $F_1 - F_2 = 0$

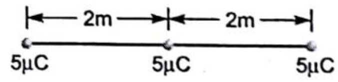
$$\Rightarrow F_1 = F_2$$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0} \times \frac{(3\,\mu\text{C}) \times (2\,\mu\text{C})}{x^2} = \frac{1}{4\pi\varepsilon_0} \times \frac{(6\,\mu\text{C}) \times (2\,\mu\text{C})}{(10 - x)^2}$$

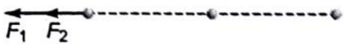
$$\Rightarrow \sqrt{2}x = 10 - x$$

$$\Rightarrow x = \frac{10}{\sqrt{2} + 1} = 4.14\text{ m}$$

Illustration | 5 *Three identical point charges each having charge 5 μC are kept fixed on a line as shown in the figure. Determine the net electric force experienced by the leftmost charge.*



Solution The net electric force experienced by any charged particle is equal to the vector sum of the electric force experienced by it due to two charges individually.



For the leftmost charge, let F_1 be the force experienced by it due to the rightmost charge, and F_2 due to the middle charge. As all the charges are of same sign both these forces are repulsive in nature, hence the net electric force experienced by the leftmost charge is,

$$F = F_1 + F_2 \text{ (towards left)}$$

$$F = \frac{1}{4\pi\varepsilon_0}\left[\frac{5 \times 5 \times 10^{-12}}{4^2} + \frac{25 \times 10^{-12}}{2^2}\right]$$

$$= 0.0741\text{ N.}$$

Electric Field

In this section we are going to see how two charges which are kept at certain distance apart *ie*, not in contact are attracting or repelling each other. In general any force (action) which is coming into the existence because of any object at a distance is explained in two steps, and in the same way the electric force is explained.

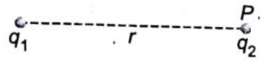


Fig. 13.12

Consider two point charges q_1 and q_2 placed at a distance r apart in vacuum, then q_2 will experience a force $F = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r^2}$ due to q_1.

It means q_1 is exerting a force on q_2. Let us ask a question from ourself—What will happen if q_2 is not present at point P ? Is there any effect of q_1 at the point P if q_2 is not present there ? Answer to these questions could be found by following explanations :

As the two point charges are placed at certain distance apart, both will experience an electric force due to the other, this can be understood in two steps :

1. Any charge creates an electric field in its surrounding region.
2. Any other charge when comes in this region experiences an electric force.

Thus, we can say $q_1(q_2)$ is creating an electric field at the location of $q_2(q_1)$ as a result of which both will experience an electric force. The Electric field region can be considered as the region surrounding any charge distribution in which its electrical effects can be experienced.

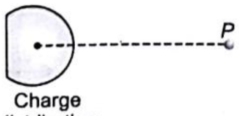


Fig. 13.13

One very important point to keep in mind about electric field is that, its existence is independent of the presence or absence of other charges to experience the effect of the electric field. For example, even if q_2 is not present in above case, electric field at P would be there due to q_1.

The strength of electric field can be expressed by any of the two physical quantities

1. Electric field intensity ($\vec{E}$)
2. Electric potential (V)

By strength of electric field we mean the power/extent of force which it can exert on other charges.

Electric Field Intensity

Electric field intensity is a vector quantity and is denoted by $\vec{E}$. At any point in space it is defined as—"The electric force experienced by a test charge divided by the test charge".

Mathematically, $\vec{E}$ at any point is $\vec{E} = \frac{\vec{F}_e}{q_0}$ where q_o is the test charge. Test charge means, we are taking a particle whose charge is so small that if it can't change the electric field due to other charge distributions appreciably.

Let us understand the concept of $\vec{E}$ in detail. Consider any charge distribution as shown in the figure, let us say we are interested in finding the electric field intensity at P due to this charge distribution. For doing so, we will place a test charge q_0 at P and see what is the electric force experienced by it, say it to be $\vec{F}_e$, then $\vec{E}$ at P it is $\frac{\vec{F}_e}{q_0}$.

Electric field intensity is a vector quantity and its direction is same as that of $\vec{F}_e$ and its SI unit is NC^{-1}.

Electric Field Intensity Due to a Point Charge

In this section we are going to find the value of $\vec{E}$ at any point due to a point charge. Consider a point charge Q placed at A, and we are interested in finding out the electric field intensity at P as shown in Fig. 13.14. Then by using the basic concepts, we will bring a test charge q_0 (+ve) at P, that will experience a force F due to point charge Q.

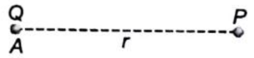


Fig. 13.14

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq_0}{r^2}$$

The direction of F is away from Q if Q is +ve and would be towards Q if it is –ve.

Electric field intensity at P would be,

$$E = \frac{F}{q_0} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

So, the electric field intensity due to a point charge Q at a distance r from it is $\frac{Q}{4\pi\varepsilon_0 r^2}$ which is away from the charge if the charge is +ve and towards the charge if charge is –ve.

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Concept Building Illustrations

Illustration | 6 *A charge of* 3 μC *is fixed at origin. Determine the electric field at* $x = 5$ m *due to this charge.*

Solution From the expression, $E = \frac{q}{4\pi\varepsilon_0 r^2}$

$$E = 9 \times 10^9 \times \frac{(3 \times 10^{-6})}{5^2} = 1080\ \text{Nm}^{-1}$$

Direction of $\vec{E}$ is along +ve X-axis.

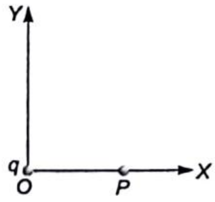
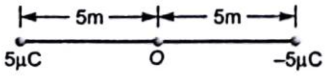


Illustration | 7 *Two point charges of* 5 μC *and* –5μC *are placed along a line as shown in figure. Determine the electric field intensity vector at O.*



Solution As principle of superposition is applicable to an electric force, it would be also be applicable for electric field intensity. So $\vec{E}$ at O is equal to the vector sum of $\vec{E}$ at O due to 5 μC and –5 μC charges.

$\vec{E}$ at O due to 5 μC charge is,

$$E_1 = \frac{q}{4\pi\varepsilon_0 r^2}$$

$$= \frac{9 \times 10^9 \times 5 \times 10^{-6}}{25}$$

$$= 1800\ \text{NC}^{-1} \text{ towards right.}$$

$\vec{E}$ at O due to –5 μC charge is,

$$E_2 = \frac{q}{4\pi\varepsilon_0 r^2}$$

$$= \frac{9 \times 10^9 \times 5 \times 10^{-6}}{25} \text{ towards right}$$

$$= 1800\ \text{NC}^{-1} \text{ towards right.}$$

So $\vec{E}$ at O is,

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$= 3600\ \text{NC}^{-1} \text{ towards right.}$$

Electric Potential Energy

As we studied in the chapter work, energy and power that PE is defined only for internal conservative forces, and as electric force is a conservative force, so correspondingly electric PE could be defined. From basic definition of PE, ΔU (change in PE) = –ve of work done by an internal conservative force in changing the configuration from initial to final.

Similarly, change in electric PE is equal to –ve of work done by electric force in changing the configuration from initial to final. If we consider a particular configuration as a reference one, then absolute electric PE can be defined as the –ve of work done by electric force in assembling the system from reference configuration to desired configuration.

$$\Delta U = -W_{\text{initial to final}}$$

$$U = -W_{\text{reference to final}}$$

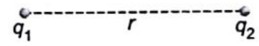


Fig. 13.15

For a two-charged particle system, taking infinite separation as reference configuration, the electric PE can be written as

$$U = \frac{q_1 q_2}{4\pi\varepsilon_0 r}$$

where r is the separation between the point charges.

**Detailed analysis of this topic requires the use of calculus.*

C-BIs

Concept Building Illustrations

Illustration | 8 *Two charged particles having charges 3 μC and +2 μC are located at a distance of 2 m. Determine the electric PE of this system.*

Solution From the relation

$$U = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r}, \text{ we have}$$

$$U = \frac{9 \times 10^9 \times 3 \times 10^{-6} \times 2 \times 10^{-6}}{2} = 27 \text{ mJ}$$

Illustration | 9 *Two point charges of magnitudes 3 μC and – 2μC are placed at a separation of 2 m. Determine the electric PE of this system.*

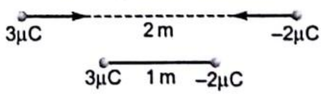
Solution $U = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$

$$= \frac{9 \times 10^9 \times (3 \times 10^{-6}) \times (-2 \times 10^{-6})}{2}$$

$$= -27 \text{ mJ}$$

Illustration | 10 *Two point charges of 3 μC and –2 μC are released from rest when they are at a separation of 2 m. Determine the total KE of the system when the charged particles are at a separation of 1 m. Assume no other forces except the mutual electric force to be acting between them.*

Solution When the charged particles are released from rest, they will start moving towards each other due to the electrostatic force, and as a result both particles will acquire some KE. This question can be solved by using work-energy theorem. Let the required KE of system be K.



Initial KE = 0

Initial PE $= \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r_i}$

$$= \frac{9 \times 10^9 \times 3 \times 10^{-6} \times (-2 \times 10^{-6})}{2} = -27 \text{ mJ}$$

$$\text{Final PE} = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r_f}$$

$$= \frac{9 \times 10^9 \times 3 \times 10^{-6} \times (-2 \times 10^{-6})}{1} = -54 \text{ mJ}$$

From work-energy theorem,

$$\Delta K = W_{\text{electric force}}$$

$$K_f - K_i = -\Delta U = -(U_f - U_i)$$

$$K - 0 = -[-54 - (-27)] \text{ mJ}$$

$$K = 27 \text{ mJ}$$

Electric Potential

In general, the word potential is only associated with conservative **force fields** just like potential energy is associated with conservative forces. Electric potential is defined as—"The electric potential difference between two points A and B in an electric field is equal to the –ve of work done by the electric force in bringing a unit positive test charge from B to A".

Mathematically

$$V_A - V_B = -\frac{W_{B \text{ to } A}}{q_0} = \frac{\Delta U}{q_0}$$

where $W_{B \text{ to } A}$ represents the work done by an electric force in bringing the test charge q_0 from B to A.

Force field means a force which creates field in which the effect of force can be experienced just like the electric field.

Let us consider an electric field region in which we want to find the potential difference between two points A and B, then the potential difference between these two points A and B *ie*, difference of potentials at A and B is given by

$$V_A - V_B = -\frac{W_{B \text{ to } A}}{q_0} = \frac{\Delta U}{q_0}$$

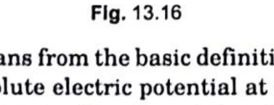


Fig. 13.16

It means from the basic definition we can't define absolute electric potential at a point but if we take some reference point and assign the potential to be zero at this point, then we can get the absolute potential. Generally, we take reference point to be at infinity. So absolute electric potential at any point is defined as, "the –ve of work done by electric force in bringing a unit positive test charge from infinity to some desired point".

$$V = -\frac{W_{\infty \text{to desired point}}}{q_0} = \frac{U}{q_0}$$

The electric potential is a scalar quantity and its SI unit is volt. It is clear from the definition of potential that 1 volt = 1 J/C.

Electric potential at any point due to a point charge : Consider a point charge Q, relative to which we want to find the potential at a distance r from it.

The required potential at point P is given by,

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

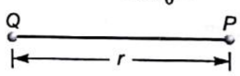


Fig. 13.17

Principle of superposition is also valid for computation of electric potential. Let us consider three point charges as shown in Fig. 13.18. For this situation we are interested in finding the potential at P.

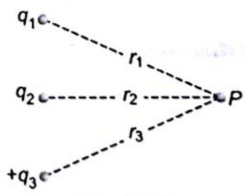


Fig. 13.18

Potential at P = Potential at P

[due to q_1 + due to q_2 + due to q_3]

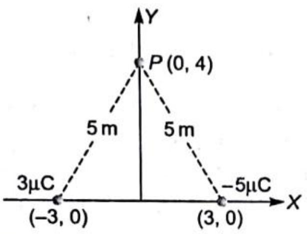
$$V = \frac{q_1}{4\pi\varepsilon_0 r_1} + \frac{q_2}{4\pi\varepsilon_0 r_2} + \frac{q_3}{4\pi\varepsilon_0 r_3}$$

C-BIs

Concept Building Illustrations

Illustration | 11 *Two point charges* 3 μC *and* −5μC *are located at* (−3m, 0) *and* (3m, 0) *respectively. Determine the potential at the point* (0, 4 m).

Solution The given situation is shown clearly in figure. Potential at point P due to 3 μC charge is,



$$V_1 = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1}{r_1}$$

$$= \frac{9 \times 10^9 \times 3 \times 10^{-6}}{5}$$

$$= 5.4 \text{ kV}$$

Potential at point P due to −5 μC charge is

$$V_2 = \frac{1}{4\pi\varepsilon_0} \times \frac{q_2}{r_2}$$

$$= \frac{9 \times 10^9 \times (-5 \times 10^{-6})}{5}$$

$$= -9 \text{ kV}$$

Potential at point P.

$$\Rightarrow \quad V = V_1 + V_2 = -3.6 \text{ kV}$$

Behaviour of Conductors in Electrostatics

In this chapter we are mainly concerned with stationary charges. In this section we are going to deal with a conductor's behaviour in electrostatics. We already know that in conductors the electrons are not bounded to a particular atom and are free to move throughout the conductor volume randomly (not in ordered manner). Now, we are going to discuss the case when a neutral conductor is placed in an **external** electric field.

External electric field means the electric field produced by some external charge distribution.

Fig. 13.19 Free electron of a conductor placed in an electric field rearrange themselves to make $\vec{E}$ inside zero.

Consider a neutral conductor placed in an external uniform (same throughout the region) electric field, as a result of electric field the free electrons experience a force in a direction opposite to the direction of electric field intensity and move in the direction of force. Due to this motion of free electrons the front surface (surface towards the direction of electric field) of the conductor acquires +ve charge as electrons get removed from here *ie*, due to deficiency of electrons and back surface of the conductor acquires −ve charge due to accumulation of electrons on this side of conductor. In other words, we can say that free electrons of the conductor move within the conductor when it is placed in an external electric field *ie*, charge gets redistributed and some charge appears on its surface. This charge is termed as induced charge, remember in this case only the charge of the conductor is redistributed but net charge of the conductor is not changing *ie*, in present case

it remains zero. Now, the question arises, this redistribution of charge continues up to what time ? Will it continue for forever or will it stop after some time ? To answer these questions fully, we require understanding of some concepts of higher mathematics, so we will try to answer these questions without rigorous reasoning. As the conductor is kept in an external electric field, the motion of free electrons takes place and the redistribution of charge occurs. This redistribution takes place for a very small time-interval of the order of 10^{-6} s and this redistribution of charge takes place in such a way that the electric field intensity at any point in the bulk of the material of the conductor becomes zero, *ie*, wherever any conducting medium is present the electric field intensity is zero in electrostatic equilibrium condition. [Electrostatic equilibrium means the motion of free electrons stops *ie*, when a conductor is placed in an external electric field, the motion of free electrons takes place *ie*, redistribution of charge takes place and when this redistribution of charge stops *ie*, free electrons come to rest, then electrostatic equilibrium condition is achieved.]

Here, the important points in above discussion have been summarized :

1. In electrostatics, the charges remain at rest.
2. When a conductor is placed in an external electric field, the redistribution of charge takes place and when this redistribution stops, an electrostatic condition is reached.
3. In electrostatics, the electric field intensity in the bulk of the material of conductor is zero.
4. Whatever excess charge we give to a conductor it resides on its outer surface.
5. In electrostatics all the points of the conductor are at same potential.
   If we consider a charged hollow conductor or conducting shell, and no charge is placed inside the conductor, then the potential at all the points on the conductor surface is the same and also the potential inside the conductor at any point remains the same.
6. The inside of any conducting shell is shielded by the electrical effect of any outside charge. Let us consider a conducting shell having charge q_1, and another point charge q_2 placed outside the conductor, then the electric field intensity at any point P inside the conductor due to all outside charges is zero.

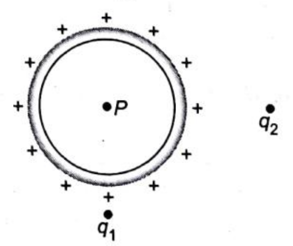


Fig. 13.20

$$\vec{E}_P = \vec{E}_{q_1} + \vec{E}_{q_2} = 0$$

Electric field at P = Electric field at P (due to q_1 + due to q_2) = 0. Here, it is to be noted that no charge is kept inside the conductor. If some charge is kept inside the conductor, then net electric field intensity at P won't be zero, but $\vec{E}$ at P (due to q_1 + due to q_2) would be zero.

Proficiency in Concepts (PIC)

Problems

Problem | 1 *How many electrons are there in 1 C of –ve charge?*

Solution We know that –ve charge is due to presence of excess of electrons since they carry –ve charge.

From $q = ne$

$\Rightarrow \quad -1\text{C} = n \times (-1.6 \times 10^{-19}\ \text{C})$

$$\Rightarrow \quad n = \frac{1}{1.6 \times 10^{-19}} = 6.25 \times 10^{18}$$

Problem | 2 *Two conductors initially having charges 3 C and –5 C are placed far away from each other. Now they are connected by a conducting wire and finally A acquires –1C charge. What would be the charge of B finally ?*

Solution This question can be solved by using the conservation of charge. Initially, the total charge of the system (comprising of A and B) is given by,

$$Q = 3\ \text{C} - 5\ \text{C} = -2\ \text{C}$$

When they are connected by a conducting wire transfer of charge takes place from one to the other, but total charge of the system remains the same as the initial one. Let q be the final charge on B, then

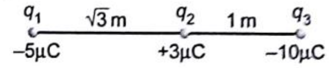
$$q - 1\ \text{C} = Q$$
$$q = -1\ \text{C}$$

Problem | 3 *Two objects having charges +1 C and –1 C are separated by 1 km. The dimensions of the object are very small compared to 1 km. Determine the electric force of attraction between these two charges.*

Solution From Coulomb's law, electric force is given by

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} = \frac{9 \times 10^9 \times 1 \times 1}{(10^3)^2} = 9000\ \text{N}$$

Problem | 4 *Three point charges are kept on a straight line as shown in the figure. Determine the magnitude and direction of net electric force experienced by +3 μC charge.*



Solution To determine the net electric force experienced by +3 μC charge, we will use the principle of superposition *ie*, we will first find the force experienced by 3 μC charge due to –5 μC and –10 μC charges individually, and then add them vectorially.

The force experienced by +3 μC due to –5 μC charge is,

$$F_1 = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r_1^2}$$
$$= \frac{9 \times 10^9 \times (5 \times 10^{-6})(3 \times 10^{-6})}{(\sqrt{3})^2}$$
$$= 45\ \text{mN towards left.}$$

The force experienced by +3 μC due to –10 μC charge is,

$$F_2 = \frac{1}{4\pi\varepsilon_0} \times \frac{q_2 q_3}{r_2^2}$$
$$= \frac{9 \times 10^9 \times (10 \times 10^{-6})(3 \times 10^{-6})}{(1)^2}$$
$$= 270\ \text{mN towards right.}$$

So, total force experienced by 3 μC charge is,

$F = F_2 - F_1$ (towards right)

$F = (270 - 45) = 225$ mN towards right.

Problem | 5 *The electric field intensity at a point is 10 NC^{-1}. If a point charge of 1 μC is placed at this point, then what will be the electric force experienced by this charged particle ?*

Solution From definition of electric field intensity,

$$\vec{E} = \frac{\vec{F}_e}{q}$$

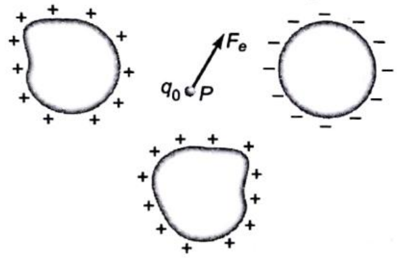
So, $\vec{F}_e = q\vec{E}$

For given situation,

$$F = 1\times10^{-6}\times10\text{ N} = 1\times10^{-5}\text{ N}$$

Problem | 6 *A positive test charge $q_0 = 3\times10^{-9}$ C is placed at a point P in an electric field region. If this test charge experiences a force of 6×10^{-8} N at this point, then what will be the magnitude of electric field intensity at point P? If we place a charge $q_1 = 6\,\mu C$ at P, then what will be the electric force experienced by it ?*

Solution In the diagram we have shown some charge distribution due to which electric field is there in the region. As the test charge q_0 is experiencing a force of 6×10^{-8} N when placed at point P, we can determine the electric field intensity at P, which is given by



$$E = \frac{F_e}{q_0} = \frac{6\times10^{-8}}{3\times10^{-9}} = 20\text{ NC}^{-1}$$

When another point charge q_1 is placed at point P, it will experience an electric force due to the electric field. The force experienced by q_1 when placed at P is given by

$$F_1 = q_1E = 6\times10^{-6}\times20 = 1.2\times10^{-4}\text{ N}$$

Problem | 7 *The work done by an electric force on the charge $q = 1\,\mu C$, as it moves from point A to point B in the region of an electric field is 5×10^{-4} J. Determine the change in electric potential energy as the point charge moves from A to B and also determine the potential difference between two points A and B.*

Solution From the definition of electric potential energy

$$\Delta U = U_f - U_i = -W_{i\,to\,f}$$

where U_f represents the final electric PE, U_i represents the initial electric PE, and $W_{i\,to\,f}$ represents work done by electric force in bringing the charge from initial to final position.

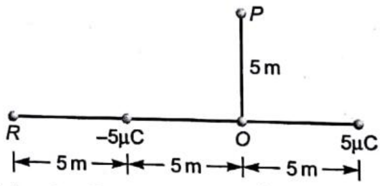
Here in given situation, initial position is A, and the final position is B, so

$$U_B - U_A = -W_{A\text{ to }B} = -(5\times10^{-4})\text{ J}$$

From the definition of potential difference,

$$\Delta V = V_B - V_A = -\frac{W_{A\text{ to }B}}{q} = \frac{-5\times10^{-4}}{10^{-6}}\text{ JC}^{-1}$$
$$= -500\text{ volt}$$

Problem | 8 *Two point charges of $5\,\mu C$ and $-5\mu C$ are placed along the x-axis as shown in the figure. Three points O, P and R are marked as shown in the figure. Determine the electric potential at*



(a) *point O* (b) *point P* (c) *point R*

Solution The potential at any point is given by sum of potentials at that point due to individual charges.

(a) At point O,

$$V = V_{\text{due to }-5\,\mu C} + V_{\text{due to }+5\,\mu C}$$
$$= -\frac{1}{4\pi\varepsilon_0}\times\frac{5\,\mu C}{5} + \frac{1}{4\pi\varepsilon_0}\times\frac{5\,\mu C}{5} = 0$$

(b) At point P,

$$V = \frac{1}{4\pi\varepsilon_0}\times\frac{q_{-5}}{r_1} + \frac{1}{4\pi\varepsilon_0}\times\frac{q_{+5}}{r_2}$$
$$= 9\times10^9\left[\frac{-5\times10^{-6}}{5\sqrt{2}} + \frac{5\times10^{-6}}{5\sqrt{2}}\right] = 0$$

(c) At point R,

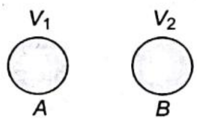
$$V = \frac{1}{4\pi\varepsilon_0}\times\frac{q_{-5}}{r_1} + \frac{1}{4\pi\varepsilon_0}\times\frac{q_{-5}}{r_2}$$
$$= 9\times10^9\left[\frac{-5\times10^{-6}}{5} + \frac{5\times10^{-6}}{15}\right]$$
$$= -6000\text{ volt} = -6\text{ kV}$$

Towards Proficiency Problems

Exercise 1

A. Subjective Discussions

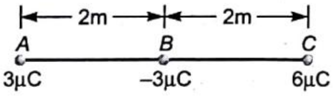
1. We have two types of charges—positive and negative. Is the positive charge more stronger than the negative charge ? Explain your answer.
2. Determine the smallest value of electric force between two charged particles when they are separated by 1 m. Would this electric force be attractive or repulsive ?
3. Two charged conducting bodies A and B having potentials V_1 and V_2 $(> V_1)$ respectively are kept far away from each other. Now they are being joined together by a conducting wire. Are the electrons are transferred from A to B or from B to A ? Explain your answer.



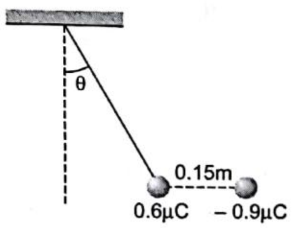
4. Attraction is not a true test of electrification (non-zero net charge). Comment on this statement.
5. A positive point charge and a negative point charge having equal magnitudes are fixed at two corners of a square. At which corner these charges must be placed so that the potential at the vacant corners should be the same ?
6. What do you mean by a point charge, and a test charge ? Are these two the same ?
7. Three point charges have identical magnitudes, but two of the charges are positive and one is negative. These charges are fixed at three corners of a square (one at each corner). In whichever way these charges are arranged, the potential at the vacant corner is always positive. Give an explanation for this fact.
8. The potential at a point in space has a certain non-zero value. Is the electric potential energy the same for every charge that is placed at that point ? Explain your reasoning.
9. A rod made from an insulating material carries a net charge, while a copper sphere is neutral. The rod and the sphere don't touch each other.
   Is it possible for the rod and the sphere to
   (a) attract each other.
   (b) repel each other ? Explain.
10. You blow up a balloon and rub it against your skins. As a result, the balloon gets charged. Now make the balloon to touch the ceiling of the room and release it. Upon being released, the balloon remains stuck to the ceiling. Explain why ? While answering, don't consider air friction or buoyancy force etc.
11. Three point charges are fixed to the corners of a square, one at each corner, in such a way that the net electric field intensity at the empty corner is zero. Do these charges all have
    (a) the same sign, and
    (b) the same magnitude but possibly different signs ?
    Explain your answer.

B. Numerical Answer Types

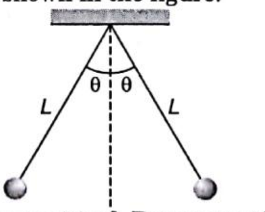
1. How many excess electrons must a body have in order to possess an electric charge of $-3C$?
2. If a body is having 3×10^{20} protons and 1×10^{20} electrons, then determine the charge of the body.
3. A system is consisting of two charged bodies having charges 3 μC and 5 μC. When these two bodies are kept in electrical contact the charge on one body is 4 μC, then what would be the charge on other body ?
4. Two point charges 3 μC and $-6 \mu C$ are kept at a distance of 2 m. Determine the electrostatic force of interaction between them. Assume there is no medium in-between the charges.
5. In above question if region in between the charges is filled with a medium having relative permittivity 4, then what would be the new electric force between them ?
6. Three point charges A, B and C are placed along a straight line as shown in the figure. Determine the magnitude and direction of net electric force experienced by all of A, B and C.



7. Four point charges each of magnitude q are placed at the corners of a square of side a. If a point charge Q is placed at centre so that all 5 charges are in equilibrium, then determine the value of Q in terms of q. Also determine the value of Q if only the charge placed at the centre has to be in equilibrium.
8. A particle having charge 3 C and mass 2 kg is revolving along a circle of radius 1 m, at the centre of which another point charge of $-5 \mu C$ is fixed. Assume only mutual electric force to be acting on two charges. Will the particle perform uniform circular motion or non-uniform circular motion ? Give your answer with proper explanations. If the particle is performing uniform circular motion, then determine its speed and if it is performing non-uniform circular motion, then determine its net acceleration.
9. Two objects are identical and small enough such their sizes can be ignored as compared to distance between them, which is 0.2 m. In vacuum each object carries a different charge, and they attract each other with a force of 1.2 N. The objects are brought into contact, so the net charge is shared equally, and then they are returned to their initial position. Now it is found that the objects repel one another with a force whose magnitude is equal to that of the initial attractive force. Find the initial charge on each object.
10. A small spherical ball made up of an insulating material having a mass of 8×10^{-2} kg and charge $+0.6 \mu C$ is hung by a thin wire of negligible mass. A charge of $-0.9 \mu C$ is held 0.15 m away from the sphere as shown in the figure. In equilibrium the string makes an angle θ with the vertical. Find out
(a) the angle θ, and
(b) the tension in the wire.
[*Take* $g = 10\, ms^{-2}$]



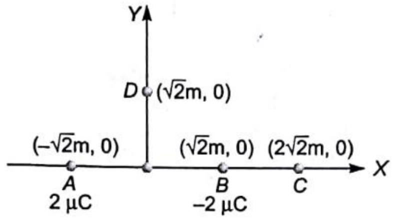
11. Two identical small balls each having mass m and charge q are suspended by two threads each of length L and negligible mass as shown in the figure.



[*The figure shows the equilibrium position*]. Determine the angle θ.

12. Determine the electric field at a point (3, 0) due to a point charge $-3\ \mu C$ placed at origin.

13. Two point charges $2\ \mu C$ and $-2\ \mu C$ are placed at points A and B as shown in the figure. Determine the electric field intensity at points C and D.



14. Calculate the electric field intensity which would be just sufficient to balance the weight of a particle of charge $-10\ \mu C$ and mass 10 mg. [Take $g = 10\ ms^{-2}$]

15. The two charged particles having charges $3\ \mu C$ and $5\ \mu C$ are initially at a separation of 5 m and are at rest. Then they are allowed to move under their mutual electric force as a result the separation between them increases to 10 m. Determine

(a) the initial electric PE of the system

(b) the final electric PE of the system

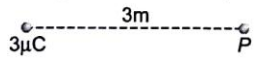
(c) work done by the electric force on system

(d) final KE of the system

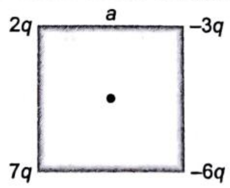
16. A charge of $2\ \mu C$ is taken from infinity to a point in an electric field, without changing its speed. If work done by electric force is $40\ \mu J$, then determine the potential at specified point.

17. If potential at a point P is 20 V, then determine the work done by electric force on a point charge $-3\ \mu C$ in bringing it from infinity to point P.

18. Determine the electric potential at point P due to $3\ \mu C$ charge as shown in the figure.



19. Four point charges are fixed at the corners of a square of edge length a as shown in figure. Determine the electric potential at the centre of square.



20. A particle with a charge of $-1.5\ \mu C$, and mass of 2.5×10^{-6} kg is released from rest at point A and accelerates toward point B, arriving there with a speed of $42\ ms^{-1}$.

(a) What is the potential difference between two points A and B *ie*, $V_B - V_A$?

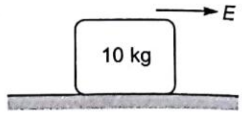
(b) Which point is at a higher potential ?

Assume only electric force to be acting.

21. Two protons are moving directly towards one another. When they are very far apart their initial speeds are $1.5 \times 10^6\ ms^{-1}$. Determine the minimum separation between them during their course of motion.

22. Two particles are each having a mass of 6×10^{-3} kg. One has a charge of $+5 \times 10^{-6}$ C while the other has a charge of -5×10^{-6} C. Initially they are held at rest at a distance of 0.8 m apart. Both are then released and accelerated towards each other. How fast is each particle moving when the separation between them is half of its initial value ?

23. A block of mass 10 kg is placed at rest on a rough horizontal surface. The coefficient of friction between the block and the surface is 0.25. The block is having a charge of 0.25 C. If an electric field E is switched ON in the region towards right, then determine the friction force between the block and surface, and acceleration of block for



(a) $E = 50\ NC^{-1}$ (b) $E = 100\ NC^{-1}$ (c) $E = 150\ NC^{-1}$

24. A particle of mass m and charge q is thrown at a speed u in the region of uniform electric field E. The particle's initial velocity is in direction opposite to direction of electric field. Determine the time which the particle takes to come momentarily to rest. Determine the time in which the particle reaches its projection position. Analyse the motion of particle.

C. Fill in the Blanks

1. The smallest charge on a body which can exist is ....
2. The charge on any body is equal to integral multiple of electronic charge. This statement is termed as ....
3. When two charged conducting bodies are brought together, then transfer of charge takes place till both acquire the same ....
4. A rod has a charge of $-2\ \mu C$, the number of electrons that must be removed from it so that it acquires a charge of $6\ \mu C$ is ....
5. Charges $-Q$ and $2Q$ are placed as shown in the figure. The electric field intensity is zero at a point which is lying somewhere to the ....

6. At a certain distance from a point charge the electric field intensity and potential are 500 Vm^{-1} and 3000 V, respectively. The distance and charge of particle are .... and ...., respectively.
7. Electrical insulator is a material, through which ....

D. True/False

1. Excess charge on a body is due to excess of electrons or due to excess of protons.
2. A –2C charge is smaller than a +2 C charge.
3. In induction (when a charged object is placed nearby to uncharged object without touching it), the net charge of both the bodies remain the unchanged.
4. In induction, the charge of an uncharged body gets redistributed, but the net charge on the body is zero.
5. In conductors, free electrons are not attached (bounded) to a particular atom.
6. Existence of an electric field in a region is independent of the presence or absence of other charges to experience its electrical effect.
7. A positive point charge is brought near to a neutral conductor, then the conductor becomes negatively charged.
8. A positively-charged metal sphere A is kept in contact with another neutral metal sphere B. Finally, A would be positively-charged and B would be negatively-charged.

High Skill Questions

Exercise 2

A. Only One Option Correct

1. Two very small spheres are initially neutral and separated by a distance of 0.5 m. Now 3×10^{13} electrons are transferred from one sphere to the other. The electrostatic force between them is

(a) Zero (b) 0.33 N
(c) 1.26 N (d) 5.2 N

2. Two point charges A and B are kept in vacuum at a certain distance. A attracts B with an electric force F. Now another point charge C is placed near to A and B. What would be the new electric force between A and B?

(a) F
(b) $2F$
(c) $F/2$
(d) Information insufficient

3. If a glass rod is rubbed with silk it acquires a positive charge because

(a) protons are added to it
(b) protons are removed from it
(c) electrons are added to it
(d) electrons are removed from it

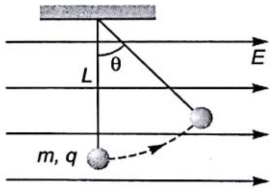
4. A electric field with a magnitude of 180 NC^{-1} exists at a spot that is 0.15 m away from a charge. At a place that is 0.45 m away from this charge, the electric field will have a magnitude of

(a) 20 NC^{-1}
(b) 60 NC^{-1}
(c) 35 NC^{-1}
(d) None of the above

5. A 3 μC point charge is kept in an external uniform electric field of 1.6×10^4 NC^{-1}. At what distance from the point charge the net electric field would be zero?

(a) 3 m (b) 1.3 m
(c) 2 m (d) 4.6 m

6. A ball of mass m and charge q is suspended from a thread of length L as shown in the figure. When a horizontal uniform electric field E is switched ON in the region, the ball deflects and in equilibrium, the string makes an angle θ with the vertical. The value of θ is given by



(a) $\tan\theta = \frac{qE}{mg}$ (b) $\tan\theta = \frac{mg}{qE}$
(c) $\sin\theta = \frac{qE}{mg}$ (d) $\sin\theta = \frac{mg}{qE}$

7. A charge 3 μC is released at rest from a point P where electric potential is 20 V, then its KE when it reaches to infinity is

(a) 20 μJ (b) 36 μJ
(c) 60 μJ (d) $\frac{20}{3}$ μJ

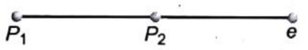
8. Two charges A and B are fixed in place, at different distances from a certain point. At this point the potential due to two charges are equal. Charge A is 0.18 m from the point while charge B is at 0.36 m from point. Then ratio of their charges *ie*, $\frac{q_A}{q_B}$ is

(a) 2
(b) 1
(c) 1/2
(d) None of the above

9. A proton and an electron are placed in an uniform electric field
(a) the electric forces acting on them will be equal
(b) the magnitudes of the electric forces acting on them will be equal
(c) their acceleration will be equal
(d) the magnitudes of their acceleration will be equal

10. Two charged particles X and Y are 4 m apart. X has a charge of $2Q$ and Y has a charge Q. The ratio of the magnitudes of the electrostatic force on X to that on Y is
(a) 4 : 1 (b) 2 : 1
(c) 1 : 2 (d) 1 : 1

11. Two protons (P_1 and P_2) and an electron (e) lie on a straight line as shown in figure. The direction of the net electric force on P_1, P_2 and e are shown. Mark the correct option.



(a) $\longleftarrow, \longleftarrow, \longrightarrow$
(b) $\longleftarrow, \longrightarrow, \longleftarrow$
(c) $\longrightarrow, \longrightarrow, \longleftarrow$
(d) $\longrightarrow, \longrightarrow, \longrightarrow$

12. A particle with a charge of 5 μC and mass of 20 g moves uniformly with a speed of 7 ms^{-1} in a circular orbit around a stationary particle with a charge of −5 μC. The radius of the orbit is
(a) 0.23 m (b) 0.62 m
(c) 1.24 m (d) 4.4 m

13. As used in the definition of electric field intensity, a test charge
(a) has zero charge
(b) has a charge of 1.6×10^{-19} C
(c) has a charge of 1 C
(d) None of the above

14. Experimenter A uses a test charge q_0, and the experimenter B uses a test charge $2\,q_0$ to measure the electric field intensity at a point produced by a stationary charge. For this situation mark out the correct statement(s).
(a) Both find the same value of E.
(b) A finds a value of E that is double of that found by B.
(c) A finds a value of E that is half of that found by B.
(d) Both measure different values.

15. Two point charges q_1 and q_2 are placed a distance r apart. The electric field is zero at a point P between the charges on the line segment connecting them. We can conclude that
(a) q_1 and q_2 must have same magnitude and signs
(b) P must be lying midway between q_1 and q_2
(c) q_1 and q_2 must have same sign but may have different magnitudes
(d) q_1 and q_2 must have opposite signs and may have different magnitudes

16. An electron travelling north enters a region where the electric field is uniform and points east. The trajectory of the motion of the electron is
(a) a straight line
(b) parabola
(c) ellipse
(d) circle

17. A 5.5×10^{-8} C charge is fixed at the origin. Another charge -2.3×10^{-8} C is moved from $x = 3.5$ cm on X-axis to $y = 4.3$ cm on Y-axis. The change in electrostatic PE of the two-charge system is
(a) 3.1×10^{-3} J (b) -3.1×10^{-3} J
(c) 6×10^{-5} J (d) -6×10^{-5} J

18. If 500 J of work is required to carry a 40 C charge slowly from one point to another in an electric field, then the potential difference between these two points is
(a) 12.5 V
(b) 20 kV
(c) 0.08 V
(d) Depends upon the path

19. An electron is accelerated from rest through a potential different V. Its final speed is proportional to
(a) V (b) V^2
(c) $\sqrt{V}$ (d) $\frac{1}{\sqrt{V}}$

B. More Than One Options Correct

1. Mark out the correct statement(s).
   (a) Only two types of charges exist in the universe.
   (b) The smallest charge possible is having the value 1.6×10^{-19} C.
   (c) Like charges repel each other.
   (d) For an electrically isolated system, the charge remains conserved.
2. Charge is quantized in nature—this statement can be explained by,
   (a) the charge on a body is an integral multiple of electronic charge
   (b) the charge is due to excess or deficiency of electrons
   (c) the charge can neither be created nor be destroyed
   (d) a charge smaller than electronic charge can't exist
3. In charging by rubbing
   (a) the mass of both the bodies changes slightly
   (b) the bodies acquire equal and opposite charges if initially both are uncharged
   (c) the body which loses electrons acquires a +ve charge
   (d) the mass of both the bodies changes by the same amount
4. In charging by conduction
   (a) the masses of the bodies change slightly
   (b) the mass of the body having higher potential increases
   (c) both the bodies acquire equal charges
   (d) both the bodies acquire same nature of charge
5. Mark out the correct statement for Coulomb's force/law.
   (a) It is an action at a distance
   (b) Its value depends upon the medium
   (c) It is valid only for point charges
   (d) It is an action-reaction pair
6. A positively charged body A attracts a body B, then the charge on body B may be
   (a) positive
   (b) negative
   (c) Zero
   (d) Can't say
7. A positively charged insulating rod is brought close to a light ball that is suspended by a string. It has been observed that ball is attracted towards the rod, from this observation we can say the
   (a) that ball may be neutral
   (b) that ball may be negatively charged
   (c) that ball may be of conducting material
   (d) there may be rearrangement of charge in the ball
8. Two point charges are located at two vertices of an equilateral triangle and the electric field is zero at the third vertex. We can conclude
   (a) that at least one other charge must be present
   (b) the two given charges may have different signs
   (c) the two given charges must have different signs
   (d) None of the above
9. Mark out the correct statement(s) regarding behaviour of conductor in electrostatics.
   (a) Electric field intensity in the bulk of the material of a conductor is zero.
   (b) Inside of a conductor is shielded by electric field.
   (c) $\vec{E}$ at any inside point of a hollow conductor is zero.
   (d) $\vec{E}$ at any inside point of a hollow conductor due to outside charges is zero.

C. Assertion & Reason

Directions (**Q. Nos. 1 to 7**) *Some questions (Assertion-Reason type) are given below. Each question contains* ***Statement I (Assertion)*** *and* ***Statement II (Reason)****. Each question has 4 choices (a), (b), (c) and (d) out of which* ***ONLY ONE*** *is correct. So, select the correct choice.*

Choices are

(a) **Statement I** is True, **Statement II** is True; **Statement II** is a correct explanation for **Statement I**

(b) **Statement I** is True, **Statement II** is True; **Statement II** is **NOT** a correct explanation for **Statement I**

(c) **Statement I** is True, **Statement II** is False

(d) **Statement I** is False, **Statement II** is True

1. **Statement I** For a body to have –ve charge, number of electrons in the body is greater than the number of protons in the body.
   Statement II Electrons are –vely charged and protons are +vely charged, both having same the amount of charge.
2. **Statement I** If we say that a body is having $-2\,\mu C$ charge, then it means the excess charge of the body is $-2\,\mu C$.
   Statement II Total +ve or total –ve charge of a body is generally much greater than the excess charge of the body.
3. **Statement I** In charging by rubbing, the mass of the body which gains the electrons increases.
   Statement II The electron has a mass equal to 9.1×10^{-31} kg.
4. **Statement I** When two charged conducting spheres having different potentials are joined by a plastic wire, then no transfer of charge takes place from one to the other.
   Statement II Charge can't be transferred through an insulating medium.
5. **Statement I** A charged object can attract a neutral object.
   Statement II If a charged body is brought near to an uncharged body, then a redistribution of charges takes place within the uncharged body.
6. **Statement I** An electron and a proton are held at $x = -a$ and $x = +a$ respectively on X-axis. Now both are released simultaneously and are allowed to move freely. The only force which effects their motions is the electrostatic force between them. During their course of motion the electron crosses the origin first.
   Statement II The electrostatic force between the two charged particle is an action-reaction pair.
7. **Statement I** In electrostatics, any excess charge given to a conductor resides on its outer surface.
   Statement II In electrostatics, $\vec{E}$ inside the bulk of a material of conductor is zero.

D. Comprehend the Passage Questions

Passage I

Four point charges all having the same magnitude are fixed at the four corners of a square (one at each corner). The charges are taken and arranged in such a way that both the electric field intensity and electric potential are zero at the centre of square. Assume infinity as reference for zero potential. *Based on above information answer the following questions :*

1. For above situation to be fulfilled out of 4 charges
   (a) two must be positive and two must be negative
   (b) three must be positive and one must be negative
   (c) All the four must be positive
   (d) Above described situation is not possible

2. Mark out the correct statement if above situation has to be fulfilled.
   (a) Like charges must be placed at diagonally opposite corners.
   (b) Like charges must be placed at the corners of the same edge.
   (c) Unlike charges must be placed at diagonally opposite corners.
   (d) Above situation is not possible.
3. If instead of four charges having same magnitude we take four charges of different magnitudes, then above situation can be realised.
   (a) Yes (b) No

Passage II

Two particles, with identical positve charges and a separation of 2.60×10^{-2} m, are released from rest. Immediately after the release the acceleration of particle 1 is having magnitude of 4.6×10^{3} ms^{-2} while particle 2 is having an acceleration of 9.2×10^{3} ms^{-2}. It is known that particle 1 is having mass of 6×10^{-6} kg. Neglect the gravitational force.

Based on above information, answer the following questions :

4. The mass of particle 2 is
   (a) 6×10^{-6} kg (b) 3×10^{-6} kg
   (c) 12×10^{-6} kg (d) 1.5×10^{-6} kg
5. The charge of particle 1 is
   (a) 4.5×10^{-6} C (b) 4.5×10^{-9} C
   (c) 4.5×10^{-8} C (d) None of these
6. The KE of the two-particle system when separation between particles increases to 5.2×10^{-2} cm would be
   (a) 3.6×10^{-4} J (b) 8×10^{-3} J
   (c) 7.2×10^{-4} J (d) 14.4×10^{-4} J

Passage III

Five styrofoam balls *A, B, C, D* and *E* are used in an experiment. The following observations were made in this experiment :

I. Ball *A* repels *C* and attracts *B*.
II. Ball *D* attracts *B* and has no effect on *E*.
III. A negatively charged rod attracts both *A* and *E*.

Based on above information, answer the following questions :

7. The ball *A* must be
   (a) positively charged
   (b) negatively charged
   (c) neutral
   (d) Can't say
8. The ball *B* must be
   (a) positively charged
   (b) negatively charged
   (c) neutral
   (d) Can't say
9. The ball *C* must be
   (a) positively charged
   (b) negatively charged
   (c) neutral
   (d) Can't say
10. The ball *D* must be
    (a) positively charged
    (b) negatively charged
    (c) neutral
    (d) Can't say
11. The ball *E* must be
    (a) positively charged
    (b) negatively charged
    (c) neutral
    (d) Can't say

Passage IV

Two charged particles each having a charge of 3 μC and mass 1×10^{-4} kg are joined by a light insulating string of length 1 m. The entire arrangement is placed on a smooth horizontal table.

Based on above information, answer the following questions :

12. The tension in the string is
    (a) 81 N (b) 0.81 N
    (c) 0.081 N (d) Zero
13. If suddenly string is cut, then
    (a) both move away from each other with constant velocity
    (b) both move away from each other with constant acceleration
    (c) both move away from each other with varying acceleration, and the acceleration of both at any instant would have same magnitude
    (d) both move away from each other with constant velocity and velocities of both at any instant may have different magnitudes
14. Just after the string is cut, the magnitude of acceleration of two particles are
    (a) 8.1×10^{5} ms^{-2}, 8.1×10^{5} ms^{-2}
    (b) 810 ms^{-2}, 810 ms^{-2}
    (c) 810 ms^{-2}, 1620 ms^{-2}
    (d) 1620 ms^{-2}, 810 ms^{-2}

E. Match the Columns

1. Match the entries of Column I with the entries of Column II.

	Column I		Column II
(A)	Electric field intensity	(P)	Scalar quantity
(B)	Electric potential	(Q)	Vector quantity
(C)	Electric potential energy	(R)	Absolute value can't be written from the basic definition
(D)	Electrostatic force	(S)	Absolute value can be written from the basic definition

2. Match the entries of Column I with the entries of Column II.

	Column I		Column II
(A)	In charging by rubbing	(P)	The two bodies acquire equal and opposite charge.
(B)	In charging by contact	(Q)	The charge of two bodies have same sign.
(C)	In induction	(R)	The mass of two bodies changes.
		(S)	There is only redistribution of charge on the body which has to be charged.

Answers

Towards Proficiency Problems
Exercise 1

B. Numerical Answer Types

1. 1.875×10^{19}
2. 32 C
3. 4 μC
4. 0.0405 N
5. 0.010125 N
6. $F_A = 0.010125$ N, $F_B = 0.02025$ N, $F_C = 0.030375$ N
7. $-q\left[\frac{2\sqrt{2}+1}{4}\right]$, Any value
8. 212.13 ms^{-1}
9. 5.574 μC and $-0.956\ \mu$C
10. (a) $\tan^{-1}(0.27)$, (b) 0.83
11. $\frac{\sin^3\theta}{\cos\theta} = \frac{q^2}{4\pi\varepsilon_0(4L^2mg)}$
12. 3000 NC^{-1}
13. $E_c = -8000\,\hat{i}\ NC^{-1}$, $E_0 = \frac{9000}{\sqrt{2}}\hat{i}\ NC^{-1}$
14. 10 NC^{-1}
15. (a) 0.027 J, (b) 0.0135 J, (c) 0.0135 J, (d) 0.0135 J
16. -20 V
17. 60 μJ
18. 9000 V
19. 0
20. (a) 1470 V, (b) B
21. 0.247 μm
22. 6.85 ms^{-1}
23. (a) 12.5 N, (b) 25 N, (c) 25 N
24. $\frac{mu}{qE}, \frac{2mu}{qE}$

C. Fill in the Blanks

1. 1.6×10^{-19} C **2.** Quantization of charge **3.** Potential **4.** 5×10^{13}
5. Left of $-Q$ **6.** 6 m, 2 μC **7.** Electrons don't flow easily

D. True/False

1. F **2.** F **3.** T **4.** T **5.** T **6.** T **7.** F **8.** F

High Skill Questions
Exercise 2

A. Only One Option Correct

1. (b) **2.** (a) **3.** (d) **4.** (a) **5.** (b) **6.** (a) **7.** (c) **8.** (c) **9.** (b) **10.** (d)
11. (b) **12.** (a) **13.** (d) **14.** (a) **15.** (c) **16.** (b) **17.** (c) **18.** (a) **19.** (c)

B. More Than One Options Correct

1. (a, b, c, d) **2.** (a, b) **3.** (a, b, c, d) **4.** (a, b, d) **5.** (a, b, c, d)
6. (b, c) **7.** (a, b, c, d) **8.** (a) **9.** (a, b, d)

C. Assertion & Reason

1. (a) **2.** (b) **3.** (a) **4.** (a) **5.** (a) **6.** (b) **7.** (b)

D. Comprehend the Passage Questions

1. (a) **2.** (a) **3.** (b) **4.** (b) **5.** (c) **6.** (a) **7.** (a) **8.** (b) **9.** (a) **10.** (c)
11. (c) **12.** (c) **13.** (c) **14.** (b)

E. Match the Columns

1. A → Q, S; B → P, R; C → P, R; D → Q, S
2. A → P, R; B → Q, R; C → S

Explanations

Towards Proficiency Problems
Exercise 1

Numerical Answer Types

1. $q = ne$

$\Rightarrow \quad 3\text{ C} = (n \times 1.6 \times 10^{-19})\text{ C}$

$\Rightarrow \quad n = \dfrac{3}{1.6 \times 10^{-19}} = 1.875 \times 10^{19}$

2. $q = n_p q_p + n_e q_e$

$= (3 \times 10^{20}) \times (1.6 \times 10^{-19}) + (1 \times 10^{20}) \times (-1.6 \times 10^{-19})$

$= 32\text{ C}$

3. From conservation of charge,

$3\ \mu\text{C} + 5\ \mu\text{C} = 4\mu\text{C} + q$

$\Rightarrow \quad q = 4\mu\text{C}$

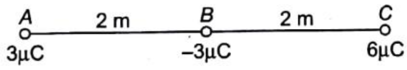
4. $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{r^2}$

$= \dfrac{9 \times 10^9 \times (3 \times 10^{-6})(6 \times 10^{-6})}{2^2} = 0.0405\text{ N}$

As charges are of opposite signs the force is attractive in nature.

5. $F_{\text{net}} = \dfrac{F}{\mu_r} = \dfrac{F}{4} = 0.010125\text{ N}$

6. Each charge experiences an electric force due to other two charges and the resultant force is a vector sum of these two forces.



$$F_A = \frac{1}{4\pi\varepsilon_0}\left[\frac{3 \times 3 \times 10^{-12}}{2^2} - \frac{6 \times 3 \times 10^{-12}}{4^2}\right]$$

towards right

$= 0.010125\text{ N}$

$$F_B = \frac{1}{4\pi\varepsilon_0}\left[-\frac{3 \times 3 \times 10^{-12}}{2^2} + \frac{3 \times 6 \times 10^{-12}}{2^2}\right]$$

towards right

$= 0.02025\text{ N}$

$$F_C = \frac{1}{4\pi\varepsilon_0}\left[\frac{3 \times 6 \times 10^{-12}}{4^2} - \frac{6 \times 3 \times 10^{-12}}{2^2}\right]$$

towards right

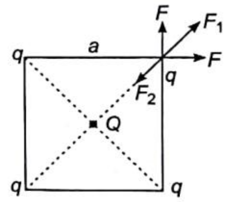
$= -0.030375\text{ N}$

–ve sign shows that the force is towards left.

7. For all charges to be in equilibrium, the resultant force on all the charges must be zero.

F_1 is the force experienced due to a charge placed on diagonally opposite corner.

F_2 is due to the charge placed at centre. Charge Q must have an opposite nature to q.



For equilibrium of q,

$$\sqrt{2}F + F_1 = F_2$$

$$\Rightarrow \quad \sqrt{2} \times \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2} + \frac{1}{4\pi\varepsilon_0} \times \frac{q^2}{2a^2} = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{(a^2/2)}$$

$$\Rightarrow \quad Q = -q\left[\frac{2\sqrt{2}+1}{4}\right]$$

For central charge to be in equilibrium Q can have any value.

8. The particle performs a uniform circular motion as magnitude of electric force which is providing necessary centripetal force is constant. Moreover, the force is always acting along the radius.

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r^2}$$

$$\Rightarrow \quad 3\text{ kg} \times v^2 = \frac{9 \times 10^9 \times 3 \times 5 \times 10^{-6}}{1}$$

$$\Rightarrow \quad v = 212.13\text{ ms}^{-1}$$

9. Let initially charges are q_1 and $-q_2$.

Then, $12 = \dfrac{1}{4\pi\varepsilon_0} \times \dfrac{q_1 q_2}{(0.2)^2}$

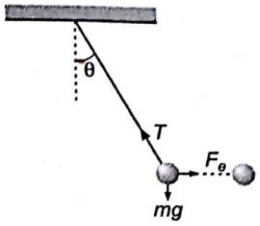
$$12 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 - q_2}{2}\right]^2 \times \frac{1}{(0.2)^2}$$

Solving above equation, we get $q_1 = 5.574\,\mu C$ and $q_2 = 0.956\,\mu C$.

Thus two charges are $5.574\,\mu C$ and $-0.956\,\mu C$.

10. $F_e = \frac{1}{4\pi\varepsilon_0} \times \frac{0.6 \times 10^{-6} \times 0.9 \times 10^{-6}}{(0.15)^2}$

$= 0.216$ N



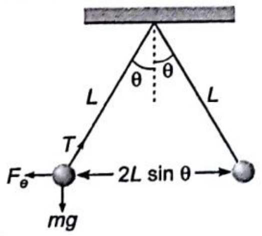
In equilibrium, $T\cos\theta = mg = 0.8$

$T\sin\theta = F_e = 0.216$

$\Rightarrow \quad \theta = \tan^{-1}(0.27)$,

and $\quad T = 0.83$ N

11. Since situation is symmetrical, the tension in two strings would be same.



$T\cos\theta = mg,$

and $\quad T\sin\theta = F_e = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{(2L\sin\theta)^2}$

$$\frac{\sin^3\theta}{\cos\theta} = \frac{Q^2}{(4\pi\varepsilon_0)(4L^2 mg)}$$

12. $E = \frac{q}{4\pi\varepsilon_0 r^2} = \frac{9 \times 10^9 \times 3 \times 10^{-6}}{9}$

$= 3000\ \text{NC}^{-1}$ towards –ve X-axis.

13. Use electric field due to point charge concept and then use vector addition.

14. $qE = mg$

$E = \frac{10 \times 10^{-6} \times 10}{10 \times 10^{-6}} = 10\ \text{NC}^{-1}$

in vertical downward direction.

15. (a) $U_i = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r_i} = 0.027$ J

(b) $U_f = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r_f} = 0.0135$ J

(c) $W = -(U_f - U_i) = 0.0135$ J

(d) From work-energy theorem,

$K_f - 0 = W = 0.0135$ J

16. Work done by electric force $= -[U_f - U_i]$

$= -[qV - 0]$

$\Rightarrow \quad 40 \times 10^{-6}\ \text{J} = -2 \times 10^{-6} \times V$

$\Rightarrow \quad V = -20$ volt

17. $W = -[qV_p - 0]$

$= -[-3 \times 10^{-6} \times 20]$

$= 60\,\mu$J

18. $V_P = \frac{q}{4\pi\varepsilon_0 r}$

$= \frac{3 \times 10^{-6} \times 9 \times 10^9}{3}$

$= 9000$ volt

19. $V = \frac{1}{4\pi\varepsilon_0}\left[\frac{2q}{a/\sqrt{2}} - \frac{3q}{a/\sqrt{2}} + \frac{7q}{a/\sqrt{2}} - \frac{6q}{a/\sqrt{2}}\right]$

$= 0$

20. Apply work-energy theorem

$K_f - K_i = W_{el} = -[qV_f - qV_i]$

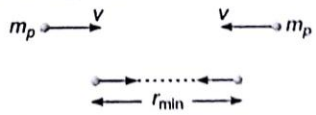
$\Rightarrow \quad \frac{2.5 \times 10^{-6} \times 42^2}{2} - 0$

$= [(-1.5 \times 10^{-6} \times V_B) - (-1.5 \times 10^{-6} \times V_A)]$

$\Rightarrow \quad V_B - V_A = 1470$ V

B is at higher potential, –ve charge moves from low potential to high potential.

21. At minimum separation, the two protons come to rest instantaneously. Applying the work- energy theorem.



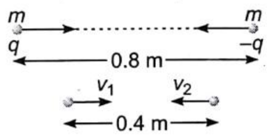
$$0 - \left[2 \times \frac{m_p v^2}{2}\right] = -\left[\frac{1}{4\pi\varepsilon_0} \times \frac{q^2}{(r_{min})^2} - 0\right]$$

$\Rightarrow \quad r_{min} = 0.247\ \mu$m .

22. Applying momentum conservation,

$$0 = mv_1 - mv_2$$

$$\Rightarrow \quad v_1 = v_2 = v_0$$



Applying work-energy theorem,

$$\frac{mv_0^2}{2} + \frac{mv_0^2}{2} - 0 = -\left[\frac{1}{4\pi\varepsilon_0} \times \frac{-q^2}{0.4} - \left(\frac{1}{4\pi\varepsilon_0} \times \frac{-q^2}{0.8}\right)\right]$$

$$\Rightarrow \quad 6 \times 10^{-3}\, v_0^2 = \frac{9 \times 10^9 \times 25 \times 10^{-12}}{0.8}$$

$$\Rightarrow \quad v_0 = 6.85\ \text{ms}^{-1}$$

23. $f_L = \mu\, mg = 0.25 \times 100 = 25$ N

(a) For $E = 50\ \text{NC}^{-1}$

$$F_e = 0.25 \times 50$$

$$= 12.5\ \text{N}$$

As $\quad F_e < f_L$

So, friction is static in nature.

$$f = 12.5\ \text{N}$$

(b) $f = 25$ N

(c) $f = 25$ N

Chapter 14

Current Electricity

The First Steps' Learning

- The Flow of Charge
- Electric Current
- Resistance
- Current Through Conductors
- Battery
- Ohm's Law
- Kirchhoff's Laws
- How to Solve Circuit Problems
- Heating Effect of Current
- Electric Power
- Ratings of Electrical Appliances
- Electric Power for Series and Parallel Combinations

In our daily life, quiet often we use the words electric current, electricity, power etc in context with electrical energy, like there would be a power cut for 2 hour today, current is not running, no electricity is there in this particular area for last night etc. You may be tackling these and many more such situations in your daily life on regular basis, and this chapter concerns with exploring physics behind all these.

In the preceding chapter we discussed about the interactions of electric charges at rest and in continuation of our discussion about charge we are going to discuss about charges in motion in the present chapter. An ***electric current*** *consists of charges in motion from one place to another and this is what we are going to study in this chapter.*

The Flow of Charge

In the previous chapter, we dealt with electrostatic condition, in which we were concerned only with the final distribution of charges in electrostatic equilibrium. We had discussed what would happen if a conductor is placed in an external electric field. Hope you remember the concept that, when we place a conductor in an external electric field, then rearrangement (redistribution) of charges takes place within the conductor, to make it a field-free region *ie,* to acquire the electrostatic equilibrium. The redistribution of charges in this case takes place for a very small interval of time, and for this small time interval the charges (electrons) are moving within the conductor. Now the question arises, what makes the charges to move? Why the movement of charges stops after some time? To achieve a steady flow of charges, what we have to do?

Answer to all these questions will be given to you in the present and next few sections. Let us consider a conductor placed in an external electric field as shown in Fig. 141.. There are free electrons inside the conductor, which experience an electric force (due to the external electric field) in a direction opposite to the electric field applied, as a result of which they move in the direction as shown and redistribution of charges takes place, and this rearrangement takes place in such a way that electric field intensity inside the conductor becomes zero in the final electrostatic equilibrium, and hence now onwards no force is experienced by the electrons and they stop. This all happens in a very small time. From above discussion we can conclude that motion of charges (electrons) takes place because of the electric force acting on them, which is quiet obvious from Newtonian physics also. But motion of charges and solving of problems related to flow of charge becomes much more convenient if we analyse the things in terms of potential difference.

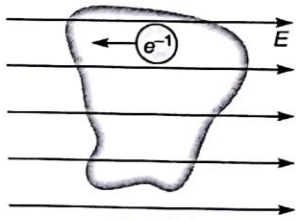


Fig. 14.1

There is no basic conceptual difference between the two ways to analyse the motion of charge, the only difference lies in the way to understand the situation. We know potential difference between any two points A and B in an electric field region is defined as negative of the work done by the electric force in bringing an unit positive test charge from B to A.

ie, $$V_A - V_B = -\frac{W_{B \text{ to } A}}{q_0}$$

We know from the definition of work that it is equal to the dot product of force and displacement.

Consider a uniform electric field [*Uniform means value of field (magnitude and direction) is same at all points in the electric field region*] of intensity E as shown in Fig. 14.2. In this electric field region we consider two points A and B at a separation of r and the line AB is parallel to the electric field direction. Now from the definition of work, work done by the electric force on a test charge q_0 if it is displaced from B to A is given by

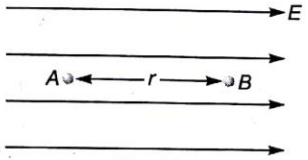


Fig. 14.2

$$W = \vec{F}_e \cdot \vec{\Delta r}$$

where, $\vec{F}_e$ is the electric force experienced by q_0 which is equal to $q_0\vec{E}$, and $\Delta\vec{r}$ is the displacement vector $\vec{BA}$.

$$W = q_0 Er \cos 180°$$

[$\because$ $|\Delta\vec{r}| = r$ *towards left and* $|\vec{F}_e| = q_0 E$ *towards right, so angle between* $\vec{F}_e$ *and* $\Delta\vec{r}$ *is* 180°]

$$W = -q_0 Er$$

So, $$V_A - V_B = -\frac{W}{q_0} = Er$$

From above expression we can conclude $V_A > V_B$ *ie*, direction of electric field is towards decreasing potential *ie*, electric field direction is from high potential to low potential. If $E = 0$ in a region, then the potential is same at all points.

If we release a positive test charge at A, then in which direction will it move? Quiet obviously you can say towards right because electric force is acting on it towards right. Same way you can say if a negative charge is released at A it will move towards left. What we can conclude from here is?

"A positive charge when freed in a region of electric field, moves from high potential to low potential region while a negative charge moves from a low potential to high potential region". Now you can think what will happen if the potential at all the points in a region is same The answer is quiet obvious-that in such a case there would not be any motion of charge which you can easily understand as follows :

If potential difference between any two points is zero *ie*, potential at all points in a region is same, then $\vec{E} = 0$ in the region and hence, charge doesn't experience any force and hence, doesn't move.

So, we can finally conclude that for charge to move from one place to another a non-zero potential difference must be there between two points. And a positive charge moves from high potential to low potential and negative charge moves from low potential to high potential region.

Now, we are able to answer the question what causes the charge to move, in two different ways although technically both methods are identical. Now the question arises - why the movement of electrons stops after some time when a conductor is placed in an external electric field? You can answer this, just by little amount of imagination. Actually within a conductor due to movement of electrons an electric field will be set up by the redistributed charges which is in opposite direction to that of applied electric field and after some time the net electric field in the conductor becomes zero thus, making all points of the conductor to be at same potential, and hence the motion of the charges stops.

This we can understand in other ways also, let us consider two charged conductors A and B having potentials V_1 and V_2 ($V_1 > V_2$) kept for apart. Now connect these conductors by conducting wire, the two ends of this wire are at different potentials with the end in contact with conductor A being at a higher potential, so a positive charge starts

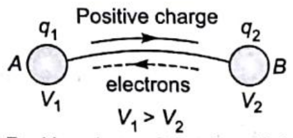


Fig. 14.3 Positive charge flows from high potential to low potential & negative charge moves from low potential to high potential.

flowing from A to B *ie*, from conductor having higher potential V_1 to conductor having lower potential V_2. But in actual as these are the electrons which move and not the positive charge, we can say that negative charge moves from a conductor having lower potential V_2 to a conductor having higher potential V_1. Due to the transfer of these electrons from B to A, the potential of B increases while the potential of A decreases which leads to the equalization of potential of conductors. When the potentials of conductors becomes equal no further motion of charge takes place. Thus, we can say—"When two conductors having different potentials are connected electrically, then the positive charge moves from a conductor having higher potential to a conductor having lower potential till both acquire the same potential." Remember that in final electrostatic equilibrium all points of the concerned region are at the same potential like in above example all points on the wire, and both conductors are at same potential in the electrostatic equilibrium. Now the next question which arises in your mind is—How can we maintain a continuous flow of charge ? You would be able to answer these questions confidently after the next two sections of this chapter.

Electric Current

Electric current is defined as the net rate of flow of charge through any cross-section. Let us try to understand the meaning of electric current in detail. Consider any cross-section or point through which n_e electrons and n_p protons are crossing in time Δt, then net charge which crosses through this point in time Δt is given by, $\Delta Q = n_p \times e - n_e \times e$ where $e = 1.6 \times 10^{-19}$ C, and the current is defined as $i = \frac{\Delta Q}{\Delta t}$.

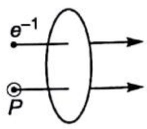


Fig. 14.4

In an electrostatic equilibrium situation there is no electric current within the conductor, but it doesn't mean that all charges within the conductor are at rest in electrostatics. In an ordinary metal (conductor) like copper or aluminium, some of the free electrons are free to move within the conducting material but these are moving randomly in all directions and hence there is no net flow of charge across any cross-section, and hence no current. *Or more specifically we can say current is defined as an ordered flow of net charge across any cross-section.*

Current is a scalar quantity and its SI unit is ampere (A). Although current is a scalar quantity but a direction is associated with it which merely represents the direction of flow of positive charge. The direction of electric current is same as that of direction of flow of positive charge and opposite to direction of flow of negative charge. For example, if 10 electrons cross any point towards right and 20 protons also cross the same point towards right then it means a net charge of 10 electrons cross the given point in right direction and hence we can say that a positive charge moves towards right and hence current.

As we have seen in the last section that a potential difference is needed for a charge to move from one point to another and current is nothing but flow of charge, so for current to be there a potential difference must exist between two points and the direction of current would be from a high potential to a low potential region, as direction of current is same as that of direction of flow of positive charge and opposite to that of direction of flow of negative charge. Hence, we can also say that potential is decreasing in the direction of current.

Although the flow of charge often takes places inside a conductor, but this is not necessary. There are many examples in which electric current is there outside the conducting wires. For example, a beam of protons from an accelerator produces a current in the direction of motion of the positively charged protons. In electrolysis the motion of negatively-charged electrons and the positively charged ions both constitute the current. Here we are going to deal only with the situations in which the electric current is set up in electric circuits.

When the ordered motion of the charge takes place within a closed conducting path, then the path is termed as an **electric circuit.** Fundamentally the electric circuits are a means of transferring electrical energy from one place to another. Electrical circuits consits of various electrical components like conducting wires, resistors, capacitors, cells, generators, switches etc. In electrical circuits, the electrical energy is transferred from a source (like cells, batteries or generator) of electrical energy to some load (like resistors, capacitors or any electrical device like fan, electric bulb, torch etc). The electrical energy supplied by source is used up or stored by/in the load in some other form. For example, in electric bulb the electrical energy is used up and is available in the form of light and heat energy, in a sound system it is available in the form of sound energy etc.

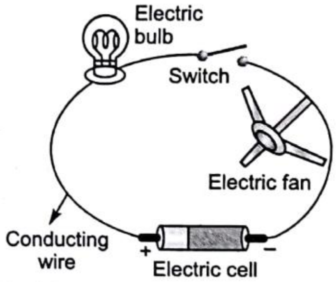


Fig. 14.5 (a) An electric circuit with open switch

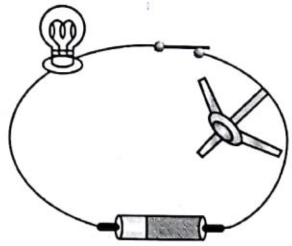


Fig. 14.5 (b) An electric circuit with closed switch

In above diagrams we have shown circuits which include a cell (source of electrical energy), an electric bulb and a fan (load), a switch (used to open or close the circuit), and a few conducting wires (to connect different electrical components). In Fig. 14.5 (a) switch is open and hence the loads are not working or are in OFF position while in Fig. 14.5 (b) switch is closed and the circuit is complete and hence the loads are operating.

Same way you can make various circuits of varying complexities, you can add any number of devices in an electric circuit.

C-BIs

Concept Building Illustrations

Illustration | 1 *A positive charge of 10 μC crosses a particular point in 1 ms. Determine the electric current through this point.*

Solution From the basic definition of current,

$$i = \frac{\Delta Q}{\Delta t} = \frac{10 \times 10^{-6}\ \text{C}}{1 \times 10^{-3}\ \text{s}} = 0.01\ \text{A}$$

Illustration | 2 *A steady current of 2A is existing in a region. Determine the amount of charge that crosses a particular point in 10 min.*

Solution From $i = \frac{\Delta Q}{\Delta t}$

$$\Delta Q = i \times \Delta t = 2 \times (10 \times 60) = 1200\ \text{C}$$

Illustration | 3 *Across a point 10^{12} electrons per ms crosses towards left and* $10^{12} \times 3$ protons per ms towards right. Determine the current through this point.

Solution The number of electrons crossing per sec is,

$$n_e = \frac{10^{12}}{10^{-3}} = 10^{15}$$

The number of protons crossing per second is,

$$n_p = \frac{3 \times 10^{12}}{10^{-3}} = 3 \times 10^{15}$$

So the positive charge which crosses the given point in 1 s is $n_p \times e$ towards right and negative charge which crosses the given point in 1 s is $n_p \times e$ towards left.

The net charge crossing sec which constitutes the current is $(n_p \times e + n_e \times e)$ towards right.

$$i = (10^{15} \times 1.6 \times 10^{-19} + 3 \times 10^{15} \times 1.6 \times 10^{-19})\ \text{A}$$
$$= 6.4 \times 10^{-4}\ \text{A}$$
$$= 0.64\ \text{mA towards right.}$$

Requirement For a Steady Current

In earlier sections we have seen that a potential difference must exist for a current to flow there, and we have also seen that when a conductor is placed in an external electric field then within a fraction of second all points of the conductor acquire the same potential and hence there is no flow of charge or current. Now, the question arises what should we need for a continuous flow of charge (continuous current or steady current) in the conductor ? There are two requirements for a steady current, these are as follows :

1. **For steady current to be there in the conductor, it must be a part of the complete circuit** : Consider a conducting rod as shown in the figure, initially the two ends of the rod are at different potentials as a result of which a positive charge flows from one end (end having higher potential) to other end which leads to the equalization of potential at all points and hence no further electric current would be there. But if this conductor is a part of complete circuit then accumulation of charge won't take place anywhere and hence steady electric current can be there in circuit. In other words the we can say for steady current to be there in circuit, it must be closed.

Fig. 14.7

2. For a steady current to be their in circuit, the circuit must contain a source of electrical energy :

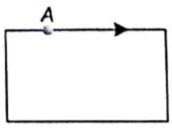


Fig. 14.8

Consider an electric circuit (closed) as shown in the Fig. 14.8 [In the figure, we have not shown any electrical component], consider any point A on the circuit if a charge q moves along the closed loop starting from A, then as the charge comes back to A its final PE must be same as that of starting PE or we can say that potential at the starting, and at end of motion of charge would be same. On the other hand we know that charge moves from high potential to low potential region, thus as charge is starting from A the potential in the direction of motion of charge must decrease and hence we can say that potential at the beginning and end of motion would be different. The above two reasoning are contradictory in nature by themselves and hence something is missing. Quiet obviously the potential at the starting and end of the trip must be same and hence we can conclude there is some part in the circuit in which the current flows from a low potential to high potential. To make the things clear, let us take an example of water fountain whose working is analogous to current in electric circuit.

You all would be familiar with the water fountain, in which the water flows down from the opening at the top (to decreasing gravitational potential energy region) and collects in a tub at the bottom. A pump then lifts the water back to top (increasing the PE) and when water reaches the top its final gravitational potential energy is same as that of gravitational potential energy at start. As we known, if we free any object from some height (here water) it has the tendency to come down to acquire minimum gravitational potential energy, and to lift it back we need some other agent which perform the work against gravitational force, here the pump serves this purpose.

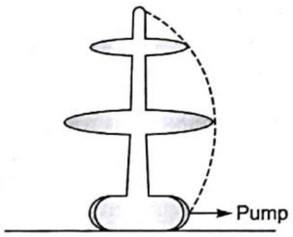


Fig. 14.9 In water fountain a pump is required to push the water up against the gravity force.

Same way you can analysis the situation for electric current in a circuit we need some external agent (battery or cell *ie,* the source of electrical energy) which can establish the current from a low potential to high potential region. Thus, we can conclude that *to establish a steady electric current in an electric circuit, it must be closed and the circuit must contain some source of electrical energy.*

Resistance

The electric current in a conductor is due to flow of electrons within the conductor. As the electrons move from one part of the conductor to the other, they collide with the other electrons and with the atoms or ions present in the body of the conductor. These collisions obstruct/ oppose the motion of electrons in the conductor, this obstruction resists the flow of electrons, and hence we can say current.

"The property of a substance due to which it resists the flow of charge or current through it is termed as resistance." Resistance is denoted by R and is a scalar quantity. Its SI unit is ohm (Ω). Numerically, the resistance for any object is equal to the ratio of potential difference applied across it to the current through it,

ie,
$$R = \frac{V}{I}$$

Remember it is only a numeric relation, it doesn't mean that resistance of an object depends upon the potential difference applied across it or the current through it.

Any device which offers some specified resistance to an electric circuit is termed as a

resistor and the symbolic representation for a resistor is

Resistance of a conductor depends on various factors like the nature of the material of conductor, its dimensions *ie,*length and area and also on temperature. For a conductor, the resistance is given by

$$R = \frac{\rho l}{A}$$

where,
ρ is the resistivity of the material of conductor, and its value depends upon the nature of material and also on temperature. For conductors, as the temperature increases, ρ increases, and hence resistance. It is a scalar quantity, and its SI unit is Ω-m. l is the length of object here-length means the length of the path through which the current travels along. A is the cross-sectional area of object *ie,* area of the cross-section through which the currents crosses.

Resistance is the property of the objects and not of the material while resistivity is the property of a substance. Although resistance of an object is independent of potential difference applied across it but is not unique for an object and it also depends on how the potential difference has been applied. Let us consider a parallelopiped whose 'dimensions' are $a \times b \times c$ as shown in Fig. 14.10.

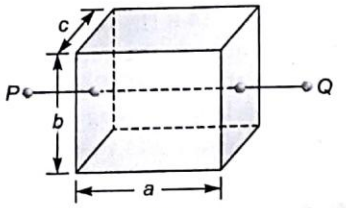


Fig. 14.10 (a) Here potential difference is applied across *PQ*

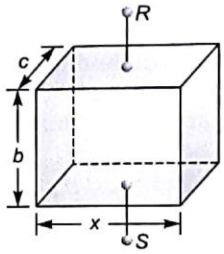


Fig. 14.10 (b) *P* is applied across *RS*

In Fig. 14.10 (a) the potential difference is applied across *PQ*, so the current covers a length of a and cross-section area through which the current passes through is $A = b \times c$, so the resistance in this connection is $R_{PQ} = \frac{\rho a}{bc}$.

Similarly for Fig. 14.10 (b) connection, resistance is,

$R_{RS} = \frac{\rho b}{ac}$ where ρ is the resistivity of the material of conductor.

Current Through Conductors

In electrostatics there is no current through the conductors as there is no ordered motion of charge in conductors in electrostatic equilibrium. In other words we can say when no potential difference is maintained across the ends of the conductor, then in this situation electrons are moving randomly in all direction as shown and as a result the net flow of charge across any section in all directions is zero, and hence no current.But in current electricity the situation is different, we maintain a constant potential difference across the ends of a conductor with the help of a battery, as a result of which an electric field is set up in the conductor and the electrons start drifting in a direction opposite to the direction of electric field. The speed with which the electrons drift is very small of the order of 10^{-4} ms^{-1} and is

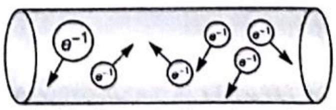


Fig. 14.11 Free electrons in an isolated conductor move randomly in a *zig-zag* path.

termed as the *drift velocity* The detailed microscopic analysis of the motion of electrons within a conductor in current electricity, you will study later on.

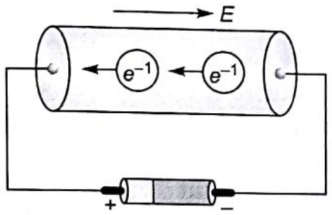


Fig. 14.12 Free electron in a conductor connected to a battery are moving in well ordered manner

It is important to note down that when there is no current through conductor *ie,* in electrostatics the electrons are moving with a very high speed of the order of $10^5 ms^{-1}$ and in spite of moving with this much high speed they don't cause any current because of their random motion, while in current electricity the electrons are moving orderly with a very small speed but still cause the current. So, again we are saying it is the **ordered motion** of charges which causes the current, and not simply the motion of charges.

Battery

A battery is a device which maintains a constant potential difference across its terminals, it can also be considered as a source of electrical energy. Battery is a 2-terminal device and its role in the circuit is to pump out positive charges from a low potential region to a high potential region and to energize the circuit..

Detailed constructional and working of a battery requires the concept of electrolysis and here we are providing only a brief discussion about battery.

Generally, battery is consisting of two plates with some medium present between them. Some internal mechanism (like electrolysis and others) exerts forces on the charges of the battery material (medium between the plates). These forces drive the positive charge to A and the negative charge towards B (remember this battery force is not an electric force), as a result of which plate A become positively-charged and B becomes negatively charged *ie,* $V_A > V_B$ (potential of A becomes greater than potential of B) as a result of which an electric field develops in between the plates whose direction is from A to B and hence charges of battery material experiences two forces :

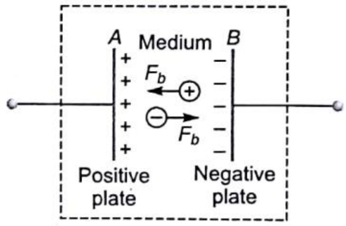


Fig. 14.13 Schematic representation of battery internal structure

1. Battery force say F_b (this is of constant magnitude) whose direction is towards A for positive charge and towards right for negative charge.
2. Electric force due to development of electric field in between the plates and direction of electric field is from A to B, so a positive charge experiences an electric force towards right and a negative charge towards left. The magnitude of electric field and hence electric force depends upon the potential difference between the plates which in turn depends upon the charge collected on plates A and B, respectively.

At the time of construction of a battery, plates are not having any charge and due to the constant battery force (F_b) the accumulation

(storage) of charge on plates takes place which leads to the appearance of electric field, and hence the electric force starts acting on the charges. After some time the equilibrium state has been reached when the battery force becomes equal to electric force and the motion of charges stops, thus maintaining the constant potential difference between the plates. This is the situation when the battery terminals are kept open.

The positively charged plate (A) is termed as the positive terminal of battery, and the negatively charged plate (B) is termed as the negative terminal of the battery.

When the battery terminals are connected by means of some load or conducting wires then also the battery force comes into existence to maintain the constant potential difference across the terminals of the battery by energizing the circuit. This situation we are not discussing in details but could be analysed in the same way as before.

The work done by a battery force in taking unit positive test charge from B to A is termed as *the electromotive force* abbreviated as emf. This is not a force as its name misleadsus but is a quantity equivalent to potential. This is a characteristic property of battery *ie,* and is a scalar quantity having SI unit same as that of potential *ie,* volt. It is denoted generally by E.

As the charges of battery material flows past the region between the plates, they encounter some resistance due to medium in between the plates which gives rise to internal resistance of the battery (r). Its value depends upon the nature of the medium in between the plates, the plates separation, cross-section area of plates etc.

Internal resistance of an ideal cell is zero while for real cells it is non-zero. Remember, potential difference across the terminals of the battery, and the emf of battery are two different things.

Symbolic Representation of Various Electrical Components/Devices

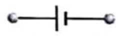
1. Resistor :

Its value determines the opposition to flow of charge *ie,* current through resistors. More is the resistance of a resistor, more opposition it offers to the current.

An open circuit possesses an infinite resistance while a short circuit possesses the zero resistance, and hence in an open circuit no current would be there and in the short-circuit path the maximum current would be there.

2. Cell/Battery :

The bigger line represents the positive terminals of the battery, and the smaller line represents negative terminal of battery.



Ideal cell

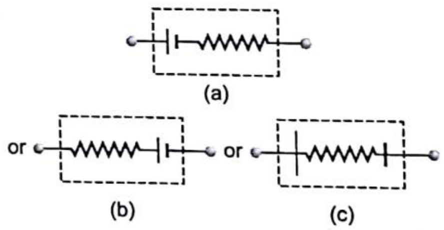


Fig. 14.14 Real Cell

3. Switch :

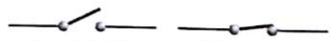


Fig. 14.15

These are used to connect various parts of circuit.

4. Conducting wires : ———

Generally, the conducting wires are taken of zero resistance and hence would be treated as a short-circuit.

Ohm's Law

According to Ohm's law "for certain substances at constant temperature the ratio of potential difference applied across it to the current through it is constant." In other words we can say that "For certain substances at constant temperature their resistance is constant."

Numerically Ohm's law can be written as,

$\frac{V}{I} = R =$ constant at constant temperature,

where, V is the potential difference across the device, I is the current through the device, and R is the resistance of device.

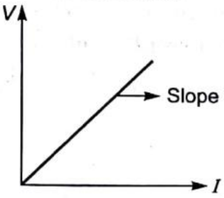


Fig. 14.16

The devices which obey Ohm's law are termed as ohmic devices or linear devices. Whatever electrical component we will encounter in our discussion, they would be ohmic devices,

For ohmic devices the graph plotted between V *versus* I is a straight line passing through origin, slope of which gives us the resistance of device.

Consider a resistor of resistance R which is connected in some closed circuit. In the diagram, we have not shown the remaining part of circuit. If a steady current I is passing through resistor R, say from A to B as shown, then as we know that some potential difference must exist for an electric current to be there so it means there is some potential drop or rise across the resistor when the current is passing through it. This potential difference is given by $V = IR$ *ie,* as we cross the resistor in the direction of current, the potential drops by IR.

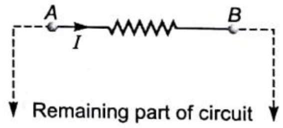


Fig. 14.17

Kirchhoff's Laws

It is very important for engineers and technicians to find out the current and potential difference across different parts of the circuit. This task can be done in many ways but here we are mentioning the most common one by using Kirchhoff's laws. Kirchhoff has given two laws for analysis and 'solving' a circuit *ie,* to find out current through various branches of the circuit, and to determine the potential difference across various branches.

Kirchhoff's First Law-Junction Rule (Current Law)

Before discussing Kirchhoff's first law, let's discuss the meaning of junction in an electric circuit. Let us consider an electric circuit as shown in Fig. 14.18. For the sake of more clarity we have not marked the values of resistances and the emfs of the batteries.

The points in the circuit where two or more branches are joined are termed as junctions or nodes. In the figure these are marked as A, B, C, D and E.

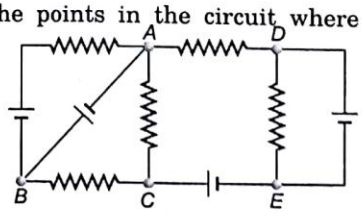


Fig. 14.18

There are four branches meeting at junction A and at all the remaining junctions B, C, D and E, there are three branches.

According to Kirchhoff's current law (KCL) : "In steady state, the algebraic sum of currents meeting at a junction in zero." In other words—"At a junction the current entering is equal to current exiting the junction in steady state." Steady state means when there is no accumulation of charge in any part of the circuit *ie,* current acquires a steady state value. KCL is simply a consequence of steady state condition *ie,* at any junction charge is conserved and hence net rate of flow of charge at any juncction is zero.

Let us consider a part of complete circuit, of which a junction A is as shown. There are five branches shown in the diagram through which currents I_1, I_2,I_3 , I_4 and I_5 are there in the direction shown. From Kirchhoff's first law :

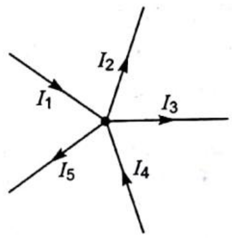


Fig. 14.19

Algebraic sum of currents is zero *ie,* $I_1 - I_2 - I_3 + I_4 - I_5 = 0$. Here, we have taken current entering into the junction as positive and current exiting the junction as negative. In other way we can write :

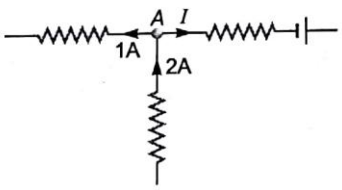
Total entering current = Total exiting current

$$I_1 + I_4 = I_2 + I_3 + I_5$$

C-BIs

Concept Building Illustrations

Illustration | 4 *The figure shows a part of the circuit, determine the value of current I in the branch shown.*

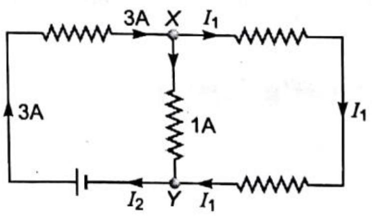


Solution Directly from KCL we can find the value of I. Total entering current = total exiting current

$\Rightarrow \quad 2 = 1 + I$

$\Rightarrow \quad I = 1\text{ A}$

Illustration | 5 *For the circuit shown in figure Determine the value of I_1 and I_2.*



Solution We can apply KCL at junctions A and B to determine the values of I_1 and I_2.

For junction $X \rightarrow$ Total entering current $= 3\text{ A}$,

and the total exiting current $= (1 + I_1)\text{ A}$

So, $\quad 3 = I_1 + 1 \Rightarrow I_1 = 2\text{ A}$

For junction $Y \rightarrow$ Total entering current $= (1 + I_1)\text{ A}$

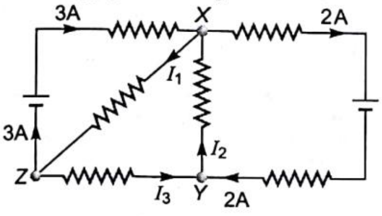
Total exiting current $= (I_2)\text{ A}$

So, $\quad 1 + I_1 = I_2$

$\Rightarrow \quad I_2 = 1 + 2 = 3\text{ A}$

Illustration | 6 *Using KCL find out the current in various branches of the circuit shown. Take* $I_1 = 0.5$ A

Solution Apply KCL at junction X –



$$3 + I_2 = 2 + I_1$$

$$\Rightarrow \quad I_1 - I_2 = 1\text{ A}$$

Applying KCL at junction Z

$$3 + I_3 = I_1$$

Applying KCL at junction Y

$$I_3 + 2 = I_2$$

Solving above equations we get

$$I_2 = -0.5\text{ A}$$

and $\quad I_3 = -2.5$ A

Negative sign of current tells that direction of current is opposite to that of the assumed direction.

Kirchhoff's Second Law (Loop Law) or Voltage Law

KVL states that the algebraic sum of potential difference along any closed loop in an electric circuit is zero.

In the adjoining circuit we have shown three loops. To use this rule, we have to start from any point on the loop and then we go along the loop either in clockwise or anticlockwise direction and we will write the potential difference across various components encountered during motion along the loop and this continues till we again reach the starting point. Now, according to loop law the algebraic sum of these potential difference what we have written along the loop would be zero.

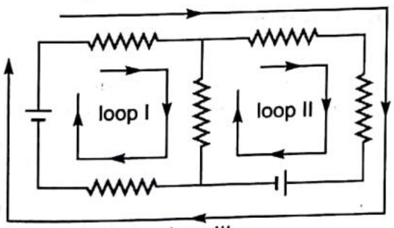


Fig. 14.20

As we are going along the loop the potential across the component can be increasing or decreasing, *ie,* the potential can drop or rise as we move along the loop across any component. Here we follow the sign convention—potential drop as positive and potential rise as negative.

Let us now see the concept related to potential drop or potential rise across various components.

For a resistor : For the resistor shown in Fig. 14.21 (a) if current through it is I, then potential difference across the resistor would be $V = IR$.

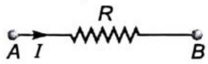


Fig. 14.21 (a)

Now we know here that A is at a higher potential than B, so if in applying KVL we are going from A to B the potential is dropping and hence the potential difference across the resistor would be written as IR, but if we are moving along the loop from B to A (*ie,* in a direction opposite to current) then the potential is rising, and hence the potential difference across the resistor would be written as —IR.

ie, drop → positive, and rise → negative.

For a battery : In the diagram we have shown an ideal battery of emf E whose positive terminal is A and negative terminal is B.

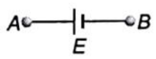


Fig. 14.21 (b)

We know that the potential difference across the battery is E. If we go from A to B, then potential is dropping and hence potential difference across the battery would be written as E, while if we go from B to A the potential difference across the battery would be written as —E.

An Illustrative example to show the sign convention used for loop law :

Consider a single loop circuit consisting of 3 batteries and 4 resistors as shown in Fig. 14.22. Let us assume that a current I is established in the circuit by the batteries. Now our aim is to write KVL (Kirchhoff's voltage law) equation for this circuit. To write KVL equation we are starting from A and we will go along the loop in ACW direction as shown in figure.

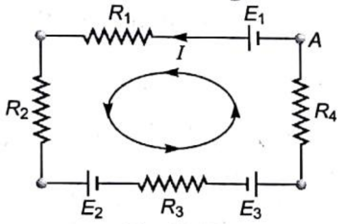


Fig. 14.22

Remember that conducting wires are of zero resistance, so the potential difference across them would be zero.

Now we start from A and move along the loop, first we encounter the battery having emf E_1 and we are moving from its negative terminal to positive terminal *ie,* in the direction of increasing potential *ie,* potential is rising as we pass this battery. The potential difference across the battery has to be written as $-E_1$.

From the diagram below you can easily understand the sign convention followed in this book and will also develop a proficency to write KVL equation.

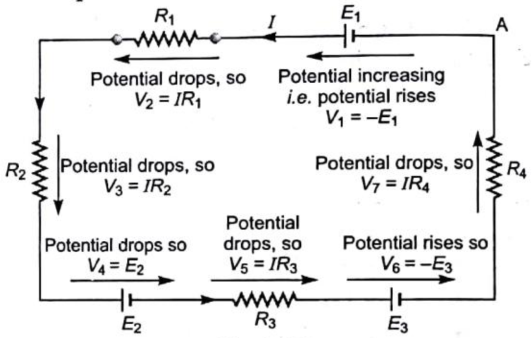


Fig. 14.23

In the above diagram we have written the potential differences across various components in accordance with the sign convention used in this book. From KVL.

$$V_1 + V_2 + V_3 + V_4 + V_5 + V_6 + V_7 = 0$$

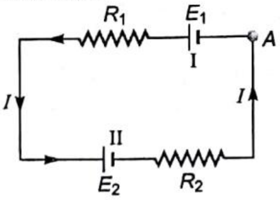
So, $-E_1 + IR_1 + IR_2 + E_2 + IR_3 - E_3 + IR_4 = 0$

$\Rightarrow \quad I(R_1 + R_2 + R_3 + R_4) = E_1 + E_3 - E_2$

C-BIs

Concept Building Illustrations

Illustration | 6 *Write the KVL equation for the given circuit.*

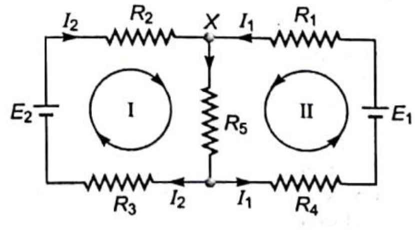


Solution Start from A and move in the anticlockwise direction.

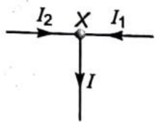
$$\underbrace{-E_1}_{\text{Rise across battery I}} + \underbrace{IR_1}_{\text{Drop across } R_1} + \underbrace{E_2}_{\text{Drop across battery II}} + \underbrace{IR_2}_{\text{Drop across } R_2} = 0$$

$\Rightarrow \quad I(R_1 + R_2) = E_1 - E_2$

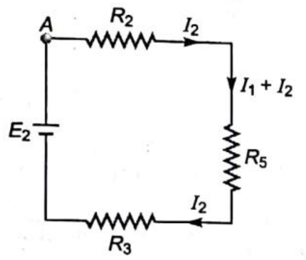
Illustration | 7 *Write KVL equation for loop I and II and use KCL at junction X.*



Solution Using KCL at junction X, the current through R_5 can be computed. Let it be I, then $I_1 + I_2 = I$.

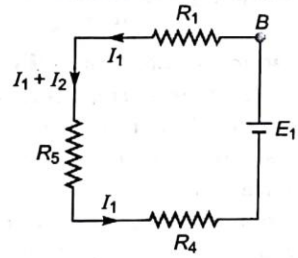


For loop I : Start from A and move in clockwise direction as mentioned.



$$\underset{\text{Drop}}{\underset{\downarrow}{I_2R_2}} + \underset{\text{Drop}}{\underset{\downarrow}{(I_1 + I_2)R_5}} + \underset{\text{Drop}}{\underset{\downarrow}{I_2R_3}} - \underset{\text{Rise}}{\underset{\downarrow}{E_2}} = 0$$

For loop II : Start from B and move in anticlockwise direction as mentioned.



$$\underset{\text{Drop}}{\underset{\downarrow}{I_1R_1}} + \underset{\text{Drop}}{\underset{\downarrow}{(I_1 + I_2)R_5}} + \underset{\text{Drop}}{\underset{\downarrow}{I_1R_4}} - \underset{\text{Rise}}{\underset{\downarrow}{E_1}} = 0$$

Grouping of Resistors

Let us consider a practical situation in which you have to draw water from a deep well say 50 m deep, but you don't have ropes of that much length.

Instead you are provided with a number of small ropes say whose lengths vary from 5 m to 10 m. Then, what you will do to manage these to serve your purpose? Simply, you will tie the smaller ropes so that you make a 50 m rope.

Same is the situation when we deal with electrical devices. Let us say we require a resistance $R\ \Omega$ for a specific purpose, but with us $R\ \Omega$ resistors are not available and we have resistors having resistances less than $R\ \Omega$ and greater than $R\ \Omega$. Then we can group the available resistances in such a specific way that through this grouped combination of resistors same current flows as in resistor of $R\Omega$ when same voltage is applied across these grouped resistors, *ie*, we can say the resistance of this combination is $R\Omega$. The resistance of these grouped resistors is known as equivalent resistance.

Resistors could be grouped/connected in two ways :

1. Series grouping 2. Parallel grouping

Series Grouping of Resistors

Two or more resistors are said to be connected in a series when they are connected one after the other and the current through all is the same. Three resistors connected in a series are shown in Fig. 14.24. Here, it is clear from the diagram that right side terminal of 1st resistor is connected to left terminal of 2nd resistor, right side terminal of 2nd to left side terminal of 3nd one and so on. This combination of three resistors is connected to an ideal cell of emf V. If this cell establishes a current I in the circuit, then the same current passes through all the three resistors.

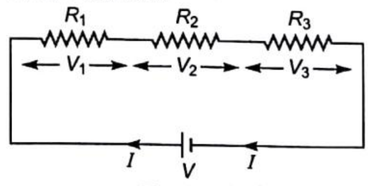


Fig. 14.24

Writing KVL for this circuit, we get

$$-V + IR_1 + IR_2 + IR_3 = 0$$

$$\Rightarrow \qquad I(R_1 + R_2 + R_3) = V \qquad \text{...(i)}$$

Let R_{eq} be the equivalent resistance of this combination, then the equivalent circuit can be drawn as shown, equivalent resistance will

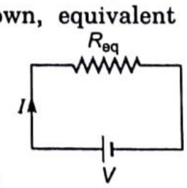


Fig. 14.25

draw the same current I from the same cell as the grouped resistors. Writing KVL for this, we get

$$-V + IR_{eq} = 0$$

$$\Rightarrow \qquad IR_{eq} = V \qquad ...(ii)$$

Comparing Eqs. (i) and (ii), we get

$$IR_{eq} = I(R_1 + R_2 + R_3)$$

$$R_{eq} = R_1 + R_2 + R_3$$

Thus we can say that in a series combination, the equivalent resistance is equal to the sum of individual resistances. Although in above discussion we have taken three resistors, but this concept is valid for any number of resistors greater than or equal to two.

Some important points to keep in mind for series combination :

1. In series combination, the current through all the resistors is the same, in present situation it is equal to I. But potential difference across various resistors can be same or different. In above discussed situation, the potential difference across the 3 resistors is given by

$$V_1 = IR_1, \quad V_2 = IR_2, \quad V_3 = IR_3$$

It the resistances of different resistors are the same, then the potential difference across them would be same-otherwise different.

2. In a series combination the equivalent resistance is equal to sum of individual resistances, and hence equivalent resistance is greater than the greatest of individual resistances. For example, if $10\,\Omega$, $20\,\Omega$ and $40\,\Omega$ resistors are connected in series, then the equivalent resistance is

$$R_{eq} = 10 + 20 + 40$$
$$= 70\,\Omega > 40\,\Omega$$

3. If n identical resistors each of resistance R are connected in series, then the equivalent resistance is nR.
4. If two resistors of resistances R_1 and R_2 are connected in series across a battery of emf E as shown, then the current through both the resistors would be same, and is given by

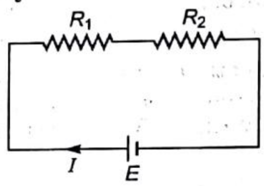


Fig. 14.26

$I = \dfrac{E}{R_1 + R_2}$, and the potential difference across the two resistors are given by,

$$V_1 = IR_1 = \frac{ER_1}{R_1 + R_2},$$

and $$V_2 = IR_2 = \frac{ER_2}{R_1 + R_2}$$

5. In a series combination the potential difference across various resistors are in direct ratio of their resistances *ie*, more the resistance, more is the potential difference across it.
6. If one component in the series circuit gets faulty *ie*, it doesn't work properly, for example, one of the resistors get broken, then the entire circuit stops functioning, because of the open path.

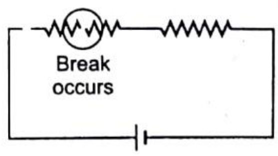


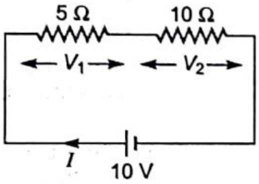
Fig. 14.27

C-BIs

Concept Building Illustrations

Illustration | 8 *Two resistors of resistances 5 Ω and 10 Ω are connected in series with an ideal battery of emf 10 V. Determine the current through both the resistors and also find out the potential difference across both.*

Solution Let the current in the circuit be I, this same current passes through both the resistors. Writing KVL equation for the circuit we have,



$$-10 + I \times 5 + I \times 10 = 0$$

$$\Rightarrow \quad I \times 15 = 0$$

$$\Rightarrow \quad I = \frac{10}{15} = \frac{2}{3} \text{ A}$$

Let V_1 and V_2 be the potential differences across 5 Ω and 10 Ω resistors respectively, then

$$V_1 = I \times 5 = \frac{2}{3} \times 5$$

$$= \frac{10}{3} \text{ volt}$$

$$V_2 = I \times 10 = \frac{2}{3} \times 10$$

$$= \frac{20}{3} \text{ volt}$$

Illustration | 9 *Three resistors having resistances 10 Ω, 5 Ω and 20 Ω are connected in series. Determine the equivalent resistance of combination.*

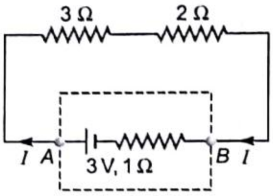
Solution The equivalent resistance is given by

$$R_{eq} = R_1 + R_2 + R_3$$

$$= (10 + 5 + 20)\,\Omega = 35\,\Omega$$

Illustration | 10 *Two resistors of resistances 3 Ω and 2Ω are connected in series with a battery of emf 3 V and internal resistance 1 Ω. Determine the current in circuit, potential difference across both the resistors and potential difference across the battery.*

Solution The circuit diagram would be as shown in the figure. A and B represent the two terminals of the battery. Let the battery establishes a current I in the circuit.



Then we can write KVL equation for the circuit as

$$3I + 2I + \underbrace{1 \times I}_{\text{potential difference across internal resistance of cell}} - 3 = 0$$

$$\Rightarrow \quad 6I = 3$$

$$\Rightarrow \quad I = 0.5 \text{ A}$$

Potential difference across 3 Ω resistor is,

$$V_1 = I \times 3 = 1.5 \text{ V}$$

Potential difference across 2 Ω resistor is,

$$V_2 = I \times 2 = 1.0 \text{ V}$$

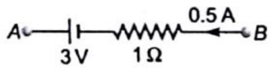
Now, our task is to find the potential difference across the battery, in all the examples discussed earlier we dealt with an ideal battery, and hence the potential difference across the battery in these cases would be equal to emf of the battery. But here the case is different, here the battery is real and therefore some potential difference would be there across the internal resistance of the battery and hence potential difference across battery terminal won't be equal to emf of battery. We can find out this in two different ways, although the basic concept in both the methods are in same.

Let V be the potential difference across the battery with terminal A at higher potential, then by applying KVL to above circuit, we have

$$-V + V_1 + V_2 = 0$$

$$\Rightarrow \quad V = V_1 + V_2 = 2.5 \text{ V}$$

Another way is to write the potential difference across the terminals of the battery directly.



$$V_B - V_A = I \times 1 - 3 = -2.5 \text{ volt}$$

$$\Rightarrow \quad V_A - V_B = V = 2.5 \text{ volt}$$

Or we can generalize as : if a current I is withdrawn from a positive terminal of the real battery then the potential difference across it would be $E - Ir$, where E and r are the emf and internal resistance of the battery, respectively.

Parallel Grouping of Resistors

When two or more resistors are connected in such a way that one end of all the resistors is connected to one point, and the other end of all resistors is connected to another common point, then resistors are said to be connected in parallel.

In Fig. 14.28 the three resistors are connected in parallel. Here, left ends of all the three resistors are connected to point A and the right ends of connected to point B. When resistors are connected in parallel then the voltage (potential difference) across all would be same and equal to the potential difference between two points A and B.

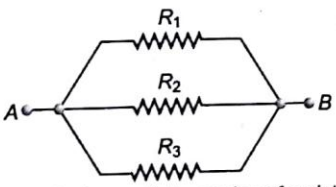


Fig. 14.28 In parallel grouping of resistors potential difference across all would be same.

Let us say the combination of three resistors in parallel is connected to an ideal cell of emf V as shown in figure 14.29. Let the battery supplies a current I to the circuit, then at junction marked A, current I would be divided into three branches. Let the amounts of current I_1, I_2, I_3 pass through branches containing resistors R_1, R_2 and R_3, respectively. Then from KCL at A

$$I = I_1 + I_2 + I_3$$

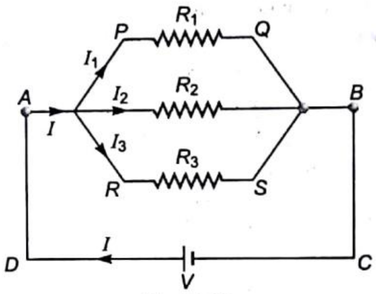


Fig. 14.29

Let us write KVL equations for $ABCDA$, $ARSBCDA$ and $APQBCDA$.

$$I_2R_2 - V = 0$$

$$I_3R_3 - V = 0$$

$$I_1R_1 - V = 0$$

From above three equations, we get

$$I_1 = \frac{V}{R_1}, \quad I_2 = \frac{V}{R_2}, \quad I_3 = \frac{V}{R_3}$$

So, $$I = I_1 + I_2 + I_3$$

$$I = V\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] \qquad \text{...(i)}$$

If R_{eq} is the equivalent resistance for this network of resistors in parallel, then the same battery must supply same current to R_{eq}. Writing KVL equation for this equivalent circuit, we have

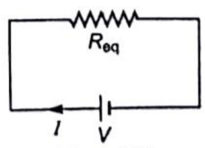


Fig. 14.30

$$-V + IR_{eq} = 0$$

$$\Rightarrow \qquad I = \frac{V}{R_{eq}} \qquad \text{...(ii)}$$

Comparing Eqs. (i) and (ii), we get

$$\frac{V}{R_{eq}} = V\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]$$

$$\Rightarrow \qquad \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

The above expression gives the equivalent resistance between two points when the three resistors are connected in parallel. Although we derived above expression for three resistors but this is valid for any number of resistors. For example, if N resistors having resistances R_1, R_2 ...... R_N are connected in parallel, then the equivalent resistance is given by $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_N}$.

Some important points to keep in mind for parallel connection :

1. In a parallel connection, the potential difference across all the resistors would be same and equal to the applied potential difference, in above described case it is equal to V. But current through different resistors would be different. In above mentioned case, the current through three resistors R_1, R_2 and R_3 would be $I_1 = \frac{V}{R_1}$, $I_2 = \frac{V}{R_2}$, and $\frac{V}{R_3}$, respectively.
2. In parallel grouping, the reciprocal of equivalent resistance is equal to sum of reciprocals of individual resistances and hence equivalent resistance is less than the least of individual resistances. For example, if three resistors of magnitudes $10\,\Omega$, $20\,\Omega$ and $40\,\Omega$ are connected in parallel, the equivalent resistance, R_{eq} is given by $\frac{1}{R_{eq}} = \frac{1}{10} + \frac{1}{20} + \frac{1}{40} = \frac{7}{40}$, and hence $R_{eq} = \frac{40}{7}\,\Omega < 10\,\Omega$.
3. If n identical resistors each of resistance R are connected in parallel, then the equivalent resistance is $\frac{R}{n}$.
4. Let two resistors are connected in parallel as shown in the Fig. 14.31. In this case, the potential difference across both the resistors would be same and equal to emf of battery.

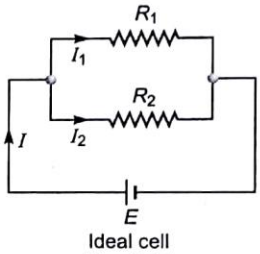


Fig. 14.31

Equivalent resistance is

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

Current I supplied by the battery is,

$$I = \frac{E}{R_{eq}} = \frac{E(R_1 + R_2)}{R_1 R_2}$$

Current through R_1 is,

$$I_1 = \frac{E}{R_1} = \frac{R_2}{R_1 + R_2} \times I$$

Current through R_2 is,

$$I_2 = \frac{E}{R_2} = \frac{R_1}{R_1 + R_3} \times I$$

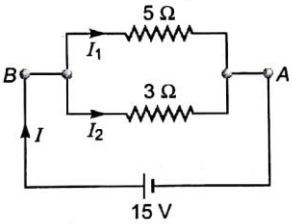
5. In parallel connection, the current through various resistors are divided according to the inverse ratios of their resistances *ie*, more is the resistance, less is the current through it. That's why if in parallel a short occurs (zero resistance) total current flows through it.
6. If in a parallel connection one of the branch gets opened then only that branch stops functioning and the remaining circuit works properly.

C-BIs

Concept Building Illustrations

Illustration | 11 *Two resistors of 5 Ω and 3 Ω are connected in parallel across an ideal cell of emf 15 V. Determine the equivalent resistance of the circuit across the terminals of battery, the current through both the resistors, and the total current supplied by battery.*

Solution Equivalent resistance across the terminals of the battery is,



$$R_{eq} = \frac{5 \times 3}{5+3} = \frac{15}{8}\ \Omega$$

The current through 5 Ω resistor is,

$$I_1 = \frac{15}{5} = 3\text{ A}$$

The current through 3 Ω resistor is,

$$I_2 = \frac{15}{3} = 5\text{ A}$$

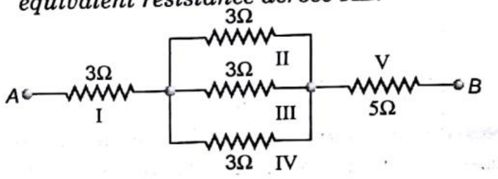
Total current supplied by the battery is,

$$I = I_1 + I_2$$

[By applying KCL at A]

$$\Rightarrow \quad I = (3+5)\text{ A} = 8\text{ A}$$

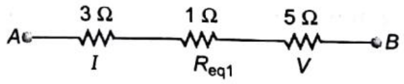
Illustration | 12 *Five resistors are connected as shown in figure. Determine the equivalent resistance across AB.*



Solution This question is based on mixed grouping *ie,* in which few resistors are connected in series while others are in parallel. In these types of questions the first step you have to take is to identify the resistors which are in series or parallel, for example, in this problem you can easily find out that resistors II, III and IV are connected in parallel. Once you identify this, replace these identified groups of resistors by their equivalent resistance, and redraw the circuit.

Here, equivalent resistance for II, III and IV is, $R_{eq_1} = \frac{3}{3} = 1\Omega$ [As 3 identical resistors each of 3 Ω are connected in parallel]

Now we are redrawing the circuit diagram, it is clear from the circuit diagram that three resistors, I, R_{eq1} and V are in series. So equivalent resistance across A and B is,



$$R_{eq} = 3 + R_{eq_1} + 5$$
$$= 3 + 1 + 5 = 9\Omega$$

How to Solve 'Circuit' Problems

Now, we shall summarize the steps to analyse/solve a circuit *ie,* to compute equivalent resistance, to find the current in various branches or to compute the potential difference across various components. A circuit can be solved in many ways, but in this book we will approach our solution through KCL and KVL. With the help of an illustrative problem you will learn the steps to solve a circuit.

For the circuit shown in Fig. 14.32, *we have to find out the potential difference across various components, and the current through various branches.*

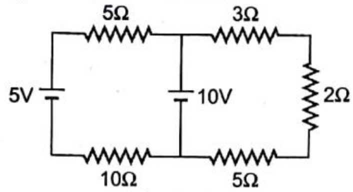
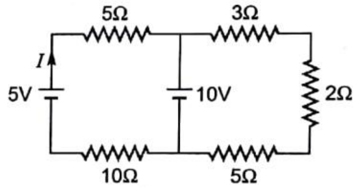
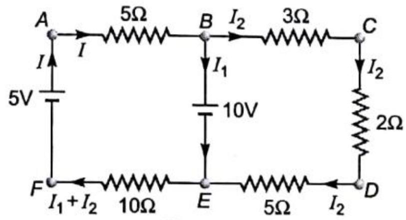
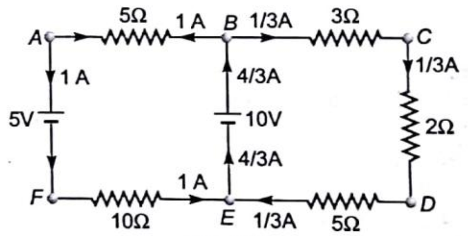


Fig. 14.32

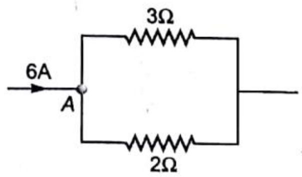
Step	Working with the Step
Step 1 : Supply/withdraw a current I from the positive terminal of battery. If there are more than one battery you can take current I from any battery.	Here in present example, two batteries are there. We can withdraw current I from positive terminal of any battery. Here, we are taking a current I from 5 V battery.  **Fig.** 14.33
Step 2 : Now the current withdrawn from the battery has to be divided amongst various branches of the circuit while distributing the current in various branches you have to keep two points in mind : (a) The division of current has to be done in accordance with KCL. (b) The current entering the negative terminal of the battery must be same as that taken from its positive terminal.	The current I coming from positive terminal of 5 V battery comes at A and the same goes through 5 Ω resistor. At B this is divided into I_1 and I_2 to branches containing 10 V battery and 3 Ω resistor respectively. The current through BC, CD and DE would be same as there is no other connection at C and D. At E the current I_1 is coming from BE branch and I_2 is coming from DE so from KCL current through 10 Ω resistor is $I_1 + I_2$.  **Fig.** 14.34 To cross check that your distribution is correct, you have to show that $I = I_1 + I_2$ in accordance with point (b) in Step 2.
Step 3 : Apply KCL at various junctions, and write the corresponding equations.	Here, only at B, we need to apply KCL equation $I = I_1 + I_2$ [From KCL equation at B] ...(i) The above equation also shows that our distribution of current in the circuit is correct.

Step	Working with the Step
Step 4 : Write KVL equation for various loops, as per requirements.	Here we have a total of 3 variables I, I_1 and I_2, so we need three equations to get the values of these variables. One equation we already have, so we need two more equations which we will get by using KVL. Apply KVL for *ABEFA* and *BCDEB*. For *ABEFA* : $5I + 10 + 10(I_1 + I_2) - 5 = 0$ $\Rightarrow \quad 5I + 10(I_1 + I_2) = -5$ ...(ii) For *BCDEB* : $3I_2 + 2I_2 + 5I_2 - 10 = 0$ $\Rightarrow \quad 10I_2 = 10$ ...(iii)
Step 5 : Solve the equation written in step 3 and step 4 to get current in various branches of the circuit. If value of any current comes out to be negative then there is nothing to worry it simply means that our assumed direction of current is wrong and in actual the current is in direction opposite to that of our assumed direction, and is of same magnitude as we found.	Solving Eqs. (i), (ii) and (iii), we get $I_2 = +1$ A $I = -\frac{1}{3}$ A $I_1 = -\frac{4}{3}$ A Here, is present case, we are getting negative values for I and I_1 and hence our assumed direction of these currents is wrong. The circuit diagram with correct current distribution in various branches is as shown in figure.  Fig. 14.35
Step 6 : Find the potential difference across various components by using $V = IR$ etc.	Using $V = IR$ we can find the potential difference across various resistors. For 5 Ω resistor in between *A* and *B*, $V_1 = 1 \times 5 = 5$ volt. For 10 Ω resistor in between *F* and *E*, $V_2 = 1 \times 10 = 10$ volt. Similarly, we can find out the potential difference across various branches of the circuit.

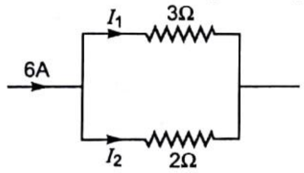
C-BIs

Concept Building Illustrations

Illustration | 13 *The figure shows a part of complete circuit. A current of* 6 A *is entering at junction A from left. Determine the current in* 3 Ω *and* 2 Ω *resistors.*



Solution Let the current through 3 Ω resistor is I_1, and through 2 Ω be I_2, then $I_1 + I_2 = 6$ A.



The potential difference across 3 Ω and 2 Ω resistors would be the same as they are connected in parallel. So,

$$3I_1 = 2I_2$$

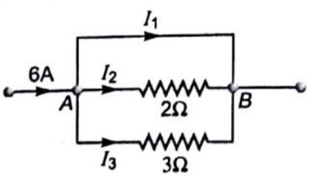
Solving above equation, we get

$$I_1 = \frac{2}{3+2} \times 6 = \frac{12}{5} \text{ A},$$

and $$I_2 = \frac{3 \times 6}{3+2} = \frac{18}{5} \text{ A}$$

The above calculation clearly shows that current through the resistor having less resistance is more, *ie*, we can say that current tries to follow the least resistive path.

Illustration | 14 *A modification has been made in previous example as shown in the figure below. Determine I_1, I_2 and I_3.*



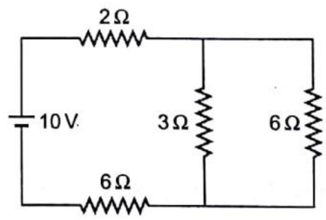
Solution Here the uppermost branch is having zero resistance because conducting wires are having zero resistance and this is termed as shorting.

As we know that current tries to follow the least resistive path, so the entire current goes through shorted branch *ie,*

$$I_1 = 6 \text{ A} \quad \text{and} \quad I_2 = I_3 = 0$$

This can be understand in this way also, if I_2 and I_3 are non-zero, then potential difference across *AB* would be non-zero. But potential difference across *AB* has to be zero because *A* and *B* are at, same potential, [The two points connected by a conducting wire are at same potential as the potential difference across zero resistance wire is zero]. So it means I_2 and I_3 are zero, and hence the total current passes through shorted branch.

Illustration | 15 *Determine the current through* 3 Ω *resistor.*



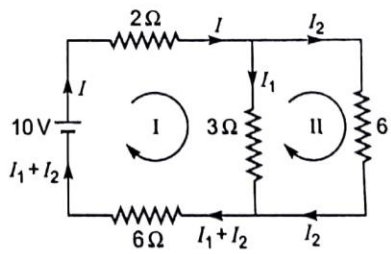
Solution Let us withdraw a current I from positive terminal of battery and then distribute it in the network as shown.

Write KVL for two loops-I and II

$$2I + 3I_1 + 6I_1 + 6I_2 - 10 = 0$$

$$6I_2 - 3I_1 = 0 \quad \text{and} \quad I = I_1 + I_2 \text{ [KCL equation]}$$

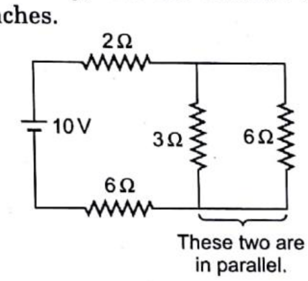
Solving above equations, we get



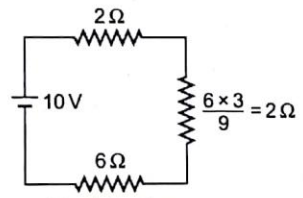
$I = 1\text{ A},\ I_1 = \frac{2}{3}\text{ A}$ and $I_2 = \frac{1}{3}\text{ A}$

So current through 3 Ω resistor is $\frac{2}{3}$ A.

Aliter : In this method we first find the equivalent resistance across the terminals of the battery and then we find the total current supplied by battery, and the current through various branches.

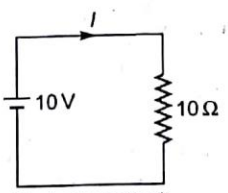


⇒



In this all the three resistors are in series

⇒



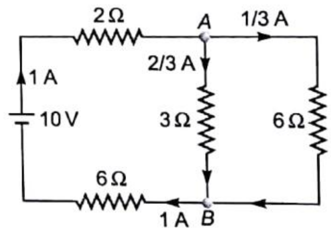
Equivalent circuit

Current supplied by the battery is,

$$I = \frac{V}{R_{eq}}$$

$$= \frac{10}{10}\text{ A} = 1\text{ A}$$

Now, draw the original circuit and redistribute the current in various branches. Divide the current of 1 A at A, and apply KCL at B.



Heating Effect of Current

When we connect a source of electrical energy say battery or generator to an electrical appliance (like electric bulb, heater, immersion rod etc.) or to a resistor, then we get some form of energy like in electric bulb it gives us light and also gets heated up, the resistor gets heated up, in speakers we get the energy in the form of sound etc. The quiet obvious question which arises in our minds from these facts is that from where do we get this energy ? Here the source is supplying the electrical energy to the device and it is getting converted to other form of energy. These all we study under heating effects of current, called as such because every electrical device possesses some resistance and when some current passes through it, then thermal (heat) energy is dissipated in the resistor which heats up the device. Now, first we are going to understand the meaning of terms electric power *ie*, the rate at which electrical energy is given to any load and then we shall consider the electric energy.

Electric Power

Consider a load (any electrical device or simply a resistor) which is connected across a battery. Let the potential difference across the load be V when current I is supplied by the battery. Then the rate at which electrical energy is supplied to the load is given by $P = VI$, this is termed as electric power of load *ie,* the rate at which it is consuming electrical energy from other parts of the circuit, here from battery.

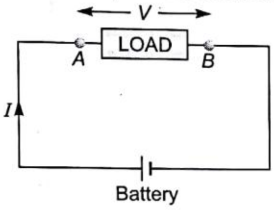


Fig. 14.36 A battery performs some work on the load by supplying energy to it.

In time t, the energy (electrical) consumed by the load is $U = Pt$

$$U = VIt$$

This energy consumed by the battery load must be used up by it and can appear in various forms as already discussed.

The SI unit of power is watt and that of energy is joule, but for commerical purposes we also use another unit for electrical energy which we call *kilowatt hour* (kWh).

Electrical energy, $U = P \times t = VI \times t$

If we take the unit of power in kW and unit of time in hour, then unit of energy is kWh. 1 kWh is the energy consumed by an electrical appliance of power rating 1 kW in 1 h, or in other words 1 kW h is the electrical energy consumed by a load in 1 h when it is receiving the electrical energy at a rate of 1 kW.

$$\begin{aligned} 1\text{ kWh} &= 1\text{ kW} \times 1\text{ h} \\ &= 10^3\text{ W} \times 3600\text{ s} \\ &= 10^3 \left(\frac{\text{J}}{\text{s}}\right) \times 3600\text{ s} = 3.6 \times 10^5\text{ J} \end{aligned}$$

Ratings of Electric Appliances

You would be very much familiar with the specifications like 40W-220 V, 100W-220 V, 1500W-220 V etc written on various electrical appliances.

Have you ever thought what these numbers in actual represent ? May be some of you may be knowing about these numbers, these are actually the power and voltage ratings of any electrical device *ie,* if P_s and V_s are the specified wattage (power rating) and voltage (voltage rating) of any device respectively, then it means when the potential difference across the device is V_s, the device will consume energy at a rate of P_s. For an 40 W-220 V electric bulb, when it is operated at 220 V, it consumes electrical energy at a rate of 40 W.

For a device having resistance R, using Ohm's law we can write $V = IR$.

From the expression of electric power, $P = VI$ now, we can write,

$$P = (IR)I = I^2R$$

or $$P = V \times \frac{V}{R} = \frac{V^2}{R}$$

Thus, power can be written in any of the forms,

$$P = VI = I^2R = \frac{V^2}{R}$$

And electrical energy consumed in time t can be written as,

$$U = VIt = I^2Rt = \frac{V^2}{R}t$$

The resistance of any electrical device having specified wattage and voltage as P_s, and V_s respectively would be given by, $P_s = \frac{V_s^2}{R}$ *ie,* $R = \frac{V_s^2}{P_s}$.

If the device is not operated at specified voltage V_s but is operated at V, then power consumed by the device is given by, $P = \frac{V^2}{R}$

where R is resistance of device. Thus,

$$P = \frac{V^2}{V_s^2 / P_s} = \frac{V^2}{V_s^2} \times P_s$$

$$\Rightarrow \quad P = \frac{V^2}{V_s^2} \times P_s$$

For example, if an electric bulb with rating 40 W-220 V is operated at 110 V, then the rate at which electrical energy is consumed by the load is $P = \frac{110^2}{220^2} \times 40 = 10$ W. If the same bulb under same operating condition is used for 2 h, then it will consume $U = 10 \times 2$ Wh $= 20$ W h $= 0.02$ kW h energy, while if the same bulb would have been used under specified condition for 2 h it will consume $U = 40 \times 2$ Wh $= 0.08$ kWh of electrical energy. Thus we can say we have to pay our electricity bill in accordance with the actual consumption of energy.

C-BIs

Concept Building Illustrations

Illustration | 16 *An electric bulb is having a potential difference of 10 V across its terminals when a current of 2 A is passing through it. Determine the power consumed by the bulb, and its resistance.*

Solution From $P = VI$

$$\Rightarrow \quad P = 10 \times 2 = 20 \text{ W}$$

$$R = \frac{V}{I} = \frac{10}{2} = 5\ \Omega$$

Illustration | 17 *An electric heater of 1.5 kW is used for 3 h at a specified voltage. Determine the energy consumed by it.*

Solution $U = Pt$

Here, the device is operated at a specified voltage, so it will consume energy at the rate given by specified wattage.

So, $U = 1.5 \times 3$ kWh $= 4.5$ kWh

Illustration | 18 *In a house, 3 fans each of 80 W-220 V and 4 tube lights each of 40 W-220 V are operated for 10 h daily. If the cost of 1 electrical unit ie, 1 kWh is 2 Rs, then what would be the monthly electricity bill of this house ? Assume a month is of 30 days and the power distribution company is always able to provide the specified voltage ie, there is no voltage fluctuation throughout the month.*

Solution Energy consumed by 3 fans in one day is,

$$U_1 = 3 \times 80 \times 10 \text{ Wh}$$
$$= 2400 \text{ Wh} = 2.4 \text{ kWh}$$

Energy consumed by 4 tube lights in one day is,

$$U_2 = 4 \times 40 \times 10 = 1600 \text{ Wh}$$
$$= 1.6 \text{ kWh}$$

Total energy consumed in one day is,

$$U = U_1 + U_2$$
$$= (2.4 + 1.6) \text{ kWh}$$
$$= 4 \text{ kWh}$$

Total energy consumed in one month is, $U_0 = U \times 30 = 120$ kWh.

For 1 kWh, cost is 2 Rs, so cost for 120 unit is 240 Rs.

So, monthly bill = 240 Rs.

Illustration | 19 *If an electric heater (1.5 kW, 220 V) is operated at 110 V for 10 h in a month, then determine the consumption of energy by the heater in one year.*

Solution The rate at which heater is consuming energy at supplied voltage is

$$P = \left(\frac{V_p}{V_s}\right)^2 \times P_s$$

$$= \left(\frac{110}{220}\right)^2 \times 1.5 \text{ kW}$$

$$= \frac{1.5}{4} \text{ kW}$$

The energy consumed in one month is,

$$U_1 = \frac{1.5}{4} \times 10 \text{ kWh}$$

The energy consumed in one year is

$$U = 12\,U_1$$

$$= \frac{12 \times 1.5 \times 10}{4} \text{ kWh}$$

$$= 45 \text{ kWh}$$

Electric Power for Series and Parallel Combinations

If we connect two resistors in series across a cell of emf E as shown, then the current in the circuit is given by,

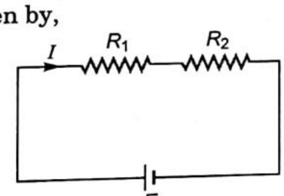


Fig. 14.37

$$I = \frac{E}{R_1 + R_2}$$

So the power across the first resistor is,

$$P_1 = I^2 R_1,$$

and the power across second resistor is,

$$P_2 = I^2 R_2$$

Total power consumed by the both the resistors is,

$$P = P_1 + P_2 = I^2(R_1 + R_2)$$

$$= I^2 R_{eq} = \frac{E^2}{R_1 + R_2}$$

Now, if the same resistors are connected in parallel across the same cell, then the power consumed by the first resistor is $P_1 = \frac{E^2}{R_1}$.

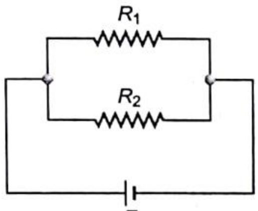


Fig. 14.38

Power consumed by second resistor is,

$$P_2 = \frac{E^2}{R_2}.$$

Thus, the total power consumed by the two resistors is,

$$P = P_1 + P_2 = E^2\left[\frac{1}{R_1} + \frac{1}{R_2}\right] = \frac{E^2}{R_{eq}}.$$

Proficiency in Concepts (PIC)

Problems

Problem | 1 *If a charge of 2 C passes across a point in 2s, then determine the electric current past this point.*

Solution From the definition of electric current,

$$I = \frac{\Delta Q}{\Delta t}$$

$$\Rightarrow \quad I = \frac{2\,\text{C}}{3\,\text{s}} = \frac{2}{3}\,\text{A}$$

Problem | 2 10^{25} *electrons cross a point in a particular direction in 1 ms, determine the electric current through this point.*

Solution $I = \frac{\Delta Q}{\Delta t}$

$$= \frac{ne}{\Delta t}$$

$$= \frac{10^{25} \times 1.6 \times 10^{-19}}{10^{-3}}\,\text{A}$$

$$= 1.6 \times 10^{9}\,\text{A}$$

Problem | 3 *Determine the resistance of a copper wire of length 3 m and cross-section area 2 cm^2. Resistivity of the copper is 1.7×10^{-8} Ω-m.*

Solution Resistance R is given by,

$$R = \frac{\rho l}{A}$$

$$\Rightarrow \quad R = \frac{1.7 \times 10^{-8} \times 3}{2 \times 10^{-4}}$$

$$= 2.55 \times 10^{-4}\,\Omega$$

Problem | 4 *If a potential difference of 10 V is applied across a resistor of resistance 20 Ω, then determine the current passing through it.*

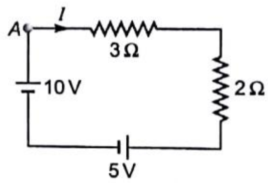
Solution We know $V = IR$

$$\Rightarrow \quad I = \frac{V}{R} = \frac{10}{20}\,\text{A}$$

$$= 0.5\,\text{A}$$

Problem | 5 *Determine the current I in the circuit shown.*

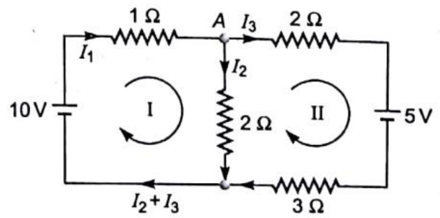
Solution Write KVL equation starting from A



$$3I + 2I + 5 - 10 = 0$$

$$\Rightarrow \quad I = 1\,\text{A}$$

Problem | 6 *Determine the currents I_1, I_2 and I_3 in the circuit.*



Solution Here, the battery of emf 10 V is supplying a current I_1 to the circuit

At A, apply KCL $\rightarrow I_1 = I_2 + I_3$ ...(i)

Apply KVL for loop I $\rightarrow$

$$-10 + I_1 \times 1 + I_2 \times 2 = 0$$

$$\Rightarrow \quad I_1 + 2I_2 = 10 \quad \text{...(ii)}$$

Apply KVL for loop II $\rightarrow$

$$I_3 \times 2 + 5 + 3 \times I_3 - I_2 \times 2 = 0$$

$$\Rightarrow \quad 5I_3 - 2I_2 = -5 \quad \text{...(iii)}$$

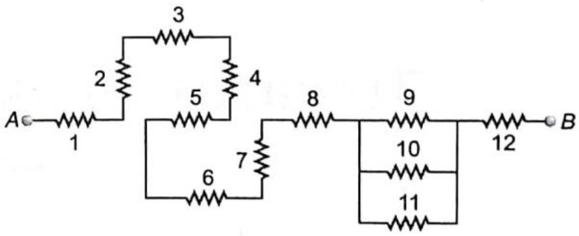
Solving above three equations, we get

$$I_1 = \frac{60}{17} \text{ A},$$

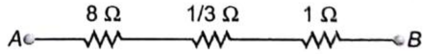
$$I_2 = \frac{55}{17} \text{ A},$$

$$I_3 = \frac{5}{17} \text{ A}$$

'roblem | 7 *Determine the equivalent resistance across AB for the shown circuit. All resistors are of* 1 Ω *each.*



Solution Here, the resistors 1 to 8 are in series, 9, 10 and 11 are in parallel. Combining these we can redraw the circuit as :



These three resistors are in series, so

$$R_{eq} = 8 + \frac{1}{3} + 1 = \frac{28}{3} \ \Omega$$

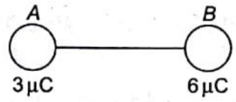
Towards Proficiency Problems
Exercise 1

A. Subjective Discussions

1. The direction of current is always opposite to the net direction of the motion of electrons. Discuss this statement.
2. In electrostatics the electric field inside the conductor is zero, but in current electricity the charge moves inside the conductor due to the non-zero electric field inside the conductor. Discuss this statement.
3. If a conductor is placed in an external electric field, then net electric field inside the conductor becomes zero. Discuss the importance and use of word 'net' in above statement.
4. Current is a scalar quantity but still a direction is associated with it. Comment on this statement.
5. In an isolated conductor, the electrons are moving with a speed of 10^5 ms^{-1} while when the same conductor is connected across a battery the electrons drift with a speed of 10^{-4} ms^{-1}. Inspite of having very high speeds in the 1st case there is no electric current but in 2nd case the current in there. Explain this statement.
6. In a closed conducting path (consisting of only conductors), is it possible to have a steady current ?
7. Two materials have different resistivities. Two wires one from each material are made up of the same length. Is it possible that both have the same resistance ?
8. Does the resistance of a copper wire increase or decrease when both the length and diameter of the wire are doubled ? Explain your reasoning.
9. Resistivity is the property of a substance while resistance is the property of an object. Comment on this statement.
10. Resistance of a given object is independent of the potential difference applied across it, but the resistance of same object can be different when connected in different ways. Discuss this statement.
11. Which statement is more appropriate ?
    I. Motion of charges constitutes the current.
    II. Net ordered motion of charges constitutes the current.
12. What is the difference between an emf and a potential difference ? Under what circumstances are the potential difference between the terminals of battery and the emf of the battery are equal to each other ?
13. We have seen that a coulomb is an enormous amount of charge, it is virtually impossible to place a charge of 1C on an object. Yet a current of 1 A (= 1 Cs^{-1}) is quiet reasonable. Explain this apparent discrepancy.
14. Two 120 V light bulbs, one of 25 W and other of 200 W were to be connected in series across a 240 V line. While doing so, one of the bulbs got damaged instantaneously. Which one would had burnt burned out and why ?
15. Is it possible to have a circuit in which potential difference across the terminals of a real battery is zero ? If possible draw the circuit.

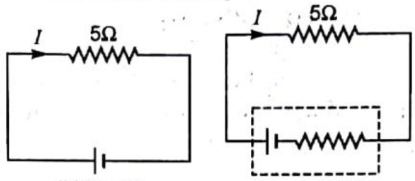
B. Numerical Answer Types

1. Two conductors having charges of 3 μC and 6 μC are placed far away from each other. Then they are joined by a conducting wire as shown in the figure. As a result of potential difference between them the charge transfer takes place and in final equilibrium situation the charge on A is 2 μC. Which one is at higher potential initially ? What is the final charge on B ?



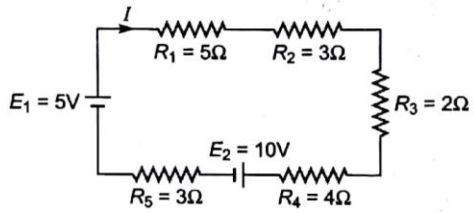
2. 10^{12} protons are crossing the origin along positive X-axis in 5 ms. Determine the current caused by this motion of protons.
3. A current of 2 A exists along positive X direction, how many electrons will cross the origin in 1 s to constitute this current and in which direction ?
4. The resistance of a copper wire is R when a potential difference V is applied across it. What would be the resistance of copper wire when a potential difference of 2 V is applied across it ? What would be the old and new current through the copper wire ?
5. The resistance of a copper wire is R, if we double the length of wire without changing its cross-section area then what would be the new resistance ?
6. A copper wire is having resistance R. Now it is stretched so that its length becomes doubled. What would be the new resistance and resistivity of wire ? Original resistivity of copper is ρ.
7. Two wires of different materials having same geometrical dimensions are having resistances of R_1 and R_2, respectively. What would be the ratio of their resistivities $\frac{\rho_1}{\rho_2}$?
8. A cylindrical copper cable carries a current of 1200 A. There is a potential difference of 1.6×10^{-2} V between two points on the cable that are 0.24 m apart. Determine the radius of the cable. Resistivity of copper is $1.72 \times 10^{-8}\ \Omega\text{-m}$.
9. A parallopiped of dimensions 2 cm × 5 cm × 1 cm is made up of a material having resistivity ρ. Determine the resistance of this object in three different configureurations when potential difference is across the pair of opposite faces ?
10. An ideal battery of emf 5 V is supplying a current of 2 A to a resistor of resistance R. Determine the value of R ?
11. A battery has an emf of 1.5 V and an internal resistance of 0.10 Ω. When the battery is connected to a resistor, the terminal potential difference across the battery is 1.3 V. Determine the resistance of the resistor and the current in the circuit ?
12. A tightly coiled spring having 75 turns, each 3.50 cm in diameter, is made of an insulated metal wire of cross section area $5 \times 10^{-6}\ m^2$. The resistance across the ends of the spring is 1.74 Ω. Determine the resistivity of the metal. ?
13. The open circuit terminal voltage of a battery is 12.6 V. When a resistance $R = 4.00\ \Omega$ is connected between the terminals of the battery, the terminal voltage of the battery is 10.4 V. What is the internal resistance of the battery ?

14. For the circuit shown in the figure, determine the current I in the circuit and the potential difference across the terminals of the battery ?



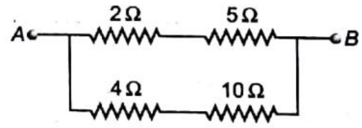
4V (Ideal) (a) (Real) 4V, 3Ω (b)

15. Determine the current I in the circuit and also determine the potential difference across battery E_1 and resistor R_1 ? [*All batteries are ideal*].

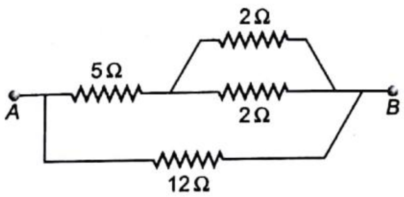


16. Determine the equivalent resistance across AB in following circuits :

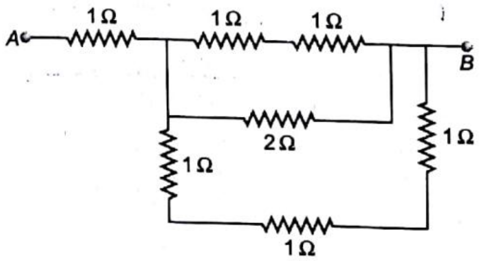
(a)



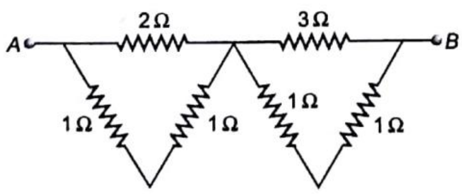
(b)



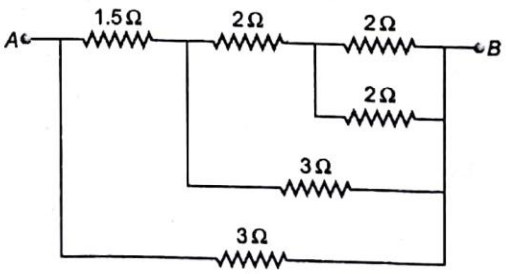
(c)



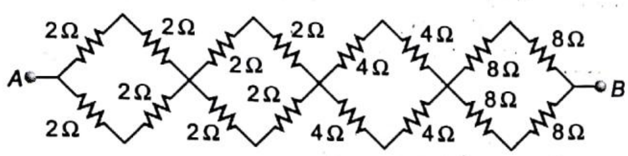
(d)



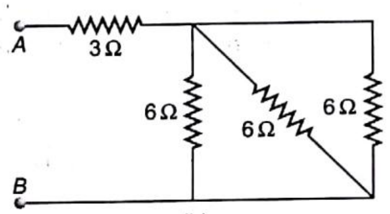
(e)



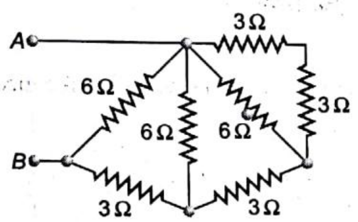
(f)



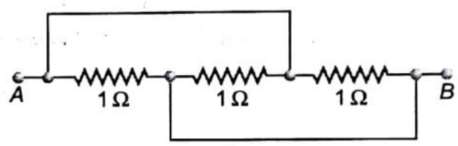
(g)



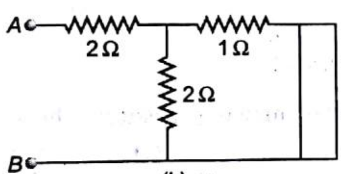
(h)



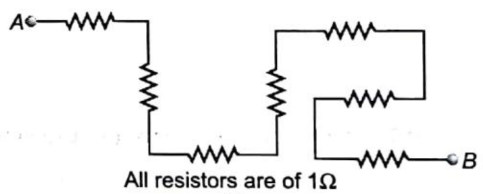
(i)



(j)

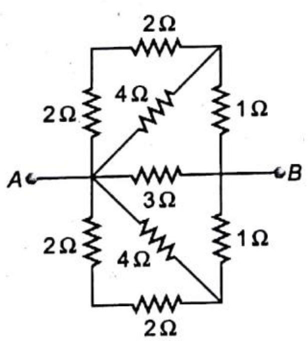


(k)

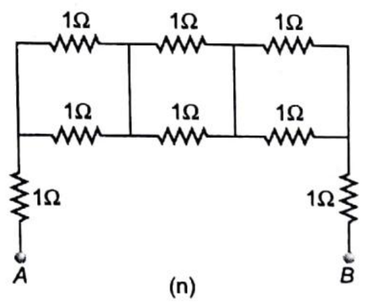


All resistors are of 1Ω

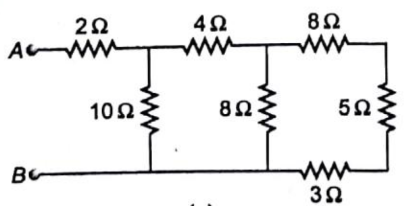
(l)



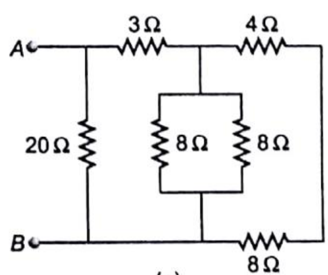
(m)



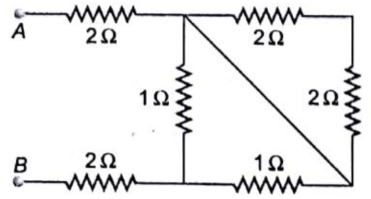
(n)



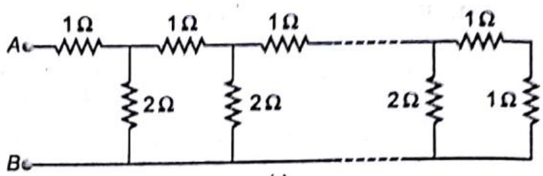
(o)



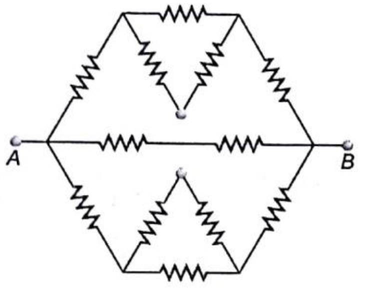
(p)



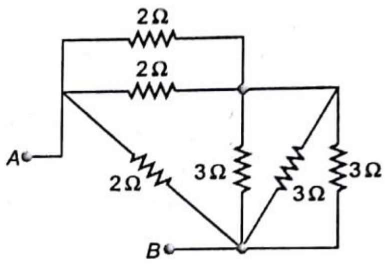
(q)



(r)



(s) All resistors are of 1 Ω



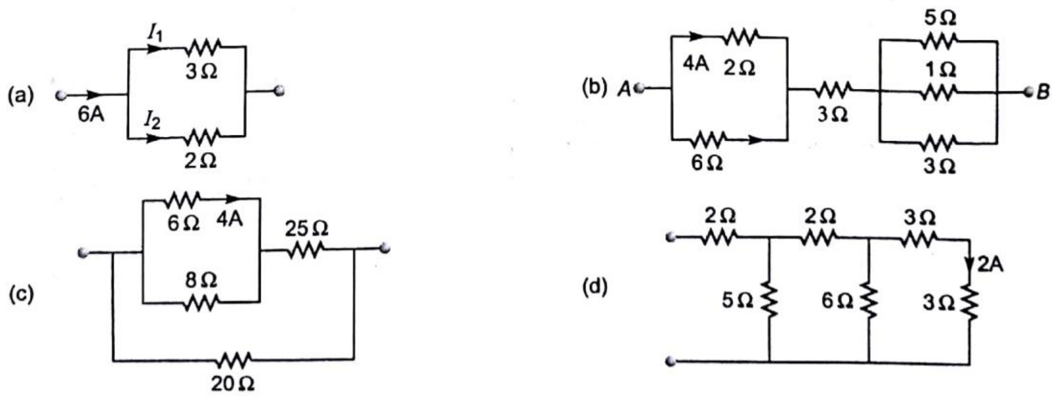
(t)

17. An electric bulb of resistance 200 Ω is connected across a battery. The battery supplies an electric current of 2 A to the bulb. Determine the rate at which the bulb is consuming electrical energy.

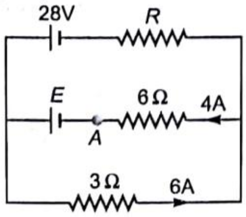
18. Three resistors having resistances of 1.6 Ω, 2.4 Ω and 4.8 Ω are connected in series to a 22 V battery (ideal). Find
(a) the current in each resistor.
(b) the potential difference across each resistor.
(c) the total current through the battery.
(d) the power dissipated by each resistor.
(e) which resistor dissipates maximum power ?

19. Repeat above questions for parallel combination of the same resistors.

20. For the given circuits, the current through one or more resistors is as shown. Determine the current through other resistors.

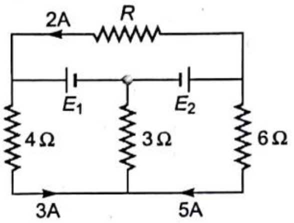


21. For the circuit shown in the figure, determine



(a) the current in resistor R.
(b) the value of resistor R.
(c) the emf E.
(d) if circuit is broken at A, then current through resistor R.

22. For the circuit shown in the figure, determine

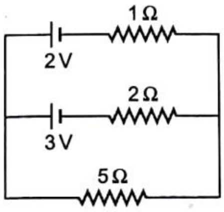


(a) the current through 3 Ω resistor
(b) value of resistance R
(c) value of emfs E_1 and E_2

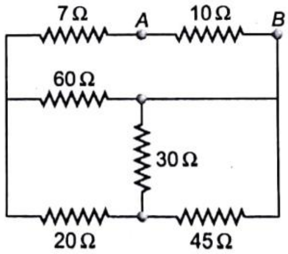
23. Two identical light bulbs are to be connected to a source (ideal) of emf 8 V. Both the bulbs have an electrical resistance of 2 Ω. Find out the current through each bulb, potential difference across each bulb, power delivered to each bulb and also the power delivered to entire network if the bulbs are connected in (a) series, and (b) parallel.

Observe the results you get. Suppose, one of the bulbs gets fused out *ie*, its filament is broken, then what happens to other bulbs in (a), and in (b) ?

24. For the circuit shown in the figure find the current in all the branches.

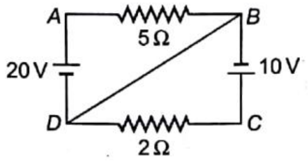


25. Determine the equivalent resistance across AB for the circuit shown in the figure.

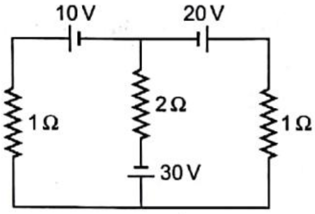


26. The rating of a bulb is 100 W and 220 V, then determine
(a) the resistance of the filament of bulb.
(b) the current through filament for 110 V.
(c) if the bulb is operated at 110 V then find power consumed by bulb.

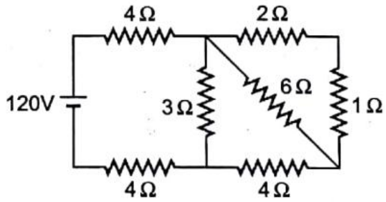
27. For the given circuit find out the current in wire BD.



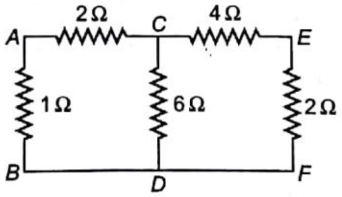
28. Find the current in each branch of the shown circuit.



29. For the circuit shown in figure determine the current through 2 Ω resistor.

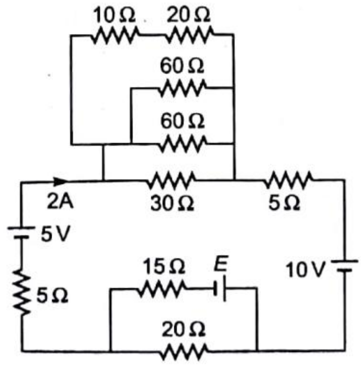


30. For the given circuit, determine the equivalent resistance between the following points :



(a) A and B (b) C and D (c) E and F

31. For the circuit shown in figure determine the emf E of the unknown battery. Is the polarity of this battery of shown correctly? Also determine the time in which the 10Ω resistor dissipates 60 J of thermal energy.



32. A resistor R_1 consumes electrical power P_1 when connected to a battery of emf E. Another resistor R_2 when connected across the same battery consumes power P_2. In terms of P_1 and P_2, determine the total power consumed when both the resistors are connected in (a) series, and (b) parallel.

33. An electric heater of 1.5 kW and 220 V is operated at 110 V, then determine
(a) the resistance of device.
(b) current through device.
(c) power consumed by device.
(d) energy consumed by device in 1 h.

34. The table shown below gives the details of usage of electrical appliances in a house. The voltage coming from supply line is 110 V. Determine the monthly electricity bill for the house. Cost of 1 unit is ₹ 6.

S. No.	Applicance	Rating	Daily Usage in Hrs	No. of Devices
1.	Tube light	40 W, 220 V	8	5
2.	Fan	60 W, 110 V	12	8
3.	Electric heater	1.5 kW, 220 V	1	1
4.	AC	2 kW, 220 V	6	1

Assume the month to be of 30 *days.*

C. Fill in the Blanks

1. If electric field intensity in a region is zero, then electric potential at all the points in the region is ....
2. KCL is based on ....
3. KVL is based on ....
4. If the terminals of a battery are connected by a conducting wire of zero resistance (this process is termed as shorting), then the potential difference across the terminals of the battery is ....
5. Nine identical wires each of diameter d and length L are connected in parallel. The combination has the same resistance as a single similar wire of length L but whose diameter is ....
6. Resistances of 2 Ω, 4 Ω and 6 Ω and a 24 V battery are all connected in series. The current in 2 Ω resistor is ....

D. True/False

1. Charge always moves from a conductor having higher potential to a conductor having lower potential.
2. The motion of charge between two points takes place only if some potential difference (non-zero) exists between the two points.
3. For a steady current to be there in a conductor, it must be a part of the complete circuit.
4. From $R = \frac{V}{I}$ we can say $R \propto V$ and $R \propto \frac{1}{I}$.
5. The points/junction in an electric circuit joined by a conducting wire are at the same potential.
6. If the operating voltage is not equal to the specified voltage for any electrical device, then the resistance of the device is different from if it is operated at the rated voltage.
7. The rate at which a battery is supplying energy to an electrical device is always same as that at which its some other form of energy is used.

High Skill Questions

Exercise 2

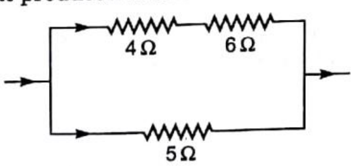
A. Only One Option Correct

1. The potential difference across the terminals of the battery when it is not connected in the circuit is
   (a) equal to its emf
   (b) greater than its emf
   (c) smaller than its emf
   (d) Zero

2. The potential difference across the terminals of an ideal battery when a current is withdrawn from it is
   (a) equal to its emf
   (b) greater than its emf
   (c) smaller than its emf
   (d) Zero

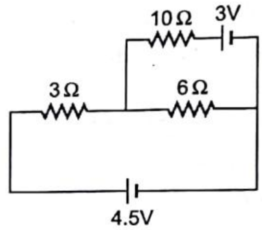
3. The potential difference across the terminals of a real battery when a current is withdrawn from it
   (a) is equal to its emf
   (b) greater than its emf
   (c) smaller than its emf
   (d) is zero

4. For the circuit shown in the figure, the heat produced in the 5 Ω resistor is 10 Js^{-1}. The heat produced in the 4 Ω resistor is



   (a) 1 Js^{-1}
   (b) 2 Js^{-1}
   (c) 3 Js^{-1}
   (d) 4 Js^{-1}

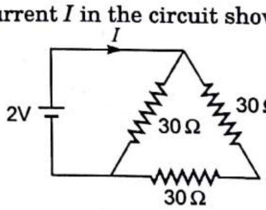
5. Find out the current through 10 Ω resistor shown in the figure.



   (a) Zero (b) 1 A (c) 2 A (d) 5 A

6. If two bulbs rated 2.5 W-110 V and 100 W-110 V are connected in series to a 220 V supply, then
   (a) 2.5 W bulb will fuse
   (b) 100 W bulb will fuse
   (c) Both will fuse.
   (d) None will fuse.

7. The current I in the circuit shown is



   (a) $\frac{1}{45}$ A (b) $\frac{1}{15}$ A (c) $\frac{1}{10}$ A (d) $\frac{1}{5}$ A

8. Three equal resistors connected in series across a source of emf together dissipate 10 W of power. If the same resistors are connected in parallel across the same source, then the total power dissipated would be
   (a) 60 W (b) 90 W
   (c) 100 W (d) None of these

9. A battery is connected across a series combination of two identical resistors. If the potential difference across the terminals is V and the current in the battery i, then

(a) the potential difference across each resistor is = will be V and current in each resistor i

(b) the potential difference across each resistor is = will be $V/2$ and current in each resistor $i/2$

(c) the potential difference across each resistor V and current in each resistor will be $i/2$

(d) the potential difference across each resistor $\frac{V}{2}$ and current in each resistor will be i

10. A battery is connected across a parallel combination of two identical resistors. If the potential difference across the terminals of the battery is V and current through it is i,

(a) the potential difference across each resistor will be V and current in each resistor i

(b) the potential difference across each resistor will be $V/2$ and current in each resistor $i/2$

(c) the potential difference across each resistor V and current in each resistor will be $i/2$

(d) the potential difference across each resistor $V/2$ and current in each resistor will be i.

11. A total resistance of 3 Ω is produced by combining an unknown resistor R with a 12 Ω resistor. What is the value of R and how it is connected to the 12 Ω resistor ?

(a) 4 Ω, parallel (b) 5 Ω, parallel
(c) 6 Ω, parallel (d) 4 Ω, series

12. By using only two resistors R_1 and R_2 a student is able to obtain resistances of 3 Ω, 4 Ω, 12 Ω and 16 Ω. The values of R_1 and R_2 (in ohms) are

(a) 3, 4 (b) 4, 12 (c) 12, 16 (d) 2, 16

13. An electric appliance withdraws a current of 3.4 A from 220 V supply, the current withdrawn by the same appliance when it is connected to 110 V supply is

(a) 3.4 A (b) 1.7 A
(c) 6.8 A (d) None of these

14. A resistor is having a resistance of 176 Ω. The number of these resistors which would be connected in parallel so that their combination withdraws a current of 5 A from a 220 V supply is

(a) 1 (b) 2 (c) 4 (d) 8

15. A current of 2 A passes through a 12 V electric bulb. The electrical energy consumed by the bulb in 20 min is

(a) 30.8 kJ (b) 30.8 kWh
(c) 8 J (d) 18 kWh

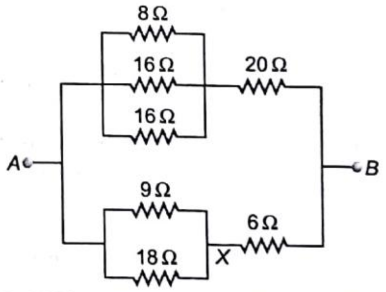
16. For a steady current to exist in a circuit, the circuit

(a) must be closed
(b) must have electrostatic forces only
(c) may be opened
(d) must have non-electrostatic forces only

17. The potential difference across the terminals of a battery is 9 V when it discharged with a current of 3 A. When the same battery is getting charged by 2 A current, the potential difference across its terminals is 12 V. The emf and internal resistance of the battery is

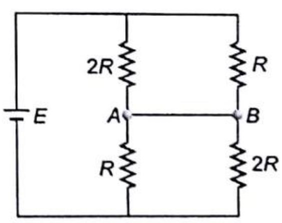
(a) $\frac{54}{5}$ V, $\frac{3}{5}$ Ω (b) $\frac{3}{5}$ V, $\frac{54}{5}$ Ω
(c) 11 V, $\frac{2}{3}$ Ω (d) None of these

18. The potential difference across AX when a current of 1 A flows through 8Ω resistor in the shown circuit is



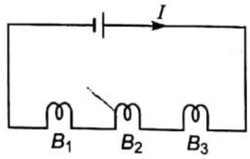
(a) 48 V (b) 18 V (c) 24 V (d) 16 V

19. The current through wire AB in the shown circuit is



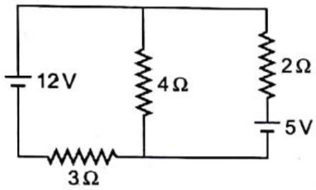
(a) Zero
(b) $E/4R$
(c) $3E/4R$
(d) $E/2R$

20. In the circuit shown, the value of I is 1 A. The bulbs B_1 and B_2 are brightly light, but B_3 is not light. The possible reason for B_3 not being light may be



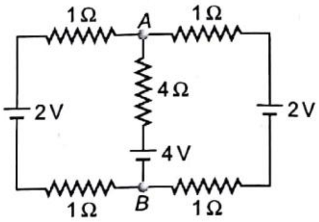
(a) the circuit may be opened
(b) the filament of B_3 is broken
(c) the connecting wire between B_2 and B_3 is broken
(d) the resistance of B_3 is very small as compared to resistance of B_1 and B_2

21. The current in 4 Ω resistor in the shown circuit is



(a) 0.5 A (b) 1.0 A
(c) 1.5 A (d) 2.0 A

22. The potential difference between points A and B for the shown circuit is



(a) 4 V (b) 2.4 V
(c) Zero (d) 2 V

23. In above question the current through 4 Ω resistor is
(a) Zero (b) 0.4 A
(c) 1 A (d) 0.5 A

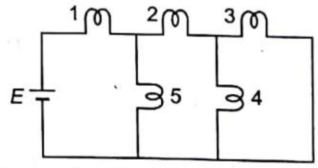
B. More Than One Options Correct

1. Mark out the correct statement(s).
(a) In an isolated conductor no steady current can be there
(b) For a steady current to be there in an electric circuit, it must contain a source of electrical energy
(c) For a steady current to be there in a conductor, it must be a part of closed path.
(d) None of the above

2. The internal resistance of a battery depends on
(a) the medium in between the plates
(b) the plates separation
(c) cross-section area of plates
(d) pressure in between the plates

3. Two 110 V light bulbs, one 25 W and other 100 W, are connected in series to a 110 V source. For the given situation, mark out the correct statement(s).
(a) Current through both bulbs is same
(b) 25 W bulb is lit with more brightness
(c) Potential difference across each both bulbs would be different
(d) None of the above

4. In the circuit shown, a fuse in one of the bulbs causes all the bulbs to go out. Which one is not that bulb ?



(a) 1 (b) 2 (c) 3 (d) 4

5. Electrical power consumed by a circuit can be written as
(a) VI (b) $\dfrac{V^2}{R}$
(c) I^2R (d) None of these
where symbols have their usual meaning.

6. Two wires made of the same material have the same length but different diameters. They are connected in parallel to a battery. The quantity that is NOT the same for the wire is
(a) the end to end potential difference
(b) the current
(c) the resistance
(d) All of the above

C. Assertion & Reason

Directions (Q. Nos. 1 to 6) *Some questions (Assertion-Reason type) are given below. Each question contains* ***Statement I (Assertion)*** *and* ***Statement II (Reason)****. Each question has 4 choices (a), (b), (c) and (d) out of which* ***ONLY ONE*** *is correct. So, select the correct choice.*

Choices are

(a) **Statement I** is True, **Statement II** is True; **Statement II** is a correct explanation for **Statement I**

(b) **Statement I** is True, **Statement II** is True; **Statement II** is **NOT** a correct explanation for **Statement I**

(c) **Statement I** is True, **Statement II** is False

(d) **Statement I** is False, **Statement II** is True

1. **Statement I** For a steady current to be in an electric circuit, it must contain a source of electrical energy.
   Statement II Source of electrical energy causes the positive charge to move from low potential to high potential region within it.
2. **Statement I** The direction of current is same as that of direction of flow of net positive charge.
   Statement II Current is a scalar quantity.
3. **Statement I** The obstruction posed by any substance in the flow of electric charge through it is termed as resistance.
   Statement II Resistance is the property of conductors only.
4. **Statement I** $V = IR$ represents mathematical form of Ohm's law.
   Statement II Resistance of an object is independent of voltage applied across it or current passing through it.
5. **Statement I** In steady state the current at any point in a closed circuit is same, even if circuit is made up of the wire having non-uniform thickness.
   Statement II In steady state the net charge is not accumulating at any of the electrical devices.
6. **Statement I** Two conducting wires of the same material and of same length are taken having different thickness. Obstruction in the flow of charge is lesser in the thick wire
   Statement II Resistance of the conducting wire is given by, $R = \frac{\rho l}{A}$.

D. Comprehend the Passage Questions

Passage I

A piece of aluminum is 3.0 cm long and has a square cross-section 0.5 cm on a side. A potential difference of 8.4 V is maintained across its long dimensions. Resistivity of aluminium is 2.82×10^{-8} Ω-m.

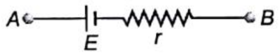
Based on above information, answer the following questions.

1. The resistance of the block is
   (a) 33.84 μΩ (b) 45.68 μΩ
   (c) 0.94 μΩ (d) None of these
2. The current through this block is
   (a) 0.25×10^6 A (b) 0.18×10^6 A
   (c) 8.94×10^6 A (d) None of these

Passage II

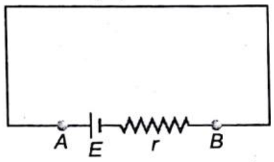
An ideal battery is having zero internal resistance, while a real battery is having non-zero internal resistance. Consider a real battery having emf E and internal resistance r, the terminal of the battery are marked A and B as shown in the figure. A is the positive terminal while B is the negative terminal. Some common things are mentioned below :

(A) When nothing is connected across the battery terminals, the battery is said to be open-circuited.



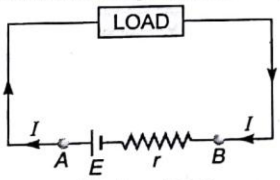
Open circuit

(B) When the terminals A and B of the battery are connected by means of zero resistance wire the battery is said to be short-circuited.



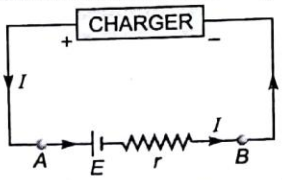
Short circuit

(C) When some load is connected across the terminals of the battery, then battery energizes the circuit and gets itself discharged. In this case the battery supplies some current to the circuit *ie,* it supplies some electrical energy to the load. This is termed as discharging of battery.



Discharging of battery

(D) When the battery gets discharged, we require it to get it charged again with the help of some charger. When the battery is consuming energy from some external source (charger), then it is said to be getting charged. In this case the current enters into the positive terminal of battery.



The above four cases are equally valid for ideal battery too.

For a real battery of emf E and internal resistance answer the following questions :

3. When the battery is open-circuited, the potential difference across its terminals is

(a) Zero (b) E

(c) $>E$ (d) $<E$

4. When the battery is open circuited, the current through battery is

(a) Zero (b) $\frac{E}{r}$

(c) $>\frac{E}{r}$ (d) $<\frac{E}{r}$

5. When the battery is short-circuited, the potential difference across its terminal is

(a) Zero (b) E

(c) $>E$ (d) $<E$

6. When the battery is short-circuited, the current through it is

(a) Zero (b) $\frac{E}{r}$

(c) $>E$ (d) $<E$

7. When the battery is short-circuited, the terminal

(a) A is at higher potential

(b) B is at higher potential

(c) Both are at same potential.

(d) Can't say anything

8. When the battery is connected across a load of resistance R, then

(a) potential difference across its terminals is $>E$

(b) potential difference across its terminal is $<E$

(c) potential difference across its terminal is $\frac{ER}{R+r}=E-Ir$ where I is current in circuit.

(d) potential difference current in the circuit is $\frac{E}{R+r}$

9. When the battery is getting charged by a current I, the potential difference across the terminals of the battery is

(a) E

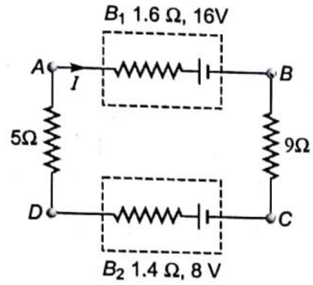
(b) Zero

(c) $E-Ir$

(d) $E+Ir$

Passage III

The circuit shown in the figure contains two batteries both being real ones. For the shown circuit, answer the following questions :



10. The value of I is

(a) $\frac{8}{17}$ A (b) $-\frac{8}{17}$ A

(c) $\frac{1}{2}$ A (d) $-\frac{1}{2}$ A

11. The potential difference across 5 Ω resistor is

(a) $\frac{40}{17}$ V with A at higher potential

(b) $\frac{40}{17}$ V with D at higher potential

(c) $\frac{5}{2}$ V with A at higher potential

(d) $\frac{5}{2}$ V with D at higher potential

12. Mark out the correct statements.

(a) Battery B_1 is getting charged and B_2 is getting discharged.

(b) Battery B_1 is getting discharged and B_2 is getting charged.

(c) Both are getting charged.

(d) Both are getting discharged.

13. The potential difference across the terminals of battery B_2 is

(a) 8.66 V (b) 7.34 V

(c) 8 V (d) None of these

Answers

Towards Proficiency Problems
Exercise 1

B. Numerical Answer Types

1. A, 7 μC **2.** 3.2×10^{-5} A **3.** 1.25×10^{19} **4.** $\frac{V}{R}, \frac{2V}{R}$ **5.** $2R$

6. $4R, \rho$ **7.** $\frac{R_1}{R_2}$ **8.** 0.01 m **9.** $250\rho, 10\rho, 40\rho$ **10.** 2.5 Ω

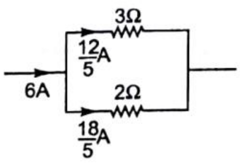
11. 0.65 Ω **12.** 2.1×10^{-6} Ωm **13.** 1.18 Ω **14.** (a) 0.8 A, 4 V, (b) 0.5 A, 2.5 V

15. $-\frac{5}{17}$ A, 5 V, $\frac{25}{17}$ V **16.** (a) 10 Ω (b) $\frac{14}{3}$ Ω (c) 4 Ω (d) $\frac{7}{4}$ Ω (e) $\frac{11}{5}$ Ω (f) $\frac{3}{2}$ Ω (g) 16 Ω (h) 5Ω (i) 3 Ω (j) $\frac{1}{3}$ Ω (k) $\frac{8}{3}$ Ω (l) 7 Ω (m) 1 Ω (n) 3.5 Ω (o) $\frac{396}{58}$ Ω (p) $\frac{60}{13}$ Ω (q) $\frac{40}{9}$ Ω (r) 2 Ω (s) $\frac{4}{5}$ Ω (t) 1 Ω

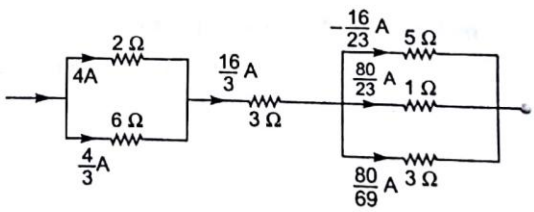
17. 800 W **18.** (a) 2.5 A (b) 4 V, 6 V, 12 V (c) 2.5 A (d) 10 W, 15 W, 30 W (e) 4.8 Ω

19. (a) 13.75 A, 9.17 A, 4.58 A (b) 22 V (c) 27.5 A (d) 302.5 W, 201.67 W, 100.83 W (e) 1.6 Ω

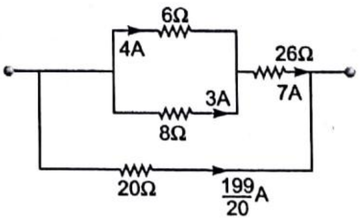
20. (a)



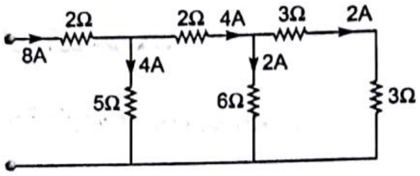
(b)



(c)



(d)



21. (a) 2 A, (b) 5 Ω, (c) 42 V, (d) 3.5 A

22. (a) 8 A, 9 Ω, 36 V, 54 V

23. (a) 2 A, 4 V, 8 W, 16 W; (b) 4 A, 8 V, 32 W, 64 W

24. $\frac{9}{17}$ A, $\frac{13}{17}$ A, $\frac{22}{17}$ A

25. 7.5 Ω

26. (a) 484 Ω, (b) 0.23 A, (c) 25 W

27. 1 A

28. 8 A, 6 A, 2 A

29. $\frac{8}{3}$ A

30. (a) $\frac{5}{6}$ Ω, (b) $\frac{3}{2}$ Ω, (c) $\frac{3}{2}$ Ω

31. 108.75 V, No, 13.5 s

32. (a) $\frac{P_1 P_2}{P_1 + P_2}$, (b) $P_1 + P_2$

33. (a) 32.27 Ω, (b) 3.41 A, (c) 375 W, (d) 0.375 kWh

34. Rs 1716.3

C. Fill in the Blanks

1. Same **2.** Conservation of charge **3.** Conservation of energy
4. Zero **5.** 3*d* **6.** 2 A

D. True/False

1. F **2.** T **3.** T **4.** F **5.** T **6.** F **7.** F

High Skill Questions
Exercise 2

A. Only One Option Correct

1. (a) **2.** (a) **3.** (c) **4.** (b) **5.** (a) **6.** (a) **7.** (c) **8.** (b) **9.** (d) **10.** (c)
11. (a) **12.** (b) **13.** (b) **14.** (c) **15.** (a) **16.** (a) **17.** (a) **18.** (c) **19.** (b) **20.** (d)
21. (c) **22.** (b) **23.** (b)

B. More Than One Options Correct

1. (a, b, c) **2.** (a, b, c) **3.** (a, b, c) **4.** (b, c, d) **5.** (a, b, c)
6. (b, c)

C. Assertion & Reason

1. (a) **2.** (b) **3.** (c) **4.** (d) **5.** (a) **6.** (a)

D. Comprehend the Passage Questions

1. (a) **2.** (a) **3.** (b) **4.** (a) **5.** (a) **6.** (b) **7.** (c) **8.** (b) **9.** (d) **10.** (b)
11. (a) **12.** (b) **13.** (a)

Explanations

Towards Proficiency Problems

Exercise 1

Numerical Answer Types

1. As the positive charge of A is decreasing after making connection, so it means charge (+ve) flows from A to B *ie*, A is at a higher potential. Let in final equilibrium situation charge on B be q μC, so from conservation of charge

$$3\ \mu C + 6\ \mu C = 2\ \mu C + q$$

$$\Rightarrow \quad q = 7\ \mu C$$

2. $i = \dfrac{\Delta q}{\Delta t} = \dfrac{10^{12} \times 1.6 \times 10^{-19}}{5 \times 10^{-3}}$ A

$$= 3.2 \times 10^{-5}\ \text{A}$$

3. $i = \dfrac{\Delta q}{\Delta t}$

$$2\ \text{A} = \frac{n \times 1.6 \times 10^{-19}}{1}$$

$\Rightarrow n = 1.25 \times 10^{19}$ in a direction opposite to current.

4. Resistance is independent of voltage applied across the resistor.

$$I_{\text{old}} = \frac{V}{R} \quad \text{and} \quad I_{\text{new}} = \frac{2V}{R}$$

5. $R = \dfrac{\rho l}{A}$

Here l is doubled keeping ρ and A constant, so R also gets doubled.

6. Resistivity is a property of the material and is independent of geometrical structures. Let l_1 and A_1 be initial lengths and cross-section area of wire and l_2, A_2 are the respective quantities after stretching.

From conservation of volume (mass),

$$l_1 A_1 = l_2 A_2$$

It is given that $l_2 = 2l_1$, so $A_2 = \dfrac{A_1}{2}$.

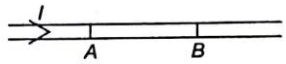
$$R_1 = \frac{\rho l_1}{A_1} = R$$

$$R_2 = \frac{\rho l_2}{A_2} = \frac{4\rho l_1}{A_1} = 4R$$

7. $R_1 = \dfrac{\rho_1 l}{A}$, and $R_2 = \dfrac{\rho_2 l}{A}$

So, $$\frac{R_1}{R_2} = \frac{\rho_1}{\rho_2}$$

8. $V_A - V_B = 1.6 \times 10^{-2} = IR = 1200 \times \dfrac{\rho l}{A}$



$$\Rightarrow \quad 1.6 \times 10^{-2} = 1200 \times \frac{1.72 \times 10^{-8} \times 0.24}{\pi r^2}$$

$$\Rightarrow \quad r = 0.01\ \text{m}$$

9. $R = \dfrac{\rho l}{A}$, in this expression l is the path-length through which charge will flow and A is the cross-sectional area normal to the flow of charge.

11. $V = E - Ir$

$$\Rightarrow \quad 1.3 = 1.5 - I \times 0.1 \quad \Rightarrow I = 2\ \text{A}$$

$I = \dfrac{E}{R + r}$, where R is load resistance.

$$\Rightarrow \quad 2 = \frac{1.5}{R + .01} \quad \Rightarrow \quad R = 0.65\ \Omega$$

12. $R = n \times \dfrac{\rho l}{A}$

where n is the number of turns in the coiled spring, and l is perimeter of one coil.

$$\Rightarrow \quad 1.74 = \frac{75 \times \delta \times 2\pi \times \frac{3.50}{2} \times 10^{-2}}{5 \times 10^{-6}}$$

$$\Rightarrow \quad \rho = 2.1 \times 10^{-6}\ \Omega\text{-m}$$

13. Open circuit voltage is same as emf, so $E = 12.6$ V.

Let r be the internal resistance, then

$$I = \frac{E}{R + r}$$

Terminal potential difference across the terminals of the battery is,

$$V = 10.4 = E - Ir$$

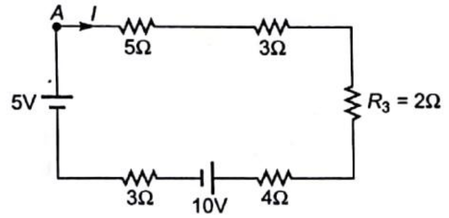
Solving above equations we get

$$1.18\ \Omega = r$$

14. (a) $I = \frac{4}{5}$, and $V = E = 4$ volt.

(b) $I = \frac{4}{5+3}$, and $V = E - Ir = 2.5$ V.

15. Start from point A and go on in clockwise direction, applying KVL.

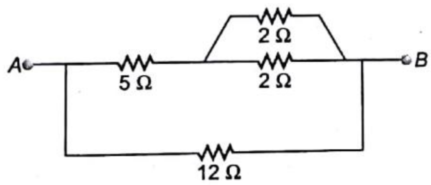


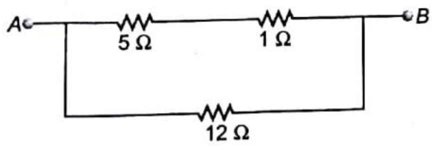
$$5I + 3I + 2I + 4I + 10 + 3I - 5 = 0$$

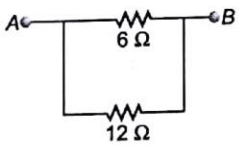
$$I = -\frac{5}{17}\text{ A}$$

–ve sign shows that current I in the circuit is in anticlockwise direction.

16. (c)



⇒ 

⇒ 

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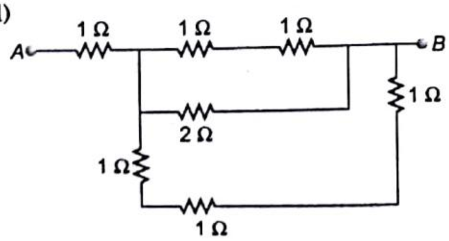
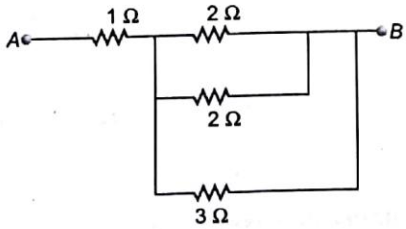
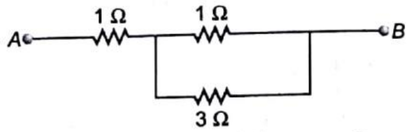
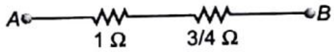
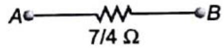
(d)



⇒ 

⇒ 

⇒ 

⇒ 

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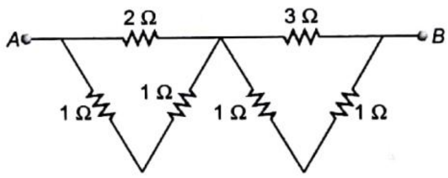
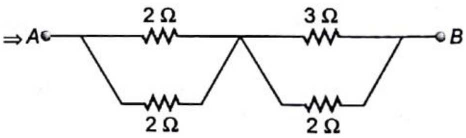


⇒ 

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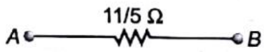
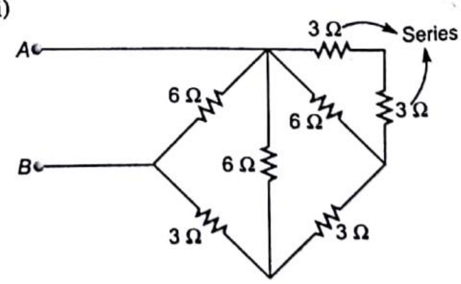
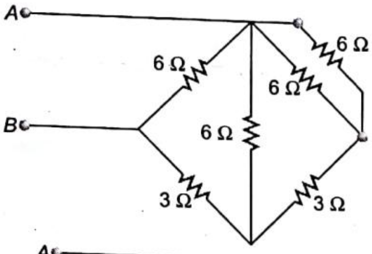
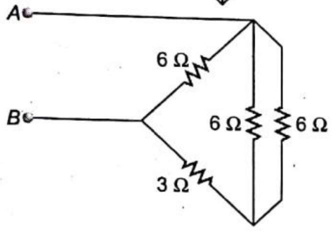


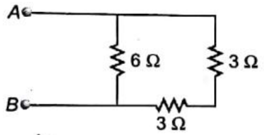
(g) In each cell, upper two and lower two are in series and these series combinations are in parallel. Further, all cells (squares) are connected in series.

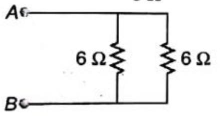
(i)

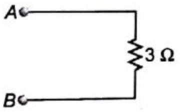


⇒ 

⇒ 

⇒ 

⇒ 

⇒ 

(m) All are in series

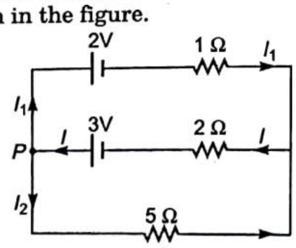
(n) Simple series parallel combination

(r) Start from right hand last loop, in which two 1Ω resistors are in series which is in parallel with 2Ω, and this continues.

17. $P = VI = I^2R = 4 \times 200 = 800$ W

20. Use KCL at various junctions and KVL for various loops.

24. Let 3V battery supply current I and the distribution of current is various branches is as shown in the figure.



Applying KCL at

$$P, I = I_1 + I_2$$

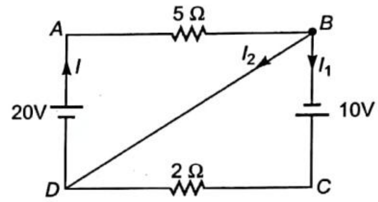
Applying KVL for upper loop,

$$-3 + 2 + I_1 + 2I = 0$$

Applying KVL for lower loop,

$$-3 + 5I_2 + 2I = 0$$

27. The current distribution in the circuit is as shown in the figure.



At B, $I = I_1 + I_2$ → from KCL

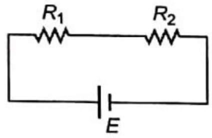
For loop $ABDA$ → $-20 + 5I = 0$

For loop $BCDB$ → $-10 + 2I_1 = 0$

⇒ $I_2 = 1\text{A}$

32. $P_1 = \dfrac{E^2}{R_1}$, and $P_2 = \dfrac{E^2}{R_2}$.

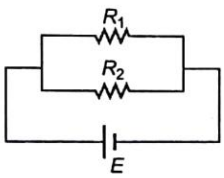
(a) When in series, $I = \dfrac{E}{R_1 + R_2}$.



$$P = I^2(R_1 + R_2) = \frac{E^2}{R_1 + R_2}$$

$$= \frac{E^2}{\dfrac{E^2}{P_1} + \dfrac{E^2}{P_2}} = \frac{P_1P_2}{P_1 + P_2}$$

(b) When in parallel,



$$P = \frac{E^2}{R_1} + \frac{E^2}{R_2} = P_1 + P_2$$

Chapter 15

Magnetism and Magnetic Effect of Current

The First Steps' Learning

- The 'Magical' Lodestone
- Magnets and Magnetism
- Magnetic Field Lines
- Compass Needle and the Earth's Magnetism
- Concept of Magnetic Force

You all would be aware about magnets in one form or the other. You all would have played with magnets during childhood. A lot of magnetic stickers may be there on your refrigerators, which you take and release from some distance and instead of falling down it gets stuck to the door of refrigerator. In one game, you might have played that one ring magnet you propel with the help of other through a zig-zag path. While doing so the two magnets stick to each other and you lose the game. But the application of magnets and magnetism is not restricted up to here only, it has a vast technological application in electric motors, TV picture tubes, loudspeakers, generation of electricity, maglev trains running at very high speeds, ATM machines, medical devices like MRI scanners etc.

In this chapter we are going to explore the development of magnetism, properties of magnets and magnetism, earth's magnetism, causes of magnetism, magnetic field etc, and in the next chapter we will explore interlinking between the electricity and magnetism.

The 'Magical' Lodestone

For thousands of years people have known the rocks which attracted each other when pulled apart in certain directions. These are called lodestone (the leading stone), and initially superstitiously believed to be possessing 'magical powers'. Chinese have known this property more than 4500 thousand years' back without naming it. They had developed the mariner's compass comprising of a splinter of lodestone. A Chinese general, according to legends, used such a compass to guide his army in the dense fog way back in 2500 BC. However, the most popular tale about the magnets is the story of the shepherd Magnes who while herding his sheep in an area of Northern Greece called Magnesia found that both, the nails in his shoes and the metal tip of the herding staff became firmly stuck to a large black rock on which he was standing. Amazed, he dug up the ground beneath to find out the stacks of lodestones. Lodestones contain the iron oxide Fe_3O_4. These rocks were later called Magnets, probably after the name of Magnes or the region of Greece, Magnesia. Even if magnets and the property of magnetism were known for centuries, a systematic study of these started only in the year 1600 with the experiments of William Gilbert in the understanding of the phenomenon of magnetism. He was the first person to have said that the Earth was a giant magnet, and we could produce magnets by beating the wrought iron. He discovered the loss of induced magnetism due to heating. 200 years later, in 1820, Oersted found the connection between magnetism and electricity by bringing a wire carrying electric current close to a magnetic compass which caused a deflection of the compass needle. This revolutionized the whole physical understanding of nature, and paved the way for a smarter life on this planet through myriad technological advancements.

Here, we shall discuss about the basic properties of magnets, and learn various terms related to the phenomenon of magnetism.

Magnets and Magnetism

It has been observed by Greeks that the lodestone of Magnesia (Now called magnetite) were having the capability to attract unmagnetized iron pieces, this property possessed by the iron ore (magnetite) is termed as magnetism and the splinters of lodestone are termed as magnets. As these stone are naturally occurring these magnets are termed

as natural magnets *ie*, the substances which occur naturally and have the ability to attract certain other substances are termed as *natural magnets*. But these natural magnets are of very less practical importance due to their weak attracting power, and not of desirable shapes. To overcome these disadvantages of natural magnets, scientists devised the artificial magnets. Artificial magnets are prepared artificially from natural magnets or from certain magnetic substances like iron. The advantages which an artificial magnet have over the natural magnets are that they have strong attracting power and are of desired shapes. Some common artificial magnets are bar magnets, horse-shoe magnet, ring magnet etc.

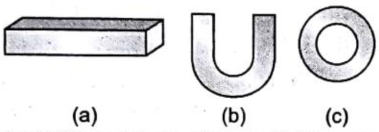


Fig. 15.1 Different types of magnet (a) Bar magnet (b) Horse shoe magnet (c) Ring magnet

The earlier magnetic observations were undoubtedly related to what we call today permanent magnets (permanent magnets are those magnets whose magnetic capabilities can be retained for a very long time) *ie*, Magnesia lodestone as well as artificial magnets which we generally see are permanent magnets. Permanent magnets are found to exert force on each other as well as on pieces of iron that were not magnetized. It was discovered that when an iron rod is brought in contact with a natural magnet, the rod also becomes magnetized. When such a rod is suspended by a string from its centre, it tends to line itself up in a north-south direction. This particular fact makes the concept of magnetism useful in navigation.

Now, before discussing earth's magnetism and the compass needle, let's first have a look at various properties of magnets.

Properties of Magnets

1. Every magnet is having two poles—the North pole, and the South pole (*For an analogy this can be considered as +ve and –ve charges*).

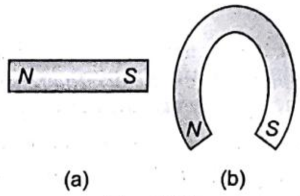


Fig. 15.2

In a ring magnet one side is the north pole, and the other would be a south pole. If a bar magnet (or other magnet) is dipped in iron dust, then the iron dust gets attracted to the magnet, the concentration of iron dust is maximum at poles (corners/ends) while minimum at the centre as shown in figure. The central region where there is no iron dust gets attracted and is termed as neutral region. In terms of magnetic field (*to be discussed soon*), the neutral region is the region where magnetic field is zero.

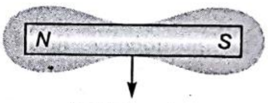


Fig. 15.3

2. When two magnets are placed closed to each other, then they attract or repel each other. The two magnets attract or repel each other depends on the fact that like poles repel each other and unlike poles attract each other (just like charges).

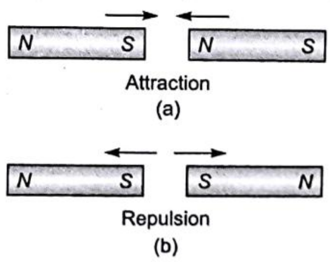


Fig. 15.4 (a) Unlike poles attract each other (b) Like poles repel each other.

3. A magnet cannot only attract other magnet but it can also attract certain other substances like iron, nickel, cobalt etc. This interaction (magnetic) can be

explained with the help of the process of magnetic induction here. When a magnet is placed near an unmagnetized (neutral) iron object, then the pole of opposite polarity gets induced in iron object and hence the magnet attracts the iron object. How these opposite polarity poles get induced is beyond our scope at this level.

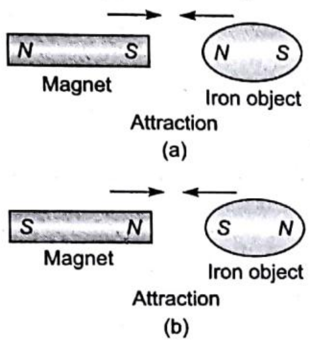


Fig. 15.5

4. Magnetic monopoles don't exist *ie*, a single north or south pole can't exist. If you try to break a magnet to get two individual poles of opposite polarities, then you won't be successful. Because, if we break a magnet, then opposite poles appear at the broken ends. This is not only when we break the magnet from centre. You break from anywhere (whatever small piece may be) you will get two poles of opposite polarities at broken end so that each piece is having two poles—one north, and the other south.

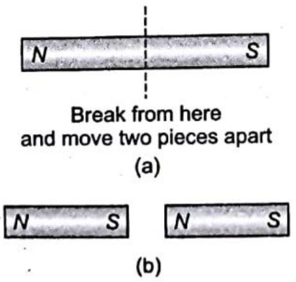


Fig. 15.6

5. Those substances like iron, nickel, steel ,etc which can be attracted by magnets are termed as magnetic substances while which can't be attracted by magnets are termed as non-magnetic substances.
6. Those magnets which can retain their magnetic properties for a long time are termed as permanent magnets while others which lose their magnetic properties after some time are termed as temporary magnets.
7. If we suspend a magnet with the help of a thread from its centre, then it will stay in north-south direction, with its north pole towards the earth's north pole, and its south pole would be aligned towards earth's south pole. Regarding this we will discuss in detail in the section earth's magnetism.

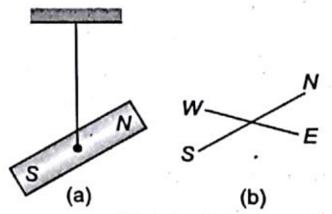


Fig. 15.7

8. The line joining north and south poles of a magnet is termed as magnetic axis.

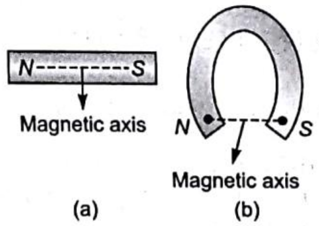


Fig. 15.8

9. Some magnets have strong attracting power while others are having weak attracting power. The attracting power or strength of a magnet is expressed in terms of the pole strength. More is the pole strength, more is the attracting power of magnet. For north pole, pole strength is denoted by $+m$ and for south pole it is denoted by $-m$. Pole strength is a

scalar quantity and its SI unit is A-m. Generally, the magnet have equal and opposite pole strengths.

Fig. 15.9

10. The length of the line joining north and south pole of a magnet is termed as the magnetic length. In actual, the poles are not exactly at the ends of the magnet but they are somewhat inside and hence the actual length of magnet and magnetic length are different with the magnetic length smaller than the actual length of the magnet.

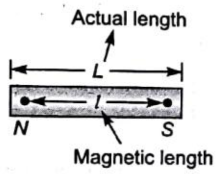


Fig. 15.10

11. With every magnet a term magnetic dipole moment ($\vec{M}$) is associated. It is equal to the product of the pole strength of either pole with the magnetic length of magnet.

ie, $M = m \times l$

It is a vector quantity whose direction is from south to north pole of magnet *ie*, along the magnetic axis.

12. Just like electric force, magnetic force is also a two-step process. Any magnet creates a magnetic field in its surrounding region and when any other magnet or magnetic material comes in this region, it experiences a magnetic force.
13. The region in which the magnetic effects of a magnet or certain magnets can be experienced is termed as magnetic field. The strength/power of the magnetic field can be expressed in terms of magnetic field intensity or by drawing magnetic field lines (*To be discussed later*).
14. Today it is a well-established fact that all magnetic phenomena are due to moving charges and/or currents. Later on, in this chapter, we shall discuss about the magnetic field produced due to current carrying conductors.

Magnetic Field Lines

We can represent any magnetic field by magnetic field lines. In any region where magnetic field is present, we draw magnetic field lines in such a way that the tangent drawn to magnetic field line at any point gives the direction of magnetic field intensity ($\vec{B}$) at that point and the number of field lines crossing any point represents the magnitude of magnetic field intensity at that point *ie*, more is the number of field lines crossing a particular point, stronger is the magnetic field at that point. It has to be kept in mind that in actual no such field lines exist, it is only an imaginary concept to visualize the magnetic field. In any region we draw only few representative lines although the magnetic field is existing in the entire region. If we won't do this, the entire region is filled up by field lines and the visualization of magnetic field won't be clear.

Important Points About Magnetic Field Lines

1. Magnetic field lines are closed curves, they come out from the north pole of the magnet and enter the south pole *ie*, outside the magnet the field lines are from the north to south pole and inside the magnet they are from the south to north pole. For a bar magnet the field lines are as shown in the figure. As the central region of the bar magnet is neutral, there are no magnetic field lines shown.

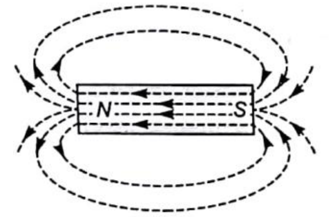
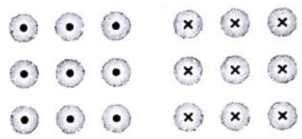


Fig. 15.11 Inside the magnet field lines are from S to N & Outside they are from N to S pole.

2. Two magnetic field lines can't intersect each other, because if they do so then there are two directions of magnetic fields at the intersection point, which is not possible.
3. Crowded lines represent a strong magnetic fields while spaced lines represent weaker a magnetic field.
4. Generally magnetic field patterns are three dimensional and it is quiet often necessary to draw magnetic field lines that point into or out of the plane of paper. We use an encircled dot (•) to represent a magnetic field which is coming out of paper and an encirded cross ⊗ to represent a magnetic field direction into the paper.



Compass Needle and the Earth's Magnetism

Compass needle is a device which is generally used for navigation purposes. It is constructed with the help of a small and light permanent magnet pivoted at its centre, this entire arrangement is enclosed in a plastic or glass case. The needle (permanent magnet) is free to rotate in a horizontal plane and it works on the principle that a freely suspended magnet from its centre always stays in north-south direction, that's why the north pole of needle (painted black) points towards earth's north pole (not exactly alligned) and the south pole of the needle points towards earth's south pole as shown in the figure.

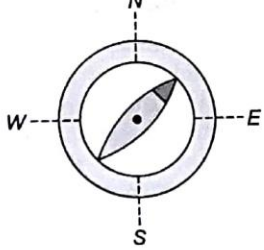


Fig. 15.12 When a magnetic compass is placed near to earth's surface in horizontal position, then the needle alligns itself in north-south direction.

Now the question arises why the compass needle or any other freely suspended magnet always stays in north-south direction ? The answer to this question lies at the heart of *earth's magnetism* or *geomagnetism*.

Today it is a well known fact that the earth itself is a magnet, although the origin of earth's magnetism is a contentious issue of discussion among the physicists. As we know, the idea that the earth is magnetized, was first suggested by Dr. William Gilbert in the end years of 16^{th} century. It is agreed upon by scientists that the earth behaves as a bar magnet (Detailed description is beyond scope here.) which is inclined at a small angle (Approx. 11.5°) to the earth's axis of rotation. The north pole of earth's magnet is towards the geographical south, and the south pole of the earth's magnet is towards the geographical north. The magnetic field lines of the earth's magnet are clearly shown in Fig. 15.13 along with the earth's magnet.

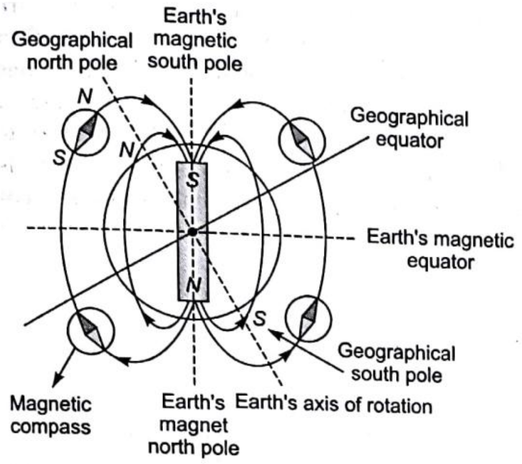


Fig. 15.13 A compass placed at any point in the earth's magnetic field points in the direction of the field lines at that point.

As the earth's magnet's south pole is almost toward north (geographical) pole, so when a magnet is suspended freely the north pole of the magnet points towards geomagnetic south pole or we can say that towards geographical north pole and other pole of magnet *ie*, south pole points towards the geographical south pole. Thus a freely suspended magnet always stays in the north-south direction.

Magnetic Field Intensity Due to Current Carrying Wires

Magnetic field intensity ($\vec{B}$) is a way to represent the strength and direction of magnetic field at any point, just like electric field intensity. It is a vector quantity whose direction is given by some specific rules which we are going to discuss soon, and its SI unit is tesla (T).

As we know the Danish Physicist Hans Christian Oersted, in 1820, first of all experimentally showed the relationship between magnetism and the moving charges. While performing experiment he found that when a magnetic compass is placed near to a current carrying wire, then the needle of the compass deflects. After getting the deflection of the needle he started thinking about why the deflection took place at all ? What he did was that he suspended a wire from two fixed supports and these two fixed supports were then connected to a battery and a rheostat as shown in Fig. 15.14, to establish a current in the wire. When a compass

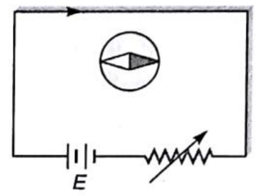


Fig. 15.14

needle is brought near to wire along its length then the needle shows the deflection which shows that some force is acting on needle. Now, the question arises—what causes the deflection of needle ? Who is exerting the force on needle ? As we know that a current carrying wire is electrically neutral, so there won't be any electric force, and the only option left is that needle experiences some magnetic force due to a magnetic field and this magnetic field is created by the current carrying wire. The various observations from this experiment which leads to the conclusion that a moving charge produces a magnetic field are :

- If the current in the wire is changed by varying the resistance of the circuit, then the deflection in the needle changes. It has been concluded that more is the current in the wire, more the needle deflects.
- The deflection in the needle when it is placed above or below the wire is in opposite direction, which confirms that the magnetic field above, and below the wire are in opposite directions.
- If the compass is placed in such a way that needle is perpendicular to the length of wire, then the needle doesn't deflect at all.

Oersted showed that a current carrying element or moving charge is the basic source of magnetic field but an expression for magnetic field intensity due to the current carrying element or moving charge was given by

Biot-Savart. We shall skip over this expression here, and will directly derive the expressions for magnetic field intensity due to various current configurations, and the moving charge.

1. Magnetic field due to an infinite long current carrying wire :

Magnetic field due to linear current carrying wire at a distance r from it is given by,

$$B = \frac{\mu_0 I}{2\pi r}$$

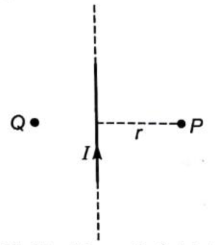


Fig. 15.15 Magnetic field due to infinite long wire

where μ_0 *is the permeability of free space and in SI units its value is* $4\pi \times 10^{-7}$, *have the SI unit T-m / amp.*

Direction of magnetic field intensity can be computed by Right Hand Palm Rule No. 1 (RHPR No. 1) or by *cross-product* rule or by *screw rule*. Here for the linear wire, we will explain the concepts by using RHPR No. 1. According to this rule if you stretch your right hand palm in such a way that your thumb points towards the current *ie*, along the direction of current, the fingers points towards point P where magnetic field has to be computed, then the outward normal to the front face of your palm gives the direction of magnetic field intensity at P.

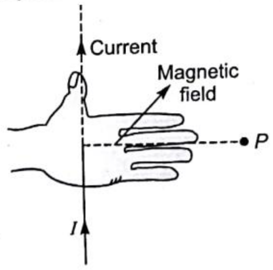


Fig. 15.16 RHPR No.1

So in above situation, at P, the magnetic field is into the plane of paper and at Q it is out of the plane of paper.

The magnetic field lines corresponding to the magnetic field produced by a current carrying wire is as shown as below which are concentric circles.

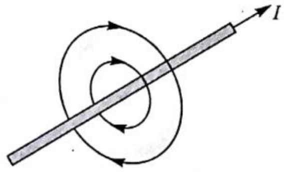


Fig. 15.17 Magnetic field lines due to an infinite long wire are concentric circles.

You can find out the direction of magnetic field due to a linear current carrying conductor in a simpler way also by using right hand thumb rule. You grab the conductor in your right hand such that the thumb points in the direction of current, then the direction in which the fingers are curled gives the direction of the magnetic field.

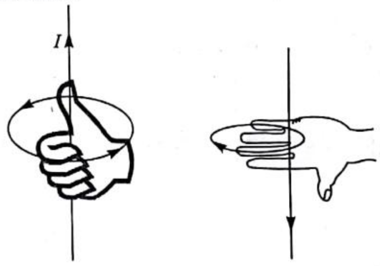


Fig. 15.18

2. Magnetic field due to a current in circular loop : The magnetic field at the centre of a circular current carrying coil of radius R is given by,

$$B = \frac{\mu_0 I}{2R}$$

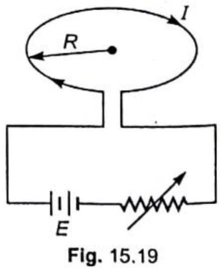


Fig. 15.19

The direction of magnetic field at the centre due to the current carrying coil is shown in Fig. 15.20.

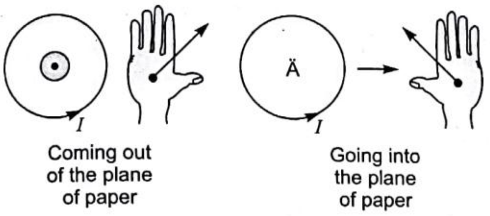


Fig. 15.20

The direction has been found by using RHPR No. 1.

If the coil is having N turns, then the magnetic field at the centre would be given by

$$B = \frac{\mu_0 NI}{2R}$$

Here, all the turns are overlapping *ie*, they all are having the same radius. Actually what happens here is that the current is passes through each turn and the each turn produces a magnetic field at the centre. The total magnetic field at the centre is vector sum of magnetic fields due to individual turns, as magnetic field due to each turn is same in magnitude and direction, so the total magnetic field would be equal to N times the magnetic field due to one turn.

The direction of magnetic field due to coil can also be found by using right hand thumb rule according to the statement—*If the fingers of the right hand are curled along the direction of the current in the loop, then the stretched thumb gives the direction of magnetic field.*

Here, we have talked only about the magnetic field at centre due to coil, but a current carrying coil produces magnetic field everywhere in its surrounding region, which is clear from the diagram of magnetic field lines.

3. Magnetic field due to a moving charge : A current carrying conductor and a moving charge are equivalent in some way because current is nothing but net rate of flow of charge, so the moving charge must also produce a magnetic field in its neighbouring region, which is also experimentally verified. Biot-Savart has given the expression of magnetic field due to a moving charge. Consider a charge $+q$ moving with velocity $\vec{v}$ as shown in figure, then according to Biot-Savart law for moving charges, the magnetic field at point P which is at position vector $\vec{r}$ *wrt* position of charge, is given by

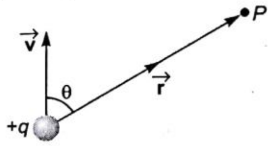


Fig. 15.21

$$\vec{B} = \frac{\mu_0}{4\pi} \times \frac{q(\vec{v} \times \vec{r})}{r^3}$$
$$= \frac{\mu_0}{4\pi} \times \frac{qvr \sin\theta}{r^3} \hat{n}$$
$$= \frac{\mu_0}{4\pi} \times \frac{qv \sin\theta}{r^2} \hat{n}$$

where θ is the angle between $\vec{v}$ and $\vec{r}$, and $\hat{n}$ is the unit vector perpendicular to both $\vec{v}$ and $\vec{r}$.

The direction of magnetic field due to moving charge can be given by RHPR No. 1 or by using cross-product rule.

Using RHPR No. 1 : Stretch your right hand palm in such a way that the thumb points in the direction of motion of charge, and the fingers point in the direction where magnetic field has to be computed, then the outward normal to the front face of your palm gives the direction of magnetic field. If the charge is –ve then the direction of magnetic field would be given by the inward normal to the front face of your palm.

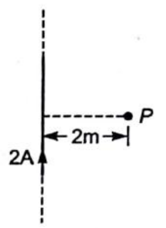
You can understand this also in this way–the direction of motion of positive charge is same as that of current, while the direction of motion of negative charge is opposite to that of current, so replace motion of charges by their equivalent current, and then use RHPR No. 1 as discussed in Current Electricity.

Using cross-product rule : As $\vec{B}$ depends upon the cross-product of $\vec{v}$ and $\vec{r}$, so direction of $\vec{B}$ is same as that of $\vec{v} \times \vec{r}$. The direction of any vector $\vec{A} = \vec{B} \times \vec{C}$ can be found by using cross product rule which states that *curl the fingers of your right hand starting from 1st vector to 2nd vector ie, from $\vec{B}$ to $\vec{C}$, then the direction to which thumb is pointing, is the direction of $\vec{A}$.*

C-BIs

Concept Building Illustrations

Illustration | 1 *An infinite long current carrying wire is as shown in the figure. A current of 2A flows in the wire. Determine the magnetic field at point P as shown.*



Solution From $B = \frac{\mu_0 I}{2\pi R}$

$$B = \frac{4\pi \times 10^{-7}}{2\pi} \times \frac{2}{2}$$

$$= 2 \times 10^{-7} \text{ T}$$

Direction of the magnetic field is into the paper.

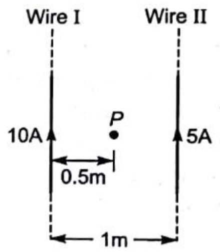
Illustration | 2 *Determine the magnetic moment of a bar magnet whose pole strength is 2 A-m and magnetic length is 0.1 m.*

Solution From, Magnetic moment = pole strength × magnetic length

$\vec{M} = m \times E$, from S to N pole

$$M = 2 \times 0.1 = 0.2 \text{ A-m}^2$$

Illustration | 3 *Two current carrying wires are placed parallel to each other at a separation of 1 m as shown in the figure. Determine the net magnetic field at a point P situated mid-way between the wires.*



Solution Here, two current elements are there, we have to compute magnetic field at point P due to both the wires individually, and then we have to add them vectorially.

Magnetic field at P due to wire I is,

$$B_1 = \frac{\mu_0 I_1}{2\pi r}$$

$$= \frac{4\pi \times 10^{-7}}{2\pi} \times \frac{10}{0.5}$$

$$= 4 \times 10^{-6} \text{ T} \otimes$$

Magnetic field at P due to wire II is,

$$B_2 = \frac{\mu_0 I_2}{2\pi r}$$

$$= \frac{4\pi \times 10^{-7}}{2\pi} \times \frac{5}{0.5}$$

$$= 2 \times 10^{-6} \text{ T} \odot$$

So, the net magnetic field at P is,

$$\vec{B} = \vec{B}_1 + \vec{B}_2$$

$$\vec{B} = 4 \times 10^{-6} \otimes + 2 \times 10^{-6} \odot$$

$$= 4 \times 10^{-6} \otimes - 2 \times 10^{-6} \otimes$$

$$= 2 \times 10^{-6} \text{ T} \otimes$$

ie, 2 μT directed into the plane of paper.

Illustration | 4 *A circular metallic loop of radius 0.1 m carries a current of 10 A. Determine the magnitude of magnetic field at its centre.*

Solution For the circular loop, magnetic field at the centre is

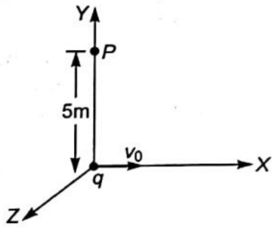
$$B = \frac{\mu_0 I}{2R}$$

$$B = \frac{4\pi \times 10^{-7} \times 10}{2 \times 0.1}$$

$$= 2\pi \times 10^{-5}\ \text{T}$$

Illustration | 5 *A charged particle having a charge +3 μC crosses the origin with a velocity of $\vec{v}_0 = 3\hat{i}\ ms^{-1}$. Determine the magnetic field at a point P (0, 5m, 0) on Y-axis at the instant when the particle crosses the origin.*

Solution For a moving charge, magnetic field at any point is given by



$$\vec{B} = \frac{\mu_0}{4\pi} \times \frac{q(\vec{v} \times \vec{r})}{r^3}$$

$$= \frac{\mu_0}{4\pi} \times \frac{q \times v \sin\theta}{r^2}\ \hat{n}$$

For present situation,

$$q = 3 \times 10^{-6}\ \text{C}$$

$$v = 3\ \text{ms}^{-1}$$

$$\theta = \frac{\pi}{2}$$

$$r = 5\ \text{m}$$

$$B = 10^{-7} \times \frac{3 \times 10^{-6} \times 3 \times \sin\frac{\pi}{2}}{5^2}$$

$$= 3.6 \times 10^{-14}\ \text{T}$$

From RHPR No. 1 direction of B is along +ve Z-axis.

Alternate **:** We can solve the above question by using vectors,

$$\vec{v} = 3\,\hat{i}\ \text{ms}^{-1}$$

$$\vec{r} = 5\,\hat{j}\ \text{m}$$

$$r = |\vec{r}| = 5\ \text{m}$$

So, $\vec{B} = 10^{-7} \times 3 \times 10^{-6}\ \frac{[(3\,\hat{i}) \times (5\,\hat{j})]}{5^3}$

$$= 3.6 \times 10^{-14}\ \hat{k}\ \text{T} \qquad [\hat{i} \times \hat{j} = \hat{k}]$$

*Remember when you use vector form of any expression, the charge has to be substituted with signs.

Concept of Magnetic Force

We know that when a charge is present in an electric field it experiences an electric force. What happens when the same charge is placed in a magnetic field. Will it experience any force? What would are the factors on which this force depends, if it is experiencing any? etc. All these things we are going to study in this section.

Experimentally, it has been found that a moving charge and current carrying elements experience a magnetic force in a magnetic field. If a charged particle having charge q is moving with velocity $\vec{v}$ in a magnetic field intensity $\vec{B}$, then the magnetic force experienced by this charged particle is given by

$$\vec{F} = q(\vec{v} \times \vec{B})$$

Thus the magnetic force acting on the charged particle is perpendicular to both $\vec{v}$ as well as to $\vec{B}$. The direction of magnetic force can be found by using cross product rule or by using Fleming's left hand rule or by using RHPR No. 2. How to find direction of any vector by using cross product rule, we have already

explained. Now we will tell you something about Fleming's left hand rule and RHPR No. 2.

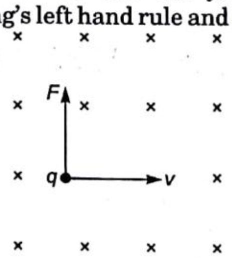


Fig. 15.22 A charge q is moving with velocity $\vec{v}$ perpendicular to a uniform magnetic field $\vec{B}$. The magnetic force acting on it would be perpendicular to both $\vec{v}$ and $\vec{B}$.

If a moving charge is experiencing a magnetic force in a magnetic field, then a current element must also experience a magnetic force when placed in a magnetic field, this is because of the fact that the current is nothing but the net motion of charge only. If a linear conductor of length l carrying current I is placed in an uniform magnetic field B, then it will experience a magnetic force given by

$$F = IlB \sin \theta$$

where θ is the angle between the direction in which the current is there and magnetic field.

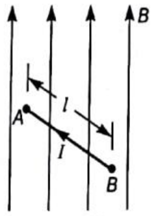


Fig. 15.23

Fleming's left hand rule

This rule gives us the direction of magnetic force experienced by a moving charge of current carrying wire in a magnetic field. According to this—"If the forefinger, second finger (middle finger) and the thumb of the left hand are stretched at right angles to each other with forefinger in the direction of magnetic field, the middle finger in the direction of motion of +ve charge, then the thumb gives the direction of magnetic force experienced by the charged particle".

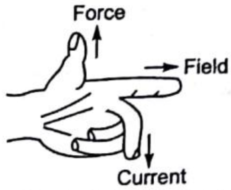


Fig. 15.24 Fleming's left hand rule is used to find the direction of magnetic force.

If instead of +ve charge, –ve charge has to be taken, then the direction of magnetic force is opposite to that found from above statement.

If we have to determine the direction of magnetic force experienced by a current carrying wire, then the middle finger must point along the direction of current while remaining all things being same.

RHPR No. 2

According to this rule—"If we stretch our right hand palm in such a way that fingers point towards magnetic field, the thumb points towards motion of charge, then the outward normal drawn to front face of your palm gives the direction of magnetic force experienced by the charged particle."

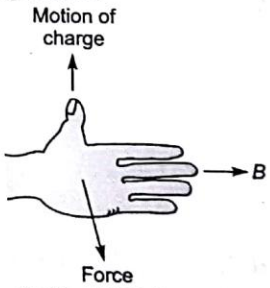


Fig. 15.25 RHPR No. 2 to find the direction of magnetic force.

For negative charge direction of force is opposite to that found from above statement.

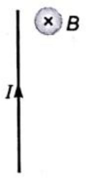
For current carrying conductor the thumb points in the direction of current while remaining things being same.

As the magnetic force is acting perpendicular to velocity of charged particles, so the work done by magnetic force on the charged particle is zero, and hence the kinetic energy of the charged particle can't be changed by a magnetic force.

Proficiency in Concepts (PIC)

Problems

Problem | 1 *A linear conductor of length 10 cm carrying a current of 2 A is placed in a magnetic field of magnitude $B = 3\ T$ as shown in the figure. Determine the magnetic force (magnitude and direction both) acting on the wire.*



Solution The magnitude of magnetic force experienced by the wire is given by

$$F = IBl \sin\theta$$

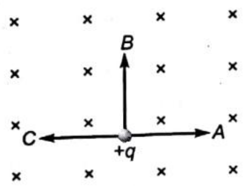
Here, $\theta = \frac{\pi}{2}$ as B is perpendicular to the length of wire

$$I = 2\text{ A}$$
$$l = 10\text{ cm} = 0.1\text{ m}$$
$$B = 3\text{ T}$$

So, $F = 2 \times 3 \times 0.1 \times \sin\frac{\pi}{2} = 0.6\text{ N}$

Direction of F is given by RHPR No. 2, it is towards left.

Problem | 2 *A charged particle having charge $+q$ is moving in a magnetic field which is directed into the plane of paper as shown. If the velocity of particle is along A, B or C then in which direction does the charged particle experiences a magnetic force ?*



Solution When the particle is moving along A, the charged particle will experience a force which acts along upward direction.

For $B \rightarrow$ *towards left*

For $C \rightarrow$ *downward*

Use RHPR No. 2 to answer the above question.

Problem | 3 *A particle of charge 3 μC is moving along a magnetic field of 2 T with a speed of 10 ms^{-1}. Determine the magnitude of magnetic force experienced by the charged particle.*

Solution As the charged particle is moving in a direction parallel to magnetic field *ie*, the angle between velocity vector of particle and $\vec{B}$ is zero, so from $F = qvB\sin\theta$, the charged particle will experience zero magnetic force.

Thus we can say that if the charged particle moves in a direction parallel to or anti-parallel to the magnetic field, then the magnetic force experienced by the charged particle would be zero.

Problem | 4 *A particle having a charge of 2 C is kept at rest in a very strong magnetic field of magnitude 10^4 T. Determine the magnetic force experienced by charge particle ?*

Solution The stationary charge particle present in a magnetic field doesn't experience any magnetic force.

Problem | 5 *A charged particle having charge 3 μC is moving in a magnetic field region having magnitude 3×10^4 T. The velocity of the charged particle is 4 ms^{-1}, and is making an angle of 30° with the field's direction. Determine the magnetic force acting on the charged particle.*

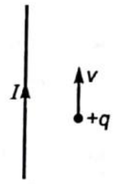
Solution From, $F = qvB\sin\theta$,

$$F = (3 \times 10^{-6}) \times (4) \times (3 \times 10^4) \times \sin 30°$$
$$= 18 \times 10^{-2}\text{ N} = 0.18\text{ N}$$

Towards Proficiency Problems
Exercise 1

A. Subjective Discussions

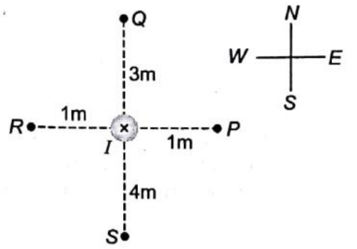
1. When a magnet is suspended freely, it stays in north-south direction. Explain why this happens so ?
2. Which are having stronger attracting power—natural magnets or the artificial magnets ?
3. A positively charged particle is moving along the length of a current carrying wire at some distance from it as shown in the figure. Discuss the subsequent motion of the charged particle.



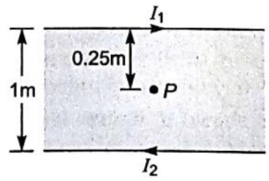
4. A circular loop carrying a current I is placed on a horizontal surface. As seen from above, the current in the loop is in clockwise direction. What is the direction of magnetic field at the centre of loop ? What would be your answer if current as seen from above is in anticlockwise direction ?
5. A wire is placed along north-south direction carrying current from north to south. The wire is placed in a horizontal plane. The region contains a magnetic field which is pointing vertically up. Determine the direction in which the wire experiences the magnetic force. If we rotate the wire in the same horizontal plane by 90°, then what would be the direction of magnetic force experienced by it ? Discuss all possibilities.
6. In above question if the direction of current is reversed, then answer the above question ? Repeat the above question if both direction of current and direction of magnetic field are reversed.
7. A beam of protons moving horizontally is approaching you, but instead of directly coming towards you, it deflects (a) towards your right (b) towards your left (c) upwards (d) downwards. Determine the direction of magnetic field in all these four situations.
8. Permanent magnets use to attract certain unmagnetised substances like iron. Explain this attraction.
9. Can a charged particle move through a magnetic field without experiencing any force ? If so, how? If not, why not ?
10. We explained how to find the direction of magnetic force by using RHPR No. 2. If instead of our right hand we want to use our left hand, then how you do this ?
11. A charged particle, passing through a certain region of space, has a velocity whose magnitude and direction remain constant. (a) If it is known that the external magnetic field is zero everywhere in this region, can you conclude that the external electric field is also zero ? Give reasons. (b) If it is known that the external electric field is zero everywhere, can you conclude that the external magnetic field is also zero ? Give reasons.

B. Numerical Answer Types

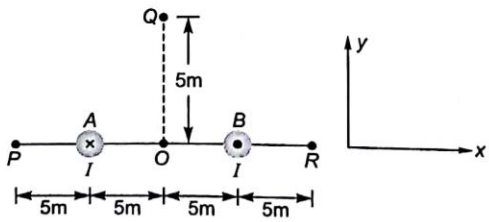
1. An infinite long current carrying wire having current $I = 3$ A is placed as shown in the figure. The wire is placed perpendicular to the plane of paper in which the current is into the plane of paper. Determine the magnitude and direction of magnetic field at four points P, Q, R and S as shown in the figure.



2. A long straight wire carries a current of 48 A. The magnetic field produced by this wire at a certain point is 8×10^{-5} T. How far is the point from the wire ?
3. In a lightning bolt, 15 C of charge flows in a time of 1.5 ms. Assuming that the lightning bolt can be represented as a long, straight wire of current, what is the magnitude of magnetic field at a distance of 25 m from the bolt ?
4. A current carrying circular loop of radius 3 m carries a current of 25 A. Determine the magnitude of magnetic field at the centre of loop ?
5. Two parallel long current carrying wires are placed at a separation of 1 m. If $I_1 = 10$ A and $I_2 = 2$ A, then determine the magnetic field at point P.

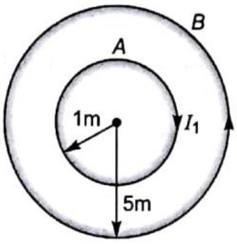


6. Two straight wires A and B are kept parallel to each other at a separation of 10 m. Both carry the same current $I = 50$ A, but are in opposite directions. In A, current is into the paper while in B it is outward. Determine the magnitude of magnetic field and the directions of both at four points O, P, Q and R as shown in the figure ?

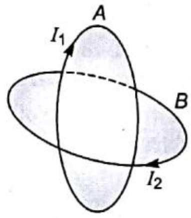


7. A circular coil having 1000 turns and radius 1 m is carrying a current of 25 A. Determine the magnetic field at its centre.

8. Two concentric coils A and B having number of turns 300 and 500 respectively are placed in a horizontal plane as shown in the figure. Current $I_1 = 20$ A and $I_2 = 5$ A are set up in A and B respectively in the directions shown. Determine the magnetic field at the common centre of coils.



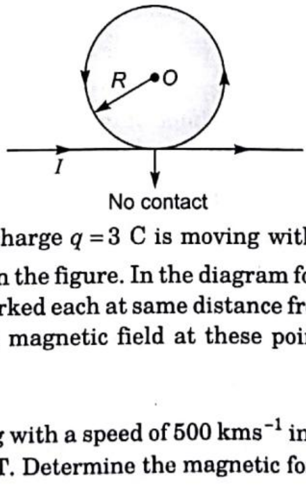
9. Two identical coils having single turn each and of radius R are placed in such a way that their centres coincide but their planes are perpendicular to each other. Currents I_1 and I_2 are there through the coils. Determine the magnitude of resultant magnetic field at their common centres.



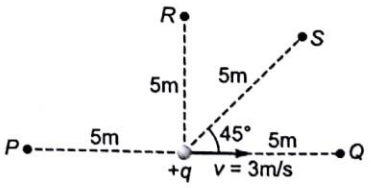
10. Two concentric coils are lying in the same plane. The inner coil has 120 turns, has a radius of 0.5 m and carries a current of 6 A. The outer coil has 200 turns and its radius is 0.75 m. Determine the current which passes through the outer coil if the net magnetic field at their common centre is zero.

11. A piece of copper wire has a resistance per unit length of $5.9 \times 10^{-3}\ \Omega\text{m}^{-1}$. The wire is then wound into a thin flat coil of many turns that has a radius of 0.05 m. The ends of the wire are connected to a 12 V battery. Determine the magnetic field at the centre of coil.

12. A wire is placed in a plane as shown in figure. If a current I passes through the wire, then determine the magnetic field at O.



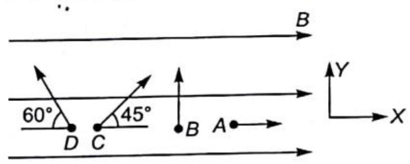
13. A charged particle having charge $q = 3$ C is moving with a velocity of 3 ms^{-1} as shown in the figure. In the diagram four points P, Q, R and S are marked each at same distance from the charge. Determine the magnetic field at these points due to the moving charge.



14. A charge of 5 μC is travelling with a speed of 500 kms^{-1} in a direction perpendicular to magnetic field of magnitude 3×10^{-3} T. Determine the magnetic force acting on the particle.

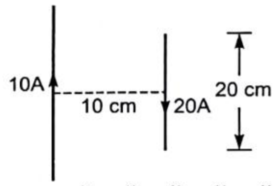
15. A charged particle of charge $8\,\mu C$ is moving perpendicular to a magnetic field with velocity $3\ ms^{-1}$. If it experiences a magnetic force of 3×10^{-3} N, then determine the magnitude of magnetic field.

16. A charged particle having charge 3 C is moving with a speed of $3\ kms^{-1}$ in a magnetic field of magnitude 3 mT. The magnetic field is along +ve x-axis. The diagram shows four paths (directions) along which the particle is moving. Determine the magnetic force (magnitude and direction both) experienced by the charged particle in all the 4 cases.

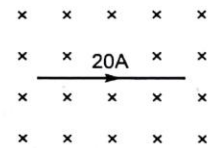


17. Instead of the charge particle, a linear current element of length 10 cm and carrying a current of 10 A is being used in the previous question. The direction of current is to be taken as same as that of velocity direction, then determine the magnetic force experienced by current element in all the four cases.

18. An infinite long conductor carrying current of 10 A is placed as shown in the figure. Another linear current element of length 20 cm and carrying a current of 20 A is placed at a distance of 10 cm from the long conductor. Determine the magnetic force experienced by the current element. If the direction of current in current element is reversed, then repeat these questions.



19. A current element of length 30 cm is carrying a current of 20 A. The region contains a magnetic field of unknown magnitude. The mass of wire is 10 g. Under the action of gravitational and magnetic forces the wire stays in equilibrium. Determine the value of magnetic field. (*Take* $g = 10\ ms^{-2}$.)



C. Fill in the Blanks

1. Crowded field lines represent .... magnetic field.
2. A stationary charge produces .... while a moving charge produces ....
3. An electron is moving from east to west in a horizontal plane where only horizontal component of the earth's electric field is present. The electron will experience magnetic force in .... direction.
4. A neutral region is that where magnetic field is ....
5. Magnetic field due to a current carrying wire at a particular point will increase if the current through wire ....
6. Fleming's left hand rule is used to find the direction of ....

D. True/False

1. Natural magnets can be termed as permanent magnets.
2. Magnetic monopoles could exist.
3. A magnetic compass when placed in a magnetic field, alligns itself in the direction of magnetic field.
4. A current carrying wire produces a magnetic field.
5. A point charge moving near to a current carrying wire deviates from its path.
6. Earth's magnetic axis and the axis about which earth rotates are the same.
7. Work done by a magnetic force on a moving charge particle is zero.

High Skill Questions

Exercise 2

A. Only One Option Correct

1. The magnetic field lines due to an infinite long straight current carrying conductor are
(a) circular (b) elliptical
(c) parabolic (d) straight

2. The unit of magnetic field intensity might be
(a) C-m/s (b) C-s/m
(c) kg/C-s (d) N/C-m

3. In the formula $\vec{F} = q(\vec{v} \times \vec{B})$,
(a) $\vec{F}$ must be perpendicular to $\vec{v}$, but not necessarily to $\vec{B}$
(b) $\vec{F}$ must be perpendicular to $\vec{B}$, but not necessarily to $\vec{v}$
(c) All three vectors must be mutually perpendicular
(d) None of the above

4. An electron moves in the –ve x direction, through a uniform magnetic field in the –ve y direction. The magnetic force on the electron is
(a) in negative x direction
(b) in negative z direction
(c) in negative y direction
(d) in positive z direction

5. A magnetic field exerts a force on a moving charge particle.
(a) Always
(b) Never
(c) If the particle moves across the field lines.
(d) If the particle moves along the field lines.

6. The direction of the magnetic field in certain region of space is determined by firing a test charge into the region with its velocity in various directions in different trials. The field direction is
(a) one of the direction of velocity when the magnetic force is zero
(b) the direction of the velocity when the magnetic force is maximum
(c) the direction of the magnetic force
(d) perpendicular to the velocity when the magnetic force is zero

7. An electron is moving north in a region where the magnetic field is south. The magnetic force experienced by the electron is
(a) Zero (b) up
(c) down (d) east

8. A magnetic field cannot
(a) exert a force on a charge
(b) accelerate a charge
(c) change the momentum of a charge
(d) change the kinetic energy of a charge

9. A proton (charge $+e$) travelling perpendicular to a magnetic field experiences the same force as an alpha particle (charge $+2e$) which is also travelling perpendicular to the same field. The ratio of their speeds, $\frac{v_{\text{proton}}}{v_{\text{alpha}}}$, is
(a) 0.5 (b) 1.0
(c) 2.0 (d) 4.0

10. A hydrogen atom that has lost its electron is moving east in a region where the magnetic field is directed from south to north. It will be deflected
(a) up (b) down
(c) north (d) south

11. A beam of electrons is sent horizontally down the axis of a tube to strike a flourescent screen at the end of the tube. On the way, the electrons encounter a magnetic field directed vertically downwards. The spot on the screen will therefore be deflected

(a) upward
(b) downward
(c) to the right as seen from the electron source
(d) to the left as seen from the electron source

12. An instant an electron is moving in the XY plane, the components of its velocity along X and Y axes are 5×10^5 ms^{-1} and 3×10^5 ms^{-1}, respectively. A magnetic field of 0.8 T is in the +ve x direction. At this instant the magnitude of the magnetic force on the electron is
(a) Zero (b) 3.8×10^{-14} N
(c) 5.1×10^{-14} N (d) 7.5×10^{-14} N

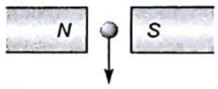
13. An electron travels due north through a vacuum in a region of uniform magnetic field $\vec{B}$ that is also directed due north. It will
(a) be unaffected by the field
(b) speed up
(c) slow down
(d) change its direction of motion

14. An electron and a proton are both initially moving with the same speed and in the same direction at 90° to the same magnetic field. They experience magnetic forces which are initially
(a) identical
(b) equal in magnitude but opposite in direction
(c) in the same direction but differing in magnitude by a factor of 1840
(d) None of the above

15. Three parallel wires carry current in the same direction and are placed in the same magnetic field. The current in wire I is due to the moving electrons, the current in wire II is due to the moving protons and in III it is due to electrons and protons moving in opposite directions. Then,
(a) the magnetic forces on all three wires are in the same direction
(b) the magnetic force on II and III are in the same direction, on I it is in opposite directions
(c) the magnetic forces on I and III are in the same direction, and on II it is in the opposite directions
(d) Information insufficient

16. For the situation of above question, mark out the correct statement(s).
(a) All the three wires experience same magnetic force (in both magnitude and direction)
(b) All the three wires experience magnetic forces whose directions are same but magnitudes are different
(c) All three wires experience different magnetic forces (both in magnitude and direction)
(d) Information is insufficient to compare the magnitude of magnetic forces experienced by three wires

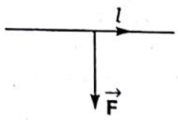
17. The diagram shows a straight wire carrying a flow of electrons into the page. The wire is kept in between the poles of an electromagnet. The direction of the magnetic force experienced by the wire is



Wire, electrons are moving into the page

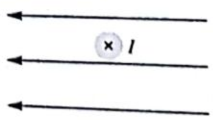
(a) ↑ (b) ↓
(c) ↗ (d) ↙

18. The diagram shows a straight wire carrying current i in the uniform magnetic field. The magnetic force acting on the wire is as shown in the figure but the magnetic field has been not shown. The probable direction of the magnetic field is



(a) upward
(b) downward
(c) into the page
(d) out of the page

19. The figure shows a uniform magnetic field directed to the left and a wire carrying current I into the page. The magnetic force acting on the wire is



(a) towards the top of the page
(b) towards the bottom of the page
(c) towards the left
(d) towards the right of the page

20. A loop of wire carrying a current of 2 A is in the shape of a right triangle with two equal sides, each 15 cm long. A 0.7 T uniform magnetic field is parallel to the hypotenuse. The resultant magnetic force on the two perpendicular sides have a magnitude of
(a) 0 (b) 0.21 N
(c) 0.30 N (d) 0.41 N

21. In the above question if the magnetic field is perpendicular to the hypotenuse, then the magnetic force on the two perpendicular sides has a magnitude of
(a) 0 (b) 0.21 N
(c) 0.30 N (d) 0.41 N

22. The magnetic field lines of a positive charge moving with constant velocity are
(a) straight lines that are parallel to the velocity
(b) straight lines that are perpendicular to the velocity
(c) circle in plane that are perpendicular to the velocity
(d) circles in plane that are parallel to the velocity

23. If r is the distance from a moving charge, the magnitude of the magnetic field produced by the charge is proportional to
(a) r (b) r^2
(c) $\frac{1}{r^2}$ (d) $\frac{1}{r^3}$

24. Two electrons are travelling with the same speed. The magnetic field is zero at the midpoint between them. This means
(a) they are travelling along same line
(b) they are travelling along parallel lines in the same direction
(c) they are travelling along parallel lines in the opposite directions
(d) they are travelling along perpendicular lines that don't intersect

25. In above question if instead of two electrons, there are one electron and one proton, then
(a) they are travelling along the same line
(b) they are travelling along parallel lines in the the same direction
(c) they are travelling along parallel lines in the opposite directions
(d) they are travelling along perpendicular lines that don't intersect

B. More Than One Options Correct

1. Mark out the option(s) which are incorrect. At any point the magnetic field lines are in the direction of
(a) the magnetic force on a moving positive charge
(b) the magnetic force on a moving negative charge
(c) the velocity of a moving +ve charge
(d) the velocity of a moving –ve charge

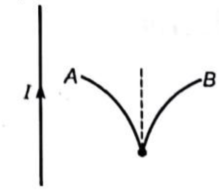
2. Mark out the incorrect option(s). The magnetic force on a charged particle is in the direction of its velocity
(a) if it is moving in the direction of the field
(b) if it is moving opposite to the direction of the field
(c) if it is moving perpendicular to the field
(d) if it is not moving

3. Mark out the correct statement(s) concerning magnetic field lines.
(a) Two magnetic field lines can't intersect.
(b) The magnetic field lines can't give the direction of the magnetic force.
(c) The magnetic field lines give the direction of magnetic field.
(d) The concept of magnetic field lines is an imaginary one.

4. RHPR No. 2 can be used to find direction of magnetic force experienced by
(a) moving charges
(b) moving +ve charges
(c) moving –ve charges
(d) current carrying wires

5. A beam of moving charges gets deflected from its original path due to magnetic force

experienced by them (due to magnetic field produced by current carrying wire), either along path A or path B. Mark out the correct statement(s).



(a) If the beam is consisting of +ve charge, it deviates to A

(b) If the beam is consisting of +ve charge, it deviates to B

(c) If the beam is consisting of –ve charge, it deviates to B

(d) If the beam is consisting of –ve charge, it deviates to A

6. A current carrying wire is placed horizontally (in air) in a magnetic field region. The wire stays in equilibrium, direction of current is not given. This situation is possible if magnetic field may be directed towards

(a) north
(b) south
(c) east
(d) west

C. Assertion & Reason

Directions (Q. Nos. 1 to 6) *Some questions (Assertion-Reason type) are given below. Each question contains **Statement I (Assertion)** and **Statement II (Reason)**. Each question has 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct. So, select the correct choice.*

Choices are

(a) **Statement I** is True, **Statement II** is True; **Statement II** is a correct explanation for **Statement I**

(b) **Statement I** is True, **Statement II** is True; **Statement II** is **NOT** a correct explanation for **Statement I**

(c) **Statement I** is True, **Statement II** is False

(d) **Statement I** is False, **Statement II** is True

1. **Statement I** Two magnetic field lines can't intersect each other.
Statement II The magnetic field is having an unique direction at a particular location.

2. **Statement I** Magnetic field lines are closed curves.
Statement II Magnetic monopoles can't exist.

3. **Statement I** There are more field lines coming out from north pole as compared to number of field lines entering into the south pole.
Statement II In general, the poles of the magnet are having equal pole strength and are of opposite nature.

4. **Statement I** If we break a magnet then we can separate two poles of the magnet.
Statement II Monopoles can't exist.

5. **Statement I** A charged particle is moving in a purely magnetic field with non-zero constant velocity.
Statement II If the direction of motion of charge particle coincides with field direction, then the magnetic force experienced by the charged particle would be zero.

6. **Statement I** A current carrying conductor at rest in a magnetic field always experiences a zero force (magnetic).
Statement II Magnetic force experienced by a stationary charge particle in magnetic field is zero.

Answers

Towards Proficiency Problems
Exercise 1

B. Numerical Answer Types

1. $B_P = 6\times10^{-7}$ T, South; $B_Q = 2\times10^{-7}$ T, East; $B_R = 6\times10^{-7}$ T, North; $B_S = 1.5\times10^{-7}$ T, West

2. 0.12 m **3.** 80 μT **4.** $1.6\pi\ \mu$T **5.** 8.533 μT $\otimes$

6. $B_O = -4\,\hat{j}\,\mu$T, $B_P = 4/3\,\hat{j}\,\mu$T, $B_Q = -2\,\hat{j}\,\mu$T, $B_R = 4/3\,\hat{j}\,\mu$T **7.** 5πmT **8.** $3.8\ \pi\mu$T $\otimes$

9. $\frac{\mu_0}{2R}\sqrt{I_1^2 + I_2^2}$ **10.** 5.4 A **11.** 0.081 T **12.** $\left(\frac{\mu_0 I}{2\pi R} + \frac{\mu_0 I}{2R}\right)\odot$

13. $B_P = B_Q = 0$, $B_R = 0.036\ \mu$T, $B_S = 0.025\ \mu$T **14.** 7.5 mN **15.** 125 T

16. $F_A = 0$, $F_B = 27$ N, $F_C = \frac{27}{\sqrt{2}}$ N, $F_D = 23.4$ N **17.** 0, 3 mN, $\frac{3}{\sqrt{2}}$ mN, $\frac{3\sqrt{3}}{2}$ mN

18. 8×10^{-5} N towards right **19.** 16.67 mT

C. Fill in the Blanks

1. Stronger **2.** Electric field only, electric and magnetic field both
3. Vertical upward **4.** Zero **5.** Increases **6.** Magnetic force

D. True/False

1. T **2.** F **3.** T **4.** T **5.** T
6. F **7.** T

High Skill Questions
Exercise 2

A. Only One Option Correct

1. (a) **2.** (c) **3.** (d) **4.** (b) **5.** (c) **6.** (a) **7.** (a) **8.** (d) **9.** (c) **10.** (a)
11. (c) **12.** (b) **13.** (a) **14.** (b) **15.** (a) **16.** (d) **17.** (a) **18.** (d) **19.** (a) **20.** (a)
21. (c) **22.** (c) **23.** (c) **24.** (b) **25.** (c)

B. More Than One Options Correct

1. (a, b, c, d) **2.** (a, b, c, d) **3.** (a, c, d) **4.** (a, b, c, d) **5.** (a, c)
6. (a, b, c, d)

C. Assertion & Reason

1. (a) **2.** (a) **3.** (c) **4.** (c) **5.** (a) **6.** (d)

Explanations

Towards Proficiency Problems
Exercise 1

Numerical Answer Types

1. At P, $B = \dfrac{\mu_0 I}{2\pi r_1} = 2 \times 10^{-7} \times \dfrac{3}{1}$

$= 6 \times 10^{-7}$ T towards South

At Q, $B = 2 \times 10^{-7}$ T towards East

At R, $B = 6 \times 10^{-7}$ T towards North

At S, $B = 1.5 \times 10^{-7}$ T towards West

2. $B = \dfrac{\mu_0 I}{2\pi r} = \dfrac{2 \times 10^{-7} \times 48}{r}$

$= 8 \times 10^{-5}$ T

$\Rightarrow \quad r = \dfrac{2 \times 10^{-7} \times 48}{8 \times 10^{-5}}$

$= 0.12$ m

3. Equivalent current,

$I = \dfrac{15}{1.5 \times 10^{-3}}$ A $= 10^4$ A

$B = \dfrac{\mu_0 I}{2\pi r}$

$= \dfrac{2 \times 10^{-7} \times 10^4}{25} = 80 \mu\text{T}$

4. $B = \dfrac{\mu_0 I}{2r}$

$= \dfrac{4\pi \times 10^{-7} \times 25}{2 \times 3} = 1.67\,\pi\,\mu\text{T}$

5. $B_p = \vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2$

$= \dfrac{\mu_0 I_1}{2\pi(0.25)} \otimes + \dfrac{\mu_0 I_2}{2\pi(0.75)} \otimes$

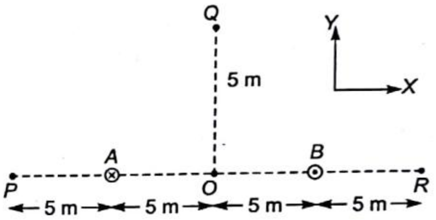
$= \dfrac{\mu_0}{2\pi}\left[\dfrac{10}{1/4} + \dfrac{2}{3/4}\right] \text{T} \otimes$

$= 2 \times 10^{-7} \left[\dfrac{128}{3}\right] \text{T} = 8.533\ \mu\text{T}$

into the plane of paper.

6. $B_O = -\dfrac{\mu_0 I}{2\pi r}\hat{\mathbf{j}} - \dfrac{\mu_0 I}{2\pi r}\hat{\mathbf{j}}$

$= -4\hat{\mathbf{j}}\,\mu\text{T}$



$B_P = \dfrac{\mu_0 I}{2\pi r}\hat{\mathbf{j}} - \dfrac{\mu_0 I}{2\pi \times 3r}\hat{\mathbf{j}} = \dfrac{4}{3}\hat{\mathbf{j}}\,\mu\text{T}$

$B_R = -\dfrac{\mu_0 I}{2\pi \times 3r}\hat{\mathbf{j}} + \dfrac{\mu_0 I}{2\pi r}\hat{\mathbf{j}} = \dfrac{4}{3}\hat{\mathbf{j}}\,\mu\text{T}$

$B_Q = 2 \times \dfrac{\mu_0 I}{2\pi(\sqrt{2}\,r)} \times \cos 45° (-\hat{\mathbf{j}}) = -2\hat{\mathbf{j}}\,\mu\text{T}$

7. $B = \dfrac{\mu_0 NI}{2r}$

$= \dfrac{4\pi \times 10^{-7} \times 1000 \times 25}{2 \times 1} = 5\pi$ mT

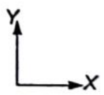
8. $B = B_{\text{due to } A} + B_{\text{due to } B}$

$= \left[\dfrac{\mu_0 I_1}{2r_1} - \dfrac{\mu_0 I_2}{2r_2}\right] \otimes$

$= 3.8\,\pi\,\mu\text{T} \otimes$

9. Magnetic field at the common centre due to coil A is

$B_1 = -\dfrac{\mu_0 I_1}{2R}\hat{\mathbf{i}},$



and due to coil B is,

$B_2 = -\dfrac{\mu_0 I_2}{2R}\hat{\mathbf{j}}$

$B = \dfrac{\mu_0}{2R}\sqrt{I_1^2 + I_2^2}$

10. For net magnetic field at the common centre to be zero, let current through outer coil be I, then

$\dfrac{\mu_0 \times 120 \times 6}{2 \times 0.5} = \dfrac{\mu_0 \times 200 \times I}{2 \times 0.75}$

$\Rightarrow \quad I = 5.4$ A

11. Let l be the length of copper wire and it is wound into a coil of n turns of radius 0.05 m.

$$n \times 2\pi (0.05) = l$$

$$\Rightarrow \quad n = \frac{l}{2\pi \times 0.05}$$

$$I = \frac{12}{5.9 \times 10^{-3}\, l}$$

$$B = \frac{\mu_0 n I}{2(0.05)}$$

$$= \frac{\mu_0 \times 12}{(2 \times 0.05)(5.9 \times 10^{-3} \times 2\pi \times 0.05)}$$

$$= 0.081 \text{ T}$$

12. $B = \left(\frac{\mu_0 I}{2\pi R} + \frac{\mu_0 I}{2R}\right) \odot$

13. Use the formula,

$$B = \frac{\mu_0 q (\vec{\mathbf{v}} \times \vec{\mathbf{r}})}{4\pi r^3}$$

14. $\vec{\mathbf{F}} = q(\vec{\mathbf{v}} \times \vec{\mathbf{B}})$

$$= 5 \times 10^{-6} [500 \times 10^3 \times 3 \times 10^{-3}] = 7.5 \text{ mN}$$

15. $F = qvB \left[\sin \frac{\pi}{2}\right]$

$$3 \times 10^{-3} = 8 \times 10^{-6} \times 3 \times B$$

$$\Rightarrow \quad B = 125 \text{ T}$$

17. Magnetic force experienced by a current carrying linear conductor in uniform magnetic field is given by,

$$\vec{\mathbf{F}} = I(\vec{\mathbf{l}} \times \vec{\mathbf{B}})$$

So, $\vec{\mathbf{F}}_A = 0$

$$F_B = 10(0.1 \times 3 \times 10^{-3}) = 3 \text{ mN}$$

$$F_C = \frac{3}{\sqrt{2}} \text{ mN}$$

$$F_D = \frac{3\sqrt{3}}{2} \text{ mN}$$

18. The magnetic field at the location of current element due to infinite long wire is,

$$B = \frac{\mu_0 \times 10}{2\pi \times 0.1} = 2 \times 10^{-5} \text{ T} \otimes$$

Force experienced by the current element is,

$$F = 20 \times 0.2 \times 2 \times 10^{-5} \text{ N}$$

$$= 8 \times 10^{-5} \text{ N rightward}$$

On reversing the direction of current, the direction of force reverses.

19. $mg = 20 \times B \times 0.3$

$$\Rightarrow \quad \frac{10}{1000} \times 10 = 6B$$

$$\Rightarrow \quad B = 16.67 \text{ mT}$$

Chapter 16

Electromagnetic Induction (EMI)

The First Steps' Learning

- Magnetic Flux
- Faraday's Law of Electromagnetic Induction
- Lenz's Law
- Motional emf

In the last chapter we have seen that a current element produces a magnetic field ie. electricity can cause the magnetism. After the establishment of this fact physicists started thinking in reverse ie. can magnetism cause the electricity? The painstaking efforts of various physicists in this direction resulted into one of the most prominent laws of physics. The Faraday's law of Electromagnetic Induction.

Today almost every modern device or machine from an electric iron to geyser, toaster to grinder, simple calculator to supercomputers, simple small drill machine to large electric cranes have some short of electric circuits at their core of working systems. In previous chapters we have studied that a battery is needed to energize the circuit, but it is not feasible in most of the cases. Can you run for a long time your mixer, your stereo system, your computer etc with the help of batteries, surely you say no. In most of the applications (industrial as well as in homes) we require an electricity generating station. In an electricity generating station some other form of energy like nuclear energy, thermal energy, wind energy or hydel energy is converted to electrical energy. Now the question arises, how this energy conversion is done ? What is the basic principle on which these power plants do work ? The answer is a phenomenon known as electromagnetic induction.

In 1831, the two eminent scientists one Michael Faraday in England and the other Joseph Henry in United States conducted a series of experiments independently which led to the today's well known principle Faraday law of electromagnetic induction.

Magnetic Flux

To understand the basic terms used in Faraday's law of electromagnetic induction, it is very important to know the meaning of term magnetic flux. Magnetic flux (ϕ_B) is a quantity which is defined through a surface. Consider a surface S, which is placed in a uniform magnetic field B and let the area of surface be A, then magnetic flux crossing this surface is defined as dot product of magnetic field with the area vector, *ie*,

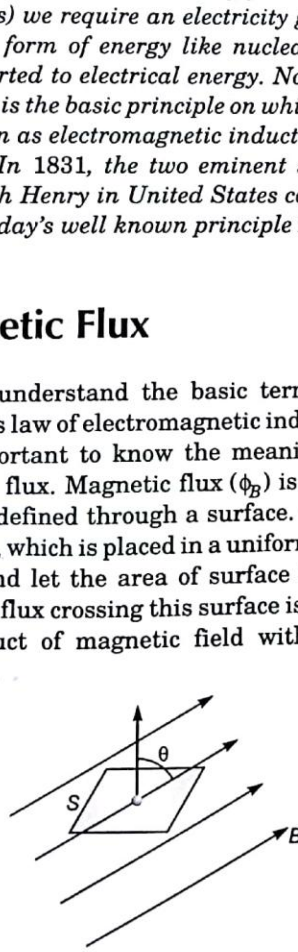


Fig. 16.1 Magnetic flux is linked with any surface when it is placed in a magnetic field.

$$\phi_B = \vec{B} \cdot \vec{A} = BA \cos \theta$$

where θ is the angle between magnetic field and the area vector.

Now the question arises, what is meant by area vector ? Until now you heard and delt with area's in Mathematics and never thought that it would be a scalar or vector. In actual we consider an area as a vector quantity. Now, you may ask–if it is a vector quantity then what would be its direction. The normal drawn to the surface is the direction of area vector. The drawn normal to surface can be in two directions –inwards and outwards to surface. As a standard we consider outwards normal to be the direction of area vector. So for a small surface whose area is ΔS, the area vector $\Delta \vec{S}$ is as shown in Fig. 16.2.

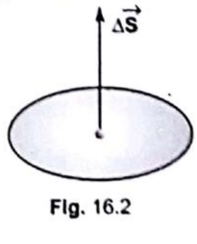


Fig. 16.2

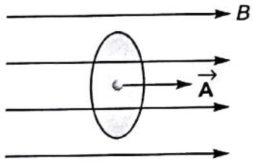
C-BIs

Concept Building Illustrations

Illustration | 1 *Compute the magnetic flux crossing through a surface of area 1 cm^2 when placed in a magnetic field of 5 T ? The plane of the surface is perpendicular to direction of magnetic field.*

Solution Here, the angle between $\vec{A}$ and $\vec{B}$ is 0° as plane of surface is perpendicular to field direction, so, normal to surface is parallel to field.

$$\phi_B = BA\cos\theta$$
$$= 5 \times 1 \times 10^{-4} \cos 0$$
$$= 5 \times 10^{-4} \text{ T-m}^2$$



Magnetic flux is a scalar quantity whose SI unit is Weber, 1 Weber = 1 T-m^2. Magnetic flux can also be interpreted as number of magnetic field lines crossing a surface, more is the number of field lines crossing through the surface more is the flux linked with the surface. Although the concept of magnetic field lines is imaginary one, the flux is a real scalar physical quantity.

If a surface is placed in a magnetic field free region *ie*, where no magnetic field is present, then the flux linked with the surface would be zero, but if flux linked with a surface is zero, then it is not necessary that magnetic field is zero in the region where surface is considered. Consider a cubical surface placed in a magnetic field region as shown in Fig. 16.3. The area vectors $\vec{A}_1, \vec{A}_2, \vec{A}_3, \vec{A}_4, \vec{A}_5$ and $\vec{A}_6$ are shown for surfaces *BCDG*, *AHEF*, *ABGF*, *CDEH*, *FGDE* and *ABCH*, respectively. The flux linked with

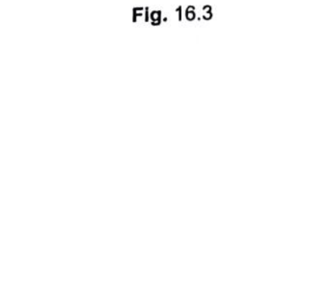


Fig. 16.3

faces having areas $\vec{A}_3, \vec{A}_4, \vec{A}_5$ and $\vec{A}_6$ would be zero because area vector and magnetic field are perpendicular to each other for these faces.

For *BCDG*, $\phi_1 = BA_1 \cos 0 = BA$

For *AHEF*, $\phi_2 = BA_2 \cos 180° = -BA$

$[\because A_1 = A_2 = A]$

So, total flux linked with the cubical surface is,

$$\phi = \phi_1 + \phi_2 + 0 = 0$$

Therefore, the flux linked with the surface is zero, although the magnetic field in the region is not zero.

Factors affecting the flux linked with a surface Magnetic flux linked with a surface of area vector $\vec{A}$ considered in a magnetic field $\vec{B}$ is given by $\phi = \vec{B} \cdot \vec{A} = BA\cos\theta$ where θ is the angle between $\vec{B}$ and $\vec{A}$. We can change the magnetic flux linked with a circuit either by changing B, A or θ or by changing any combination of this. Now the question arises — what is the need to change flux linked with a circuit ? The reason lies in Faraday's law of electromagnetic induction which we study next.

Change of magnetic flux linked with a circuit due to change in θ. Consider a coil which is placed in a uniform magnetic field as shown in Fig. 16.4. Initially, assure the plane of coil to be in the direction of field than the

magnetic flux linked with the coil is zero because the angle between the $\vec{A}$ and $\vec{B}$ is $\pi/2$. If we start rotating the coil with certain angular velocity, then the value of θ changes, and hence the value of the flux linked with the coil.

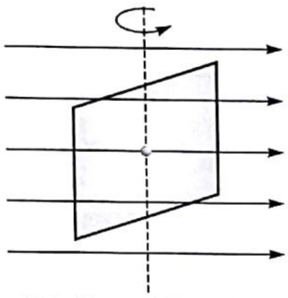


Fig. 16.4 Magnetic flux linked with a surface in a magnetic field changes when its orientation change.

Due to change in magnetic field If the field changes, then the flux linked with the circuit also changes. Consider a coil which is placed in a uniform magnetic field (field which is having same value at all locations at any instant). If the magnetic field in the region starts changing with time, the flux linked with the coil also starts changing with time.

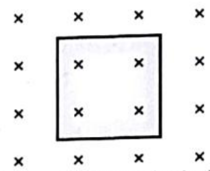


Fig. 16.5 Magnetic flux linked with a surface in a magnetic field changes when magnetic field change with time

Due to change in area Consider a square coil placed in a constant uniform magnetic field as shown Fig. 16.6. If we start deforming the square into rhombus by pulling it from its two diagonally opposite corners, then the area of the coil area changes and hence the flux linked with the coil changes.

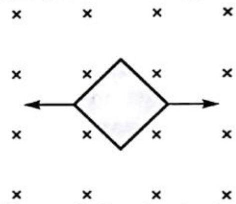


Fig. 16.6 Change in flux due to change in area.

Here, we have seen only that how flux changes when we change any one of B, A or θ, but in practice the change in flux linked with a circuit changes by any combination of this.

Faraday's Law of Electromagnetic Induction

In 1831, Joseph Henry and Michael Faraday conducted a series of experiments independently and concluded that a time varying magnetic flux linked with a closed circuit causes a current in it. This conclusion in known as Faraday's Law of Electromagnetic Inanition, (EMI). But before coming to the statement of Faraday's law of EMI, we learn about some experiments conducted by these two eminent physicists.

Experiment 1 They considered a wire loop which is connected to a galvanometer (galvanometer is a current sensitive device which shows deflection when current passes through it), and a bar magnet. If the magnet is moved along the axis of the coil, then it is observed that the galvanometer shows a deflection. It has been also observed that if the magnet is stationarly, but the coil moves, then also the galvanometer shows a deflection. Thus, we can conclude whenever there is relative motion between the magnet and coil, the galvanometer shows a deflection. It has also been observed that galvanometer shows the

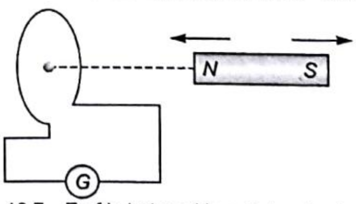


Fig. 16.7 Emf is induced in coil due to change in magnetic flux linked with the coil, due to motion of magnet which causes the change in magnetic field at the location of coil.

deflection only when there is relative motion between the coil and magnet. From all these observations we can conclude that whenever there is a change in magnetic flux linked with the coil, the galvanometer shows a deflection.

Experiment 2 In this experiment, they took two coils placed co-axially-coil 1 and coil 2 as shown in Fig. 16.8. Coil 1 is connected to a battery and a rheostat, while coil 2 is connected to a galvanometer. When a steady current is established in coil 1, then the galvanometer attached to coil 2 shows no deflection. If the resistance of coil 1 circuit is changed, then the galvanometer shows the deflection for the duration for which resistance of coil 1 circuit is changing. But as soon as we stop changing the resistance of coil 1 circuit the deflection of galvanometer disappears. If we think little about what is happening in the two circuits as we change the resistance of coil 1 circuit, then we can easily come to final conclusion. As we change the resistance of coil 1, the current through coil 1 changes, and therefore we can say that changing current in 1 causes the deflection in galvanometer connected to coil 2, further we can say that a current carrying loop element is a source of magnetic field and therefore as the current through 1 changes, the magnetic field produced by it in neighbouring region changes, and also at the location of coil 2. Thus, we can conclude that as the magnetic flux linked with coil 2 changes the galvanometer connected to it shows a deflection.

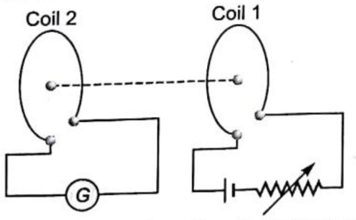


Fig. 16.8 Emf is induced in coil due to change in magnetic flux linked with it, due to variable resistance of circuit connected to coil 1.

Experiment 3 In this experiment a coil is placed into a magnetic field. The coil is connected to a galvanometer. It is observed that when coil rotates about its own axis, the galvanometer deflects. From this observation it can be directly concluded that if magnetic flux linked with the coil changes, the galvanometer attached to it shows the deflection.

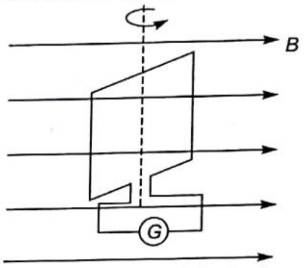


Fig. 16.9 Emf is induced due to change in magnetic flux linked with coil, which is because of change in orientation of coil.

Experiment 4 Consider two parallel conducting rails are to connected with a galvanometer as shown in Fig. 16.10. And, a conducting rod PQ is allowed to move on two parallel rails in a direction perpendicular to the length of rod as shown in figure. The entire arrangement is placed in a horizontal plane and a uniform magnetic field exists in the region perpendicular to the plane of paper directed into it. It is observed that when the rod is moving, the galvanometer shows deflection and as soon as the rod stops moving deflection of galvanometer also disappears. Actually what happens here is that the area of circuit $APQB$ is increasing as rod PQ moves in the direction shown, and hence the flux linked with this circuit is changing, as a result of which the galvanometer shows a deflection. From this again we can conclude that changing the magnetic flux linked with a circuit causes galvanometer to show a deflection.

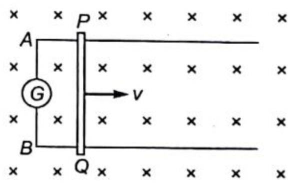


Fig. 16.10 emf is induced due to cutting of magnetic flux linked with cont cht.

Here, we have given you the idea about few of the experiments conducted by Joseph

Henry and Michael Faraday and from all these experiments you may also conclude that "whenever there is any change in magnetic flux linked with a circuit, an emf in induced in the circuit and this induced emf will last as long as the change in magnetic flux exists". This is what the descriptive conclusion Faraday and Henry had had after performing the series of experiments. The emf which induced in the circuit due to changing the flux is termed as induced emf, and this phenomenon of development of induced emf due to changing magnetic flux in a circuit is known as electromagnetic induction.

Faraday's, law of EMI states : "The emf induced in a circuit due to changing the magnetic flux is equal to –ve of time rate of change of magnetic flux linked with it".

$$\text{Induced emf} = -\frac{\text{Change in flux}}{\text{Time interval}}$$

$$\Rightarrow \qquad e = \frac{-\Delta\phi}{\Delta t} = -\left[\frac{\phi_f - \phi_i}{\Delta t}\right]$$

In the above expression, the –ve sign only tells about the polarity of the induced emf *ie*, which side would be at the higher potential, which we can find very easily by using Lenz's law which we shall discuss next.

Some important points related to EMI are as follows :

1. Whenever there is some change in magnetic flux linked with a circuit as the time passes, an emf is induced in the circuit. Remember persistence of emf is only for that much amount of time for which the flux changes.
2. In EMI, the emf will always exist whether the circuit is closed or open, but the current (due to induced emf) would be there in the circuit only when the circuit is closed, which is a necessary condition for the steady current to be there is the circuit.
3. Induced emf doesn't at all depend on how the flux has been changed *ie*, by changing area, magnetic field or any combination of three factors which can change the flux. Emf induced depends only on the time rate of change of magnetic flux.
4. Magnetic flux linked with a circuit can be changed in three basic ways which are as
   (1) by changing the magnetic field
   (2) by changing the area
   (3) by changing the orientation of circuit

In practice, generally the flux is changed by any combination of above three but on the basis of origin of the induced emf due to change, the flux is classified in two :

(A) **Motional emf** The emf which is induced in the circuit due to change in flux because of change in area or change in orientation.

(B) **Induced field emf** The emf which gets induced due to change in mangntic field. This type of induced emf we won't discuss in this text, this we are leaving for your higher classes.

Lenz's Law

Using Lenz's law we can easily find the polarity of induced emf, according to Lenz's law—"The polarity of induced emf is such that it produced a current in the circuit, whose magnetic field opposes the cause of induced emf *ie*, change in magnetic flux linked with circuit." In simpler words, Lenz's law can be stated as—"Effect takes place in such a way that it opposses the cause". Here the cause is the changing flux and effect is the induced emf. Or we can say that induced emf tries to maintain the original flux *ie*, it opposses the variation in flux.

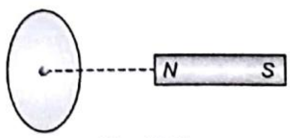


Fig. 16.11

Lenz's law is not an independent law, it is simply an intelligent interpretation of Faraday's law of EMI to determine the polarity of induced emf.

Consider a coil and a bar magnet as shown in Fig. 16.11. From Experiment 1 we know that if a magnet moves towards or away from the centre of the coil, a current is induced in the coil. Here our aim is to determine the direction of induced current using Lenz's law.

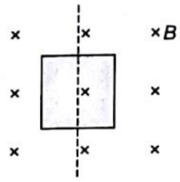
In present case the N-pole of the magnet is nearer to coil, and hence the magnetic field at the location of coil due to the magnet is towards left. If the magnet approaches the coil, the magnetic field at the location of coil increases and hence the magnetic flux linked with the coil. As the flux linked with the coil changes (increases), an emf and hence current (due to closed path) will be induced in coil which tries to decrease the increasing flux *ie*, magnetic field at the location of coil due to induced current would be in a direction opposite to magnetic field at location of coil due to magnet *ie*, magnetic field at location of coil due to induced current would be towards right. Which is possible when current in coil is in anti-clockwise direction as seen from the magnet side.

We know that a changing flux linked with a circuit causes an induced emf in the circuit and if the circuit is closed this induced emf causes a current in the circuit which we call the induced current. This induced current develops its own magnetic field, as a result some flux will be linked with the circuit. This flux which develops due to induced current would be in such a way that it tries to strengthen the original flux if the original flux was decreasing and weaken if flux was increasing.

C-BIs

Concept Building Illustrations

Illustration | 2 *A square coil of side 5 cm is placed in a magnetic field $B = 8$ T as shown in the figure. Now the coil is rotated by 180° about its axis passing through centre and parallel to one of its edges. The time taken to rotate the coil is 0.5 ms. Determine the average emf induced in the circuit and the induced current if the resistance of coil is 5 Ω.*



Solution Here due to change in flux an emf would be induced in the coil, which we have to determine.

$$= -2 \times 10^{-2} \text{ Wb}$$

Initial flux, $\phi_i = \vec{B} \cdot \vec{A} = 8 \times 25 \times 10^{-4} \cos 180°$

Final flux, $\phi_f = \vec{B} \cdot \vec{A} = 8 \times 25 \times 10^{-4} \cos 0°$

$$= 2 \times 10^{-2} \text{ Wb}$$

[Area vector is taken as +ve out of plane of paper. When the coil is turned by 180°, the area vector also rotates by 180°, and comes into the plane of paper]

$$e = -\frac{\Delta\phi}{\Delta t} = -\left[\frac{\phi_f - \phi_i}{\Delta t}\right]$$

$$= -\left[\frac{2 \times 10^{-2} - (-2 \times 10^{-2})}{0.5 \times 10^{-3}}\right] = -80 \text{ V}$$

So the induced emf would be 80 V.

Induced current,

$$i = \frac{e}{R} = \frac{80}{5}$$

$$= 16 \text{ A}$$

Motional emf

When the emf is induced due to motion of coil *ie*, due to change in θ or due to change in area, then the emf induced is termed as motional emf. Consider the circuit as already mentioned in experiment 4. If the conducting rod PQ moves, then an emf is developed across

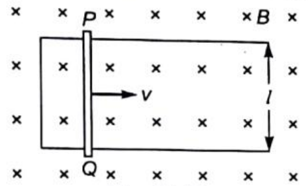


Fig. 16.12

the rod, here our aim is to find this induced emf and to know the reasoning behind the origination of emf. Let us consider that the rod moves with velocity v in the direction shown, as the rod is conducting some free electrons would be there inside the rod and along with the rod they are also having a velocity v towards right and as these charges (electrons) are moving in a magnetic field they will experience a magnetic force, $F_m = evB$ towards down as a result of this force the electron start moving towards down as

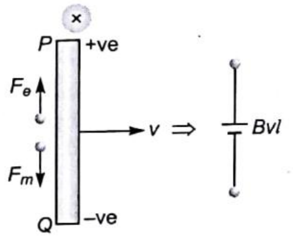


Fig. 16.13

a result the electrons accumutate at Q, and hence the end Q becomes negatively charged and end P becomes positively charged or we can say the end P is at higher potential as compared to end Q, due to this potential difference an electric field will set up in the rod from P to Q. Due to this electric field, the electron will experience an electric force $F_e = eE$ in upward direction.

In steady state these two forces balance each other, so $F_m = F_e$.

$$\Rightarrow \qquad E = vB$$

Or we can say that the potential difference between two ends of the rod is $El = vBl$ which is nothing but the induced emf.

So, the induced emf $\Rightarrow e = Bvl$ with end P at higher potential.

For solving the question, the rod can be considered as a battery of emf $e = Bvl$.

The direction of induced emf in these type of situations can be found very easily by using Fleming's right hand rule or RHPR No. 3.

Fleming's Right hand rule According to this rule if we stretch the fore finger, middle finger and thumb of our right hand in mutual perpendicular directions in such a way that the fore finger points towards magnetic field and the thumb points towards the direction of motion of conductor, then the tip of the middle finger points towards the +ve polarity of induced emf.

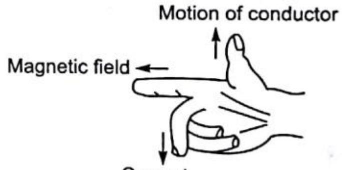


Fig. 16.14 Fleming's right hand rule is used to determine the polarity of induced emf.

RHPR No. 3 Another way to find the polarity of the induced emf for linear conductors is by using RHPR No. 3. This rule states—"if we stretch our right hand palm in such a way that fingers point towards magnetic field direction, and the thun b point towards the motion of conductor, then the end towards the front face of your palm is at higher potentia".

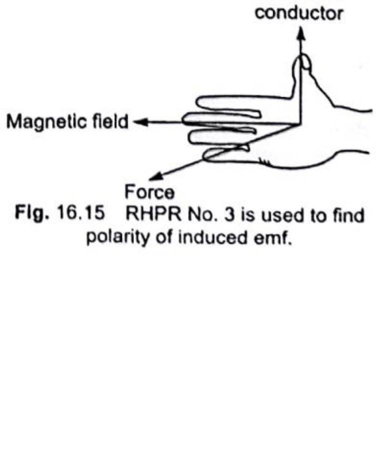
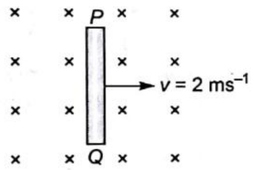


Fig. 16.15 RHPR No. 3 is used to find polarity of induced emf.

C-BIs

Concept Building Illustrations

Illustration | 3 *A conducting rod of length* 10 *cm is moving in a magnetic field of* 10 *T as shown in the figure. The speed of rod is* 2 ms^{-1}. *Determine the magnitude of emf induced across the ends of the rod and also find out which end is at the higher potential ?*



Solution We have seen in the last section that the induced emf across the ends of a rod moving perpendicular to its length in a perpendicular magnetic field is given by $e = Bvl$.

So, $$e = 10 \times 2 \times 0.1 = 2\text{ V}$$

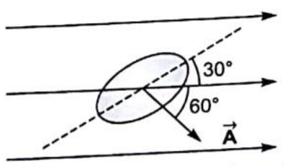
From RHPR No. 3 we can find that end P is at higher potential then, end Q by 2 V.

Proficiency in Concepts (PIC)

Problems

Problems | 1 *A circular coil of area* 10 cm^2 *is placed in a magnetic field of* 25 *T. The plane of the coil is making an angle of* 30° *with the magnetic field direction. Determine the flux linked with the coil.*

Solution As the angle between the plane of coil and magnetic field is 30°, the angle between the area vector and the magnetic field would be 60°.



The flux linked with the coil would be,

$$\phi = BA\cos\theta$$
$$\phi = 25 \times 10 \times 10^{-4} \times \cos 60° = 12.5\text{ mWb.}$$

Problems | 2 *In above question if plane of coil is parallel to magnetic field direction, then find the answer.*

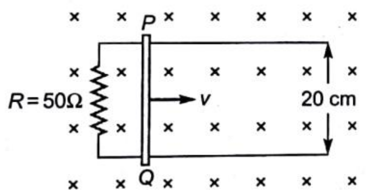
Solution In this case the angle between $\vec{A}$ and $\vec{B}$ is $\pi/2$ and hence the flux linked with coil would be zero.

Problems | 3 *If the flux linked with a circuit changes by* 80 *Wb in* 0.2 *s, then what is the emf induced in the circuit ?*

Solution Induced emf,

$$e = \left| -\frac{\Delta\phi}{\Delta t} \right|$$
$$e = \frac{80}{0.2} = 400\text{ V}$$

Problems | 4 *Two parallel conducting rails having negligible resistance are connected by a resistor of resistance $R = 50\,\Omega$ at one of their ends. A rod PQ is moving with constant velocity $v = 10\ ms^{-1}$ along the length of rails as shown in figure. The entire arrangement is placed in a perpendicular magnetic field of magnitude 2T. Determine*



(a) *the emf induced and the end of the rod which is at the higher potential*

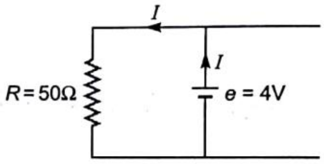
(b) *the current flowing through the circuit*

(c) *the magnetic force acting on rod*

Analyse the situation and think about all the forces acting on rod.

Solution As the conductor is moving with constant velocity v in a magnetic field an emf is developed across the ends of the rod with end P at higher potential [from RHPR No. 3]. The emf developed would be equal to,

$$e = Bvl$$
$$= 2 \times 10 \times 0.2$$
$$= 4\ \text{V}$$



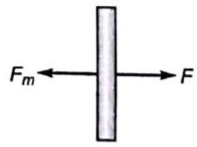
The circuit diagram would be as shown in the figure. Using KVL, we can findout the current I in the circuit.

$$-e + IR = 0$$
$$i = \frac{e}{R}$$
$$= \frac{4}{50} = 0.08\ \text{A}$$

Due to this current the rod PQ experiences a magnetic force $F = IBl$ towards left (Using RHPR No. 2).

$$F_m = 0.08 \times 2 \times 0.2$$
$$= 0.032\ \text{N towards left}$$

As the rod is moving with constant velocity it means net force acting on the rod must be zero, and hence in addition to F_m some other external force must be acting on the rod so that the net force acting on rod must be zero. Let this external force be F, then it has to be equal and opposite to F_m.



ie, $\quad F = 0.032\ \text{N towards right.}$

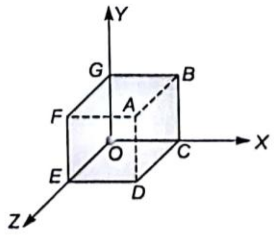
Towards Proficiency Problems
Exercise 1

A. Subjective Discussions

1. Magnetic flux is a real scalar physical quantity but it can be interpreted as number of magnetic field lines (which is an imaginary concept) crossing through the surface. Comment on this statement.
2. In experiment 1 of Faraday as mentioned in text, assume that both coil and magnet are moving with same velocity with respect to ground. Would there by any emf induced in the coil ? What will happen if both are moving with same speed but in opposite directions ?
3. A magnetic field is necessary if there is to be a magnetic flux passing through a coil of wire. Yet, just because there is a magnetic field doesn't mean that a magnetic flux will pass through a coil. Account for this statement.
4. Two circular loops lie side by side in the same plane. One is connected to a source that supplies an increasing current, the other is a simple closed ring. Is the induced current in the ring in the same direction as that in the loop connected to the source, or opposite ? What if the current in the first loop is decreasing ? Explain.
5. A square conducting loop is in a region of uniform constant magnetic field. Is it possible to rotate the loop about some axis so that no emf would be induced in the loop ? Discuss various possibilities.
6. What do you mean by area vector ? Explain.

B. Numerical Answer Types

1. A rectangular coil of wire is situated in a constant magnetic field whose magnitude is 0.50 T. The coil has an area of 2 m^2. Determine the magnetic flux for the three orientations, $\theta = 0°$, $60°$ and $90°$ where θ is the angle between area vector and magnetic field intensity.
2. The figure shows a cube of edge 2 cm, whose three edges are coinciding with X, Y and Z axes and one of vertices of cube coincides with the origin of coordinate system. A uniform magnetic field exists in this region whose components are $B_x = 0.5$ T, $B_y = 1.0$ T and $B_z = 1.5$ T. Determine the magnetic flux crossing through surfaces that lie in (a) XY plane (b) YZ plane, and (c) ZX plane ?



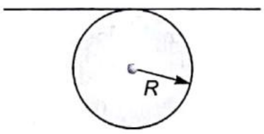
3. A coil having 100 turns is placed in a magnetic field of 3 T. The plane of the coil makes an angle of 60° with the field direction. If the area of coil is 0.5 m^2, then determine the magnetic flux linked with this coil.

4. A house has a floor area of 112 m^2, and has an outside wall that has an area of 28 m^2. The earth's magnetic field here has a horizontal component of 2.6×10^{-5} T that points due north and a vertical component of 4.2×10^{-5} T that points straight down, toward the earth. Determine the magnetic flux through the wall if the wall faces (a) north, and (b) east, and (c) also calculate the magnetic flux that passes through the floor.

5. The magnetic flux linked with a circuit changes from $\phi_i = -5$ mWb to $\phi_f = 3$ mWb in a time-interval of 0.03 s. Determine the average emf induced in the circuit for this interval. If the resistance of the circuit is 80 Ω, determine the average induced current in the circuit.

6. A 300 turn rectangular loop of wire has an area per turn of 0.05 m^2. At $t = 0$ s a magnetic field is turned on, and its magnitude increases to 0.4 T in 0.8 s without any change in direction. The field is directed at an angle of 30° *wrt* the normal of the loop. (a) Find the magnitude of the average emf induced in the loop. (b) If the loop is closed and has a resistance of 6 Ω, determine the average induced current.

7. The figure shows a straight wire, a part of which is bent into the shape of a circle. The radius of the circle is 2 cm. A constant magnetic field of magnitude 0.8 T is directed into the plane of paper. If the end of wire is held and pulled to make it taut so that the radius of circle shrinks to zero in 0.25 s, then determine the magnitude of the average emf induced between the ends of the wire.

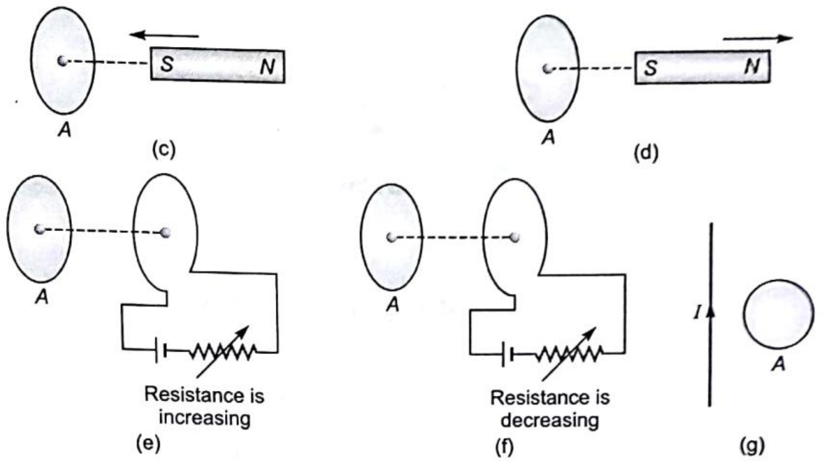


8. A circular coil (950 turns, radius 0.06 m) is rotating in a uniform magnetic field. At $t = 0$ s, the normal to the coil is perpendicular to the magnetic field. At $t = 0.01$ s, the normal makes an angle of 45° with the field. An average emf of magnitude 0.065 V appears in the coil for this duration. Find the magnitude of the magnetic field at the location of the coil.

9. Magnetic resonance imaging (MRI) is a medical technique for examining interior parts of the body. The patient is placed in a strong magnetic field. One safety concern is what would happen to the positively and negatively charged particles in the body fluids if an equipment failure caused the magnetic field to be shut off suddenly. An induced emf would cause these charges to flow, producing an electric current within the body. Suppose the surface area of the body is 0.032 m^2 through which the magnetic field passes through and the field is having magnitude of 1.5 T normal to the surface of the body. Determine the smallest time period during which the field can be allowed to vanish if the magnitude of the average induced emf is to be kept less than 0.01 V ?

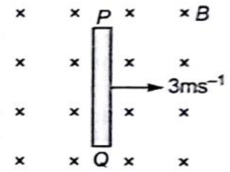
10. A piece of copper wire is formed into a single circular loop of radius 15 cm. A field is present in a direction parallel to the normal of the loop whose magnitude increases from 0 to 0.5 T in 0.25 s. The wire has a resistance per unit length of 3.3×10^{-3} Ωm^{-1}. What is the average electrical energy dissipated in the loop ?

11. Determine the direction of the induced current in coil *A* in various mentioned situations as seen from right side.



If the current is decreasing in the wire, determine the direction of induced current in A as seen from your position.

12. A conducting rod of length 0.5 m and having resistance of 3 Ω is moving in a magnetic field of 0.75 T with speed of 3 ms^{-1} as shown in figure (a). Determine the emf induced across the ends of the rod. (b) Which end of the rod is at higher potential. (c) Determine the induced current through the rod. (d) Determine the potential difference across the ends of the rod.



Repeat the question if the two ends of the rod are connected by a wire of zero resistance.

C. Fill in the Blanks

1. The tangent drawn to magnetic field lines at any point gives the direction of .... at that potential.
2. Crowded field lines represent .... magnetic field.
3. Induced emf would be there in a circuit only for that much duration for which .... linked with circuit is ....
4. In Lenz's law, the cause means ....
5. In Lenz's law, the effect means ....
6. According to Lenz's law the magnetic flux due to induced current will .... the original flux if original flux is increasing.

D. True/False

1. Magnetic flux through a surface in a region is zero-means magnetic field in that region must be zero.
2. Emf induced in a circuit depends on how the flux has been changed.
3. If the magnetic flux linked with a open circuit is changing, then an emf will be induced in the circuit.
4. If the magnetic flux linked with a open circuit is changing, then the induced current in the circuit would be zero.
5. Lenz's law has no relation with Faraday's law of EMI.

High Skill Questions

Exercise 2

A. Only One Option Correct

1. The normal to a certain 1 m^2 area makes an angle of 60° with a uniform magnetic field. The magnetic flux through this area is the same as the flux through a second area that is perpendicular to the field if the second area is
(a) 0.856 m^2
(b) 1.15 m^2
(c) 2 m^2
(d) 0.5 m^2

2. Suppose this page is perpendicular to a uniform magnetic field and the magnetic flux through it is 5 Wb. If the page is turned around 30° *wrt* an edge, the flux through it will be
(a) 2.5 Wb
(b) 4.3 Wb
(c) 5 Wb
(d) 5.8 Wb

3. Suppose this page is perpendicular to a non-uniform constant magnetic field and the magnetic flux through it is 5 Wb. If the page is turned 30° *wrt* an edge, the flux through it will be
(a) 2.5 Wb
(b) 4.3 Wb
(c) 5 Wb
(d) 5.8 Wb

4. A 2 T uniform magnetic field makes an angle of 30° with the z-axis. The magnetic flux through a 3 m^2 portion of the x - y plane is
(a) 2 Wb
(b) 3 Wb
(c) 5.2 Wb
(d) 6 Wb

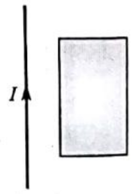
5. A uniform magnetic field makes an angle of 30° with the z-axis. If the magnetic flux through a 1 m^2 portion of the x - y plane is 5 Wb, then the magnetic flux through a 2 m^2 portion of the same plane is
(a) 2.5 Wb
(b) 4.3 Wb
(c) 10 Wb
(d) 7.5 Wb

6. The emf that appears in Faraday's law is
(a) around a conducting circuit
(b) around the boundary of the surface used to compute the magnetic flux
(c) throughout the surface used to compute the magnetic flux
(d) perpendicular to the surface used to compute the magnetic flux

7. A square loop of wire lies in the plane of the paper. A decreasing magnetic field is directed into the page. The induced current in the loop is
(a) clockwise
(b) anticlockwise
(c) Zero
(d) depends upon the rate at which field is changing

8. A square loop of wire lies in the plane of paper. A increasing magnetic field is directed into the page. The induced current in the loop is
(a) clockwise
(b) anticlociwise
(c) Zero
(d depends upon the rate at which the field is changing

9. A long straight wire is in the plane of a rectangular conducting loop. The straight wire carries a constant current I in the direction shown. If the wire is moved towards the rectangle, the current in the rectangle is



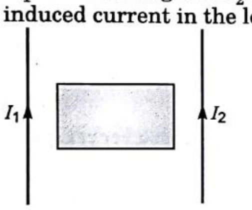
(a) Zero
(b) clockwise
(c) anticlockwise
(d) The direction can't be determined

10. Repeat above question if the current in wire is increasing.
(a) Zero
(b) clockwise

(c) anticlockwise
(d) Direction can't be determined

11. Repeat Q. 9 if current is suddenly decreased to zero.
(a) Zero
(b) clockwise
(c) anticlockwise
(d) Direction can't be determined

12. A rectangular loop of wire is placed midway between two long straight parallel conductors as shown in the figure. The conductors are carrying currents I_1 and I_2 as shown. If I_1 is increasing and I_2 is constant, then the induced current in the loop is

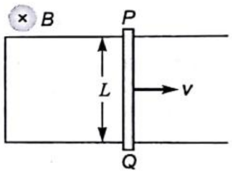


(a) Zero
(b) clockwise
(c) anticlockwise
(d) first clockwise, and then anti-clockwise

13. A circular loop of wire rotates about a diameter in a magnetic field that is perpendicular to the axis of rotation. Looking in the direction of the field at the loop the induced current is
(a) always clockwise
(b) always anticlockwise
(c) sometimes clockwise and sometimes anticlockwise
(d) None of the above

14. The emf developed in a coil X due to the current in a neighbouring coil Y is proportional to the
(a) magnetic field in X
(b) rate of change of magnetic field in X
(c) resistance of X
(d) current in Y

15. A rod PQ is moving on two parallel frictionless rails as shown in the figure. The entire arrangement is placed in a magnetic field B. The resistance of the circuit is R. The force that must be applied on the rod to move it with constant speed v is



(a) $\dfrac{BvL}{R}$
(b) $\dfrac{B^2L^2v}{R}$
(c) $\dfrac{BLv^2}{R}$
(d) $\dfrac{2B^2L^2v}{R}$

B. More Than One Options Correct

1. 1 Weber is same as
(a) 1 T-m
(b) 1 T-m^2
(c) 1 V-s/m
(d) 1 C-Ω

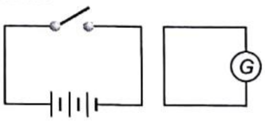
2. Faraday's law of EMI states the [Mark out the incorrect option]
(a) Induced emf is proportional to rate of change of magnetic field.
(b) Induced emf is proportional to rate of change of electric field.
(c) Induced emf is proportional to rate of change of magnetic flux.
(d) All of the above

3. If the magnetic flux linked with a circuit is changing with time.
(a) then energy may be dissipated in circuit as heat
(b) then emf must be induced in the circuit
(c) then induced current may be there in the circuit
(d) then induced current must be there in the circuit

4. As an externally generated magnetic field through a certain conducting loop increases is in magnitude, the field produced at the points inside the loop by the current induced in the loop
(a) may be increasing in magnitude
(b) must be increasing in magnitude

(c) may be decreasing in magnitude
(d) must be opposite to applied field

5. In the experiment shown mark the incorrect statements.



(a) There is a steady reading in G as long as S is closed
(b) A motional emf is generated when S is closed
(c) There is a deflection in G when S is closed or opened
(d) The current in G is always zero

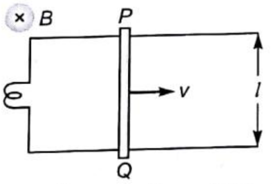
6. For above experiment shown the galvanometer shows a non-zero reading.
(a) when S is closed
(b) when S is kept closed
(c) when S is opened
(d) All of the above

7. In a region magnetic flux through a surface is zero, then it means
(a) magnetic field in the region must be zero
(b) magnetic field in the region may be zero
(c) magnetic field in the region may be non-zero
(d) magnetic field in the region must be non-zero

C. Comprehend the Passage Questions

Passage I

Consider two parallel conducting rails placed in a uniform magnetic field as shown in figure. The rails and the rod PQ are having negligible resistance but the electric bulb is having a resistance of 100 Ω. The rod is msoving towards right with constant velocity of $v = 5\ \text{ms}^{-1}$. The magnitude of magnetic field is 2 T and $l = 0.2$ m.



Based on above information answer the following questions.

1. The emf induced by the moving rod is
(a) 2 V (b) 1 V (c) 0.5 V (d) 4 V

2. The end of the rod which is at higher potential is
(a) P
(b) Q
(c) Either one, Can't be predicted
(d) None of the above

3. The electrical power delivered to the bulb is
(a) 0.02 W (b) 0.04 W
(c) 0.08 W (d) 1.46 W

4. The force applied to the rod by an external agent so that the rod moves with constant velocity is
(a) 4×10^{-3} N (b) 2×10^{-3} N
(c) 8×10^{-3} N (d) None of above

5. If the rod is having a resistance of 50 Ω, then the emf induced by the rod is
(a) 2 V (b) 1 V (c) 0.5 V (d) 1.33 V

6. If the rod is having a resistance of 50 Ω, then the potential difference across its two ends are
(a) 2 V (b) 1 V
(c) 5 V (d) 1.33 V

Passage II

A coil of wire consists of 20 turns, each of which has an area of $1.5 \times 10^{-3}\ \text{m}^2$. A magnetic field is perpendicular to the surface of each loop at all times, so that angle between the area vector and magnetic field is zero at all times. At $t = 0$ s, the magnitude of the magnetic field at the location of coil is 0.05 T and at $t = 0.1$ s, the magnetic field increases to 0.06 T.

Based on above information answer the following questions :

7. The average emf induced in the coil during 0.1 s duration is
(a) -3×10^{-3} V (b) 3×10^{-3} V
(c) 1.5×10^{-4} V (d) -1.5×10^{-4} V

8. The average emf induced in the coil if the magnetic field decreases instead of increasing, is
(a) -3×10^{-3} V (b) 3×10^{-3} V
(c) 1.5×10^{-4} V (d) -1.5×10^{-4} V

Answers

Towards Proficiency Problems
Exercise 1

B. Numerical Answer Types

1. 1 Wb, 0.5 Wb, 0 Wb **3.** 75 Wb
4. (a) 72.8×10^{-5} Wb, (b) 0, (c) 470.4×10^{-5} Wb
5. $\frac{800}{3}$ mV, $\frac{10}{3}$ mA **6.** 6.495 V, 1.0825 A **7.** 4.02 mV **8.** 8.55×10^{-5} T
9. 4.8 s **10.** 1.6 J **11.** (a) ACW, (b) CW, (c) CW, (d) ACW, (e) CW, (f) ACW, (g) CW
12. (a) 1.125 V, (b) P, (c) O, (d) 1.125 V

C. Fill in the Blanks

1. Magnetic field **2.** Strong **3.** Magnetic flux, Changing
4. Changing flux **5.** Induced emf and current **6.** Weaken

D. True/False

1. F **2.** F **3.** T **4.** T **5.** F

High Skill Questions
Exercise 2

A. Only One Option Correct

1. (d) **2.** (b) **3.** (b) **4.** (c) **5.** (c) **6.** (b) **7.** (a) **8.** (b) **9.** (c) **10.** (c)
11. (b) **12.** (c) **13.** (c) **14.** (b) **15.** (b)

B. More Than One Options Correct

1. (b, c, d) **2.** (a, b) **3.** (a, b, c) **4.** (a, c, d) **5.** (a, b, d)
6. (a, c) **7.** (b, c)

C. Comprehend the Passage Questions

1. (a) **2.** (a) **3.** (b) **4.** (c) **5.** (a) **6.** (d) **7.** (a) **8.** (b)

Explanations

Towards Proficiency Problems
Exercise 1

Numerical Answer Types

1. $\phi = BA\cos\theta$

For $\theta = 0°$, $\phi_1 = 1$ Wb
$\theta = 60°$, $\phi_2 = 0.5$ Wb
$\theta = 90°$, $\phi_3 = 0$ Wb

2. $\phi_{ABCD} = B_x \times A = 0.2$ mWb
$\phi_{OGFE} = -B_x \times A = -0.2$ mWb
$\phi_{ABGF} = B_y \times A = 0.4$ mWb
$\phi_{OECD} = -B_y \times A = -0.4$ mWb
$\phi_{ADEF} = B_z \times A = 0.6$ mWb
$\phi_{BCOG} = -B_z \times A = -0.6$ mWb

3. $\phi = N \times \vec{B}\cdot\vec{A}$
$= 100 \times 3 \times 0.5 \times \cos 60°$
$= 75$ Wb

4. (a) $\phi = 2.6 \times 10^{-5} \times 28 = 72.8 \times 10^{-5}$ Wb
(b) $\phi = 0$
(c) $\phi = 4.2 \times 10^{-5} \times 112$
$= 470.4 \times 10^{-5}$ Wb

5. $e_{av} = -\left[\dfrac{\phi_f - \phi_i}{\Delta t}\right] = -\dfrac{800}{3}$ mV

$i_{av} = \dfrac{e_{av}}{R} = \dfrac{800}{3 \times 80}$ mA $= \dfrac{10}{3}$ mA

6. $e_{av} = \dfrac{\phi_f - \phi_i}{\Delta t}$

$= \dfrac{0.4 \times 300 \times 0.05 \times \cos 30° - 0}{0.8}$

$= 6.495$ V

$i = \dfrac{6.495}{6} = 1.0825$ A

7. $\phi_i = 0.8 \times \pi \times 4 \times 10^{-4}$
$\phi_f = 0$
$\Delta t = 0.25$ s

$e = \left|\dfrac{-\Delta\phi}{\Delta t}\right| = 4.02$ mV

8. $e = 0.065 = \dfrac{950 \times \pi \times 0.06^2 \times B \times \cos 45°}{0.01}$

$\Rightarrow \quad B = 8.55 \times 10^{-5}$ T

9. $e = 0.01 = \dfrac{1.5 \times 0.032}{\Delta t}$

$\Rightarrow \quad \Delta t = 4.8$ s

10. $e = \dfrac{0.5 \times \pi R^2 - 0}{0.25}$

$I = \dfrac{e}{\rho \times 2\pi R}$

$= \dfrac{0.5 \times \pi R^2 \times 4}{\rho \times 2\pi R}$

$= \dfrac{R}{\rho} = \dfrac{0.15}{3.3 \times 10^{-3}}$ A

$H = I^2(\rho \times 2\pi R) \times \Delta t = 1.6$ J

12. (a) $e = Bvl = 0.75 \times 3 \times 0.5 = 1.125$ volt.
(b) From RHPR No. 3 or Fleming's right hand rule we can find that P is at a higher potential.
(c) 0 as circuit is open.
(d) Same as that of induced emf.
When the ends of the rod are connected by a conducting wire of zero resistance, then the current flows through rod, given by

$i = \dfrac{1.125}{3} = 1.375$ A

Potential difference across the ends of the rod is

$V = e - IR$
$= 1.125 - 0.375 \times 3 = 0$

Chapter 17

Optics

The First Steps' Learning

- What is Light ?
- The Nature of Light
- Geometrical Optics
- Laws of Reflection (Specular Reflection)
- Image Formation by a Plane Mirror
- Spherical Mirrors
- Magnification
- Refraction
- Lens Theory

You all would be very much familiar with the word light, as we use it many times a day in our daily conversations. Can you ever imagine what is the life for the one having no eyes ie, for completely blind persons ? Can you ever think of your life without instruments like cameras, telescopes, sunglasses, spectacles, mirrors etc. ? Have you ever thought, how a new born baby gets the first perception of this beautiful and colourful world surrounding him or her? Some of you may enjoy the colourful things surrounding you like blue lakes, green forests, colourful rainbows, rising and setting sun etc. Can we enjoy these colourful natural phenomena without colours say in black and white mode ? The existence of these colourful phenomena in this world is based on the concept of light, and the branch of physics which deals with the effect and behaviour of light is termed as ***optics****. The study of optics is broadly classified into two categories namely—geometrical and wave optics. Major part of our discussion in this chapter is concerned with geometrical optics.*

Principles of optics (like law of reflection, Snell's law etc.) and the optical phenomena (like reflection, refraction, diffraction, polarisation, scattering etc.) play a very crucial role in our daily lives as well as in science and technology. Knowledge of the properties of light helps us to understand the blue colour of the sky and the design of optical devices such as telescopes, microscopes, spectacles, cameras, projectors etc. Our ability to see the objects, differentiating their colours, eye pleasing rainbows and sight seeings etc are all possible because of the light only. The same principles of optics also lies at the core of modern developments like lasers, optical fibres, holograms etc.

In this chapter we shall discuss the laws of reflection and refraction at plane and curved surfaces, about mirrors, prism, lenses etc. But before coming to the central discussion of the chapter, it is required that we have a preliminary idea about the light, its properties and the terms we shall use in this chapter and that's what we are going to explain these you the first.

What is Light ?

We all are familiar about the word light, very frequently we make the statements like today sun is very bright *ie*, giving more light, there is no light pierces in dense forest, a particular region is having darkness or having no light etc. But hardly, we bother about exactly light means in physics. "Light is a part of electromagnetic spectrum, which makes objects visible to our eyes."

On the basis of wavelength/frequency electromagnetic spectrum is classified into various parts whose schematic diagram has been shown in Fig. 17.1.

The part of the electromagnetic spectrum whose wavelength is lying approximately within the range 4000 Å to 7000 Å is termed as the visible part of electromagnetic spectrum and is popularly known as light. To the left of visible part, ultra-violet rays would be there whose wavelength is smaller than the wavelength of light while on right side we have infra-red rays.

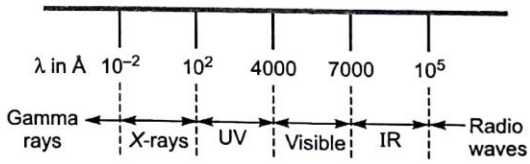


Fig. 17.1 Emf spectrum

Thus light is a part of electromagnetic spectrum whose wavelength lies between 4000 Å to 7000 Å (1 Å = 10^{-10} m) which we also call as the visible light, due to the fact that it makes objects visible to our eyes. Light carries the energy with it and hence it can also be understood as a form of energy which causes sensation to our eyes to make the objects visible to us, just like sound is a form of energy which causes us to hear.

The Nature of Light

Now the question arises, does the light consist of some particles like all physical objects (chair, pen, board etc) or is it a wave (like string waves etc) ? This particular question took centuries to be answered until about the middle of 17th century. Till then it was strongly believed that light consisted of tiny particles called corpuscles, this theory was mainly provided by Newton and this particle model of light was able to explain reflection, refraction etc. But in 1670, Christian Huygen's showed that laws of reflection and refraction could be explained by considering light as wave, he presented his theory (only theory, no experimental proof was there with him) on the basis of wave nature of light but has been discarded by society due to various reasons. It was not until 1827, when experiments of Thomas Young (this experiment is popularly known as Young's double slit experiment) demonstrated that corpuscular theory of light was unable to explain the interference. Interference is an optical phenomena which can only be expected when light behaves as wave. Since Thomas Young proved and showed experimentally that light shows interference and only waves can exhibit interference, so the conclusion made after Young's experiment was that the light behaved as waves. But the picture was not yet over, in the beginning of 19th century, another giant Albert Einstein showed and explained about photoelectric effect [when light of suitable frequency falls on a metallic surface, then electrons are ejected out from metallic surface, this phenomenon is termed as photoelectric effect], and declared that photoelectric effect can be explained only by considering particle nature of light [Although his particle theory of light was very different from Newton's corpuscular theory]. Now, the question arises, what we consider light to be, particle or of wave nature ? One experimental observation says that light must behave as wave while other says that light is a particle in nature, and we know that in development of science experimental observations and results are supreme and nothing else. The solution to all these confusion came in the form of a statement—**"Light has dual nature, it can behave as wave as well as particle—depending on the situation"**. At the moment it is not required for you to know the conditions under which light behaves as a particle or wave—you just need to know that the light has a dual nature.

Properties of Light and Some Important Terms Used in Optics

Before proceeding to understand reflection, we are making you aware of some of the basic properties and terms related to light.

1. Light is a form of energy or we can say visible part of electromagnetic spectrum which makes objects visible to our eyes, *ie*, light causes the sensation of sight.
2. The wavelength of light lies between 4000 Å to 7000 Å [1 Å = 10^{-10} m.
3. The speed of light is approximately 3×10^8 ms^{-1} in vacuum*, the speed of light is different in different media, although in vacuum all types of light travels with same speed.
4. Light possesses a dual nature *ie*, it can behave as wave as well as a particle. Phenomena like interference, diffraction etc can be explained only through wave nature of light while photoelectric effect can be explained only by particle nature of light.
5. Light doesn't require any medium for its propagation *ie*, light can propagate through empty space (vacuum) also.
6. Any (material or non-material) medium through which light can pass through is termed as an optical medium. For

* It has been proved by Einstein that no object can move faster than the speed of light.

example, vacuum, air, glass, water, gases, plastic etc.

On various bases optical media are classified into various categories.

(A) On the basis of optical properties and the composition of medium.

1. **Homogeneous medium** If the composition and optical properties of the medium are same everywhere, then the medium is said to be a homogeneous medium.
2. **Heterogeneous medium** If the composition and optical properties of the medium are different at different locations in the medium, then the medium is said to be a heterogeneous medium.

(B) On the basis of extent to which the light passes through the medium.

1. **Transparent medium** The medium through which the light can pass through almost completely is said to be transparent medium. For example, air, vacuum, glass etc.
2. **Translucent medium** The medium through which the light passes only partially is said to be a translucent medium. For example, thick glass slab, oil etc. In translucent medium intensity of light reduces.
3. **Opaque medium** The medium through which no light passes through is said to be an opaque medium. For example, bricks, woods etc.

(C) On the basis of the speed of light

1. **Rarer medium** In this medium the speed of light is less. For example, water is a rarer medium than air.
2. **Denser medium** In this medium the speed of light is more. For example, air is denser than water.

7. When light goes from one medium to another its frequency doesn't change but its speed and wavelength do change *ie*, speed of light is different in different media while frequency of a given light is same in all media, and from $\lambda = \frac{v}{f}$ we can say that wavelength of a given light is also changing as the medium changes.
8. Colour of light is determined by its frequency, and as frequency of light is same in different media, so the colour of light doesn't change as it propagates from one medium to another.
9. Whenever a light strikes a boundary separating two media, three phenomena occur simultaneously, these three phenomena are reflection, refraction and absorption. Depending on the nature of boundary any of the phenomena may dominate the others. For example, when light falls on a mirror, the light would be almost completely reflected.
10. All objects which emit light (directly or indirectly) are known as sources of light. For example, the Sun, electric bulb, your notebook in brightly lit room etc are sources of light. Source of light can be of any shape and size. Sources of light are classified into two :
    (a) Luminous source or self-luminous sources
    (b) Non-luminous sources
    (a) Luminous sources : Those objects which emit their own light are luminous sources. For example, the Sun, lamp, candle, tube light, etc.
    (b) Non-luminous source : Those objects which don't have their own light but are still visible to us are termed as non-luminous sources. For example, table, notebook, moon etc.
11. We (our eyes) are able to see the object when light coming from the object (directly or indirectly) reaches our eyes. For example, we are able to see the sun or

electric bulb (luminous sources or objects) because light from them is reaching to our eyes. The objects which are not having their own light are visible to us because of the light reflected (or scattered) from them. For example, we are able to see table or chair or books etc kept in a lighted room because light coming from the luminous sources— say the electric bulb is falling on these objects and this light is reflected back by these objects (non-luminous objects), and this reflected light when reaches to our eyes we would be able to see the object. From above discussion we can also answer the question—why we are we not able to see the objects placed in dark ?

12. The branch of optics in which the wave nature of light doesn't play an important role constitutes the subject-matter of geometrical optics. In geometrical optics the size of obstacles (optical instruments like lenses, mirrors etc) are very large as compared to wavelength of light and thus the wave nature of light is of no importance. In geometrical optics (also termed as ray optics) light can be considered as a ray which moves in a straight line unless some obstruction occurs in its path or the medium changes, this fact/principle is also termed as **"rectilinear propagation of light."**

Thus according to the rectilinear propagation of light, the light rays start from a point (object or source) and travel along straight lines until they fall on an object or a surface separating the two media.

Now we shall explain some more important terms to understand the optics.

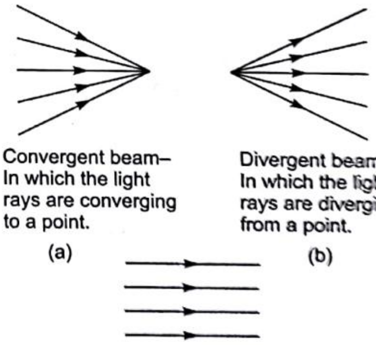
Ray of light Any straight line which can represent the path of light from one point to another—say from point A to B is termed as the ray of light. If a light ray is passing through A and B as shown in the figure, then light passes through all points on the straight line AB. The arrow on the ray of light represents the direction of propagation of light.

Fig. 17.2

Point source of light A source of light whose size is very small as compared to the concerned distance travelled by the light is termed as a point source of light. All other sources would be termed as the extended sources of light.

All the sources of light emit infinite numbers of light rays in all directions, but only few (2 to 3) are enough to understand the optical phenomena.

Beam A bundle of light rays is termed as beam or light beam. A beam can be convergent, divergent or parallel as shown in Fig. 17.3.



Convergent beam–In which the light rays are converging to a point.
(a)

Divergent beam–In which the light rays are diverging from a point.
(b)

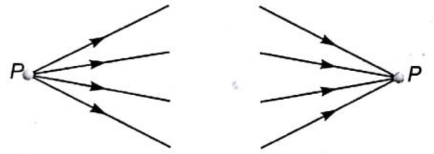
Parallel beam–In this different light rays are parallel to each other.
(c)

Fig. 17.3

The Object

In optics any point or body from which light has been emitted is termed as object. We can consider object and source to be same because in most of the cases we will encounter here, the object itself would be the source. The light rays

which an object emits are termed as incident rays, as these rays are going to be incident (fall on) some optical devices like mirrors, lenses etc. If the incident rays are diverging, then the object is said to be a **real object** while if the incident rays are converging at a point, then the point is said to be a **virtual object**.



Real point object–As rays are diverging from point *P*. (a)

Virtual point object–As the rays are converging at point *P*. (b)

Fig. 17.4

The concept of real and virtual objects would be more clear to you as you proceed this chapter.

The Image

In optics, the word image would be very frequently used and would be of utmost importance. In general the word image means where the object appears to be if seen from some optical instrument like mirror or lens. For a point object, corresponding image is defined as the point of intersection of reflected or refracted rays. If the reflected or refracted rays are intersecting in actual, then the image is said to be a **real image**, while if the reflected or refracted rays are not intersecting in actual but

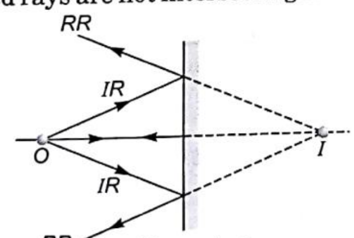


Fig. 17.5 Diagram showing real object *O*, virtual image *I*. *IR* = incident rays, *RR* = Reflected rays. Here image is virtual as *RR* are not intersecting in actual.

appear to diverge from a point when extended backwards, then this point is termed as **virtual image**.

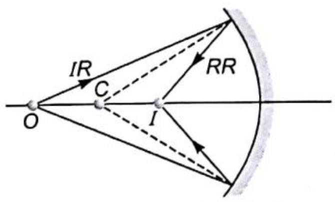


Fig. 17.6 Diagram showing real object *O*, Real image *I*. Here image is real as *RR* are intersecting in actual.

Pertinent Points Regarding *objects* and *Images*

- Human eyes can't observe virtual objects.
- A virtual image can't be taken on the screen (some cardboard, photographic film, wall etc.).
- Human eyes can't distinguish between the real and virtual images.
- Two rays are enough to locate the image of a point object.
- An object is visible from all directions due to scattering of light.
- A person is able to see the image of any object when reflected or refracted rays reach his eye, *ie*, we are able to see the object when light emitted by it or reflected by it reaches our eyes and we are able to see the image of an object if the reflected or refracted rays from some optical device reaches to our eyes.

The Ray Diagram

This diagram gives us the nature and description of image of an object formed by any optical instrument. In optics if one understands the construction of a ray diagram, half of the task would be over. While drawing a ray diagram for a point object you should keep following points in mind :

(a) Originate the incident rays from the object in all directions, and take any two incident rays which are going to fall on the optical instrument. In the diagram, there is no need to draw more than two incident rays.

(b) After striking the optical device, these incident rays will reflect or refract (depending upon the nature of

instrument), the point of intersection of these reflected or refracted rays will be the location of the image of point object formed by optical device.

If the object is an extended one (a collection of point objects), then to draw the entire image of the object, locate the image of two end points of the extended object and join them, this will give us the image of an extended object.

We will draw a lot of ray diagrams as and when required. Remember, if you have not understood the basics of object, image and ray diagrams, then don't panic. Simply proceed further and whenever required come back to this part again for a fast recap.

Geometrical Optics

In geometrical optics, we mainly deal with reflection and refraction. We have discussed in rectilinear propagation of light, that light rays propagates along a straight line until and unless some obstacle comes in its path or it crosses a boundary which is separating the two media. Actually what happens, when a light ray encounters some obstacle, all the three phenomena—reflection, refraction (transmission) and absorption will take place simultaneously. Depending upon the nature of surface or obstacle, out of these three phenomena any one can be dominating over the other two.

First we are going to see about reflection in detail, but before that we are telling you basic meaning of these three phenomena.

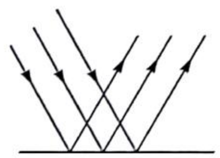
Reflection When a light ray is striking a surface, then a part of it is reflected back into the same medium, this phenomena is termed as reflection. For example, with the help of mirrors we can reflect the light.

Refraction When a light ray goes from one medium to another, then it deviates from its path, this phenomena is termed as refraction.

Absorption When a light is falling on a surface, then a part of the light gets absorbed by boundary, this phenomena is termed as absorption.

Reflection

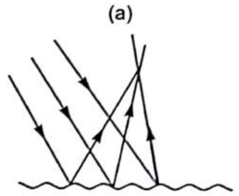
Most of the objects reflect a certain portion of the light falling on them. Consider the two surfaces—one smooth and shiny while other one is rough, a parallel beam of light is incident on both the surfaces as shown in Fig. 17.7.



Reflected light corresponding to a parallel beam of light from a smooth shining surface. For example, mirrors.

Specular Reflection

(a)



Reflected light corresponding to a parallel beam of light from rough surfaces. For example, wall.

Diffuse Reflection

(b)

Fig. 17.7

When the reflection takes place from a smooth and shiny surface the reflection is termed as **specular reflection** and when the reflection takes place from the rough surfaces, it is termed as **diffuse reflection.** Here, in your shall only talk about specular reflection and whenever we say reflection without specifying the nature of reflection, then we mean the specular reflection.

We are able to see most of the objects in our daily lives (like book, pen, plates, walls floor, roof, table, etc) only because of the reflection of light by these objects, and in general all these objects are rough and hence the reflection in all these cases is a diffuse reflection. Specular reflection takes place from perfectly smooth and shiny surfaces, and an example of one such surface is mirror.

A mirror is an optical device for which if light is incident on its reflecting surface, then it will be reflected back completely. Here, we shall study about reflection by two types of mirrors—plane and spherical.

Laws of Reflection (Specular Reflection)

Whenever a light ray strikes a smooth, polished surface *ie*, mirror, then it gets reflected in accordance with following two laws which are popularly known as laws of reflection :

1st Law of Reflection : The first law of reflection states—*"Angle of incidence is equal to angle of reflection"*. Angle of incidence means the angle between the incident ray and the normal drawn on reflecting surface at the point where the light ray is incident while angle of reflection means the angle between the reflected light and the normal to surface. *ie*, from Fig. 17.8, $\angle i = \angle r$.

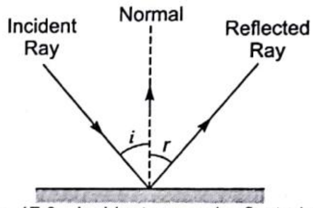


Fig. 17.8 Incident ray and reflected ray makes same angle with the normal.

2nd Law of Reflection : The second law of reflection states—*"The incident ray, the reflected ray and the normal to the surface, all lie in same plane."*

Both the laws of reflection are valid only for perfectly reflecting surfaces (*having any arbitrary shapes*).

C-BIs

Concept Building Illustrations

Illustration | 1 *A ray is incident on a plane mirror at an angle of 42° with the plane of mirror. Determine the angle made by the reflected ray wrt the normal drawn to the mirror.*

Solution Here, angle of incidence = 48°, and the angle of reflection has to be found. From 1st law of reflection. $\angle i = \angle r$. Here $\angle r = 48°$.

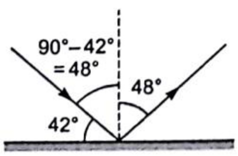


Illustration | 2 *Discuss what will happen when a ray of light is falling normally to the surface of a mirror ?*

Solution When a light ray falls normally to the surface of a mirror, then the angle of incidence is zero. From first law of reflection, angle of incidence is equal to angle of reflection, and hence angle of reflection is also zero. Thus the reflected ray would be along the normal to the mirror. Thus we can say when a light ray is incident along the normal to mirror, then it will be reflected back along the same path.

Images Formation By a Plane Mirror

Whenever we see our face in the mirror, then in the mirror, we see the image of our face. We have already discussed about object, image and ray diagram and the steps to locate the image of an object formed by an optical device just like as in above mentioned example. Now we are going to discuss the formation of image of a point object and an extended object by a plane mirror. Let us consider a point object O placed in front of a plane mirror as shown in Fig. 17.9. Now we are going to locate the image of this point object as formed by the plane mirror. The two rays I and II originating (emitted) from the point object O are considered which strikes the plane mirror at P and Q respectively and reflected as rays III and IV as shown. As the reflected rays are diverging the image formed by plane mirror is virtual and is formed at point from where the reflected rays appear to diverge. The image thus formed is represented by I in figure. From 1st law of reflection $i_1 = r_1$ and $i_2 = r_2$.

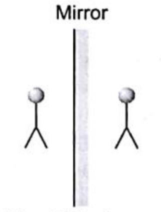


Fig. 17.9 Image formation of a real object by a plane mirror

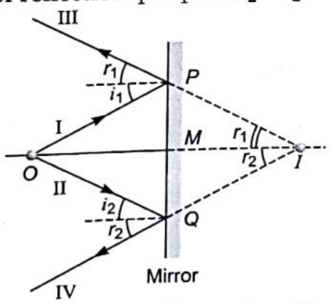


Fig. 17.10 Formation of image of a point object by a plane mirror.

If object O is at a perpendicular distance x from mirror, then image I formed would also be at distance x from the mirror. This can be verified very easily as OMP and IMP are congruent triangles and hence $OM = IM$. Thus we can conclude "For real point object, image formed by a plane mirror would be virtual, lying behind the mirror at the same distance from the mirror as the object is".

It has to be kept in mind that—"the distance of the image and object from plane mirror is the same" means distance perpendicular to the mirror plane. For example, a point object O is placed in front of mirror as shown in Fig. 17.11, then point I_1 is also at same distance from mirror as the object is, but I_1 is not the image of O. Actually, the image of object is I_2.

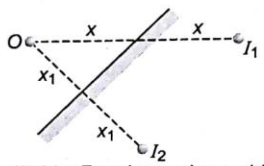


Fig. 17.11 For plane mirror, object distance is equal to image distances is valid only along the normal to mirror.

Now, we shall locate the image of an extended object formed by a plane mirror. Consider an extended object AB which is kept in front of a plane mirror as shown in Fig. 17.12. To locate the image of the complete object, it is enough to locate the image of end points A and B and then join these image points. To locate the image of end points A and B, we can follow the same procedure as for a point object.

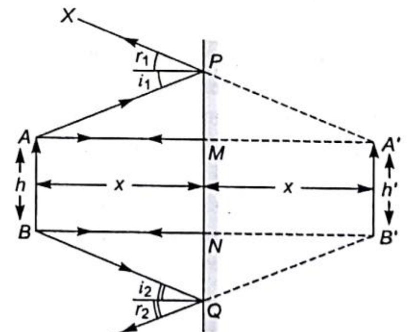


Fig. 17.12 Image formation of an extended object AB by a plane mirror.

Here to locate the image of point A, we have taken two rays AM and AP which after reflection get reflected to respectively which when produced backwards intersect at A', thus locating the image of point A. Similarly, we can locate the image of point B at B' with the help of incident rays BN and BQ. Now, to locate the image of the entire object AB, join the image points $A'B'$ which will give us the image of an extended object AB.

From geometry and previous discussion— $BN = NB'$ and $AM = MA'$ as well as AMA' and BNB' are parallel lines, so $A'B' = AB$, *ie*, size of the image is equal to the size of object.

Thus for a real extended object—the image formed by a plane mirror would be virtual, erect, at the same distance from the mirror as the object is and of same size as the object. Here erect means the image is upright. Here, diagrammatically we are explaining the words erect and inverted.

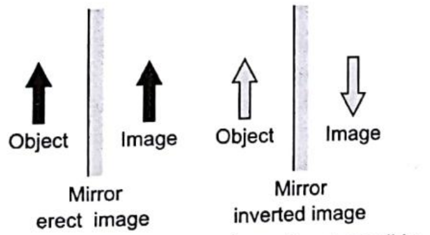


Fig. 17.13 Here, the second case is not possible.

Some important points related to the plane mirrors :

1. For a real object, a plane mirror always forms a virtual image.
2. For an extended object, the size of image formed by the plane mirror is same as that of the size of the object.
3. The perpendicular distance of the image from mirror is same as that of the object distance from mirror.
4. In general, the plane mirror forms an erect image, but if mirror is kept horizontal and object is placed erect on it then image formed by plane mirror would be inverted as shown in figure.

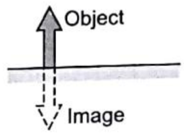


Fig. 17.14 Plane mirror can also form inverted image.

5. The plane mirror forms a laterally inverted image *ie*, person's right hand standing in front of plane mirror becomes the image's left hand. In other words, we can say that the plane mirror converts the right hand into left hand and *vice-versa*.

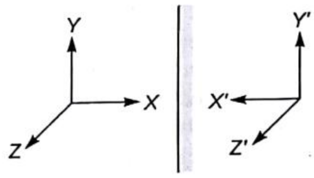


Fig. 17.15 Plane mirror converts the right handed system to a left handed system.

Deviation Produced by Reflection from a Mirror

Let us consider a ray OP which strikes the mirror at point P and due to reflection at P it bends and after reflection moves along PI. If mirror would not be there then the ray will go along OPQ but due to mirror it deviates from its path and moves along PI, the angle δ shown in the figure represents the deviation produced by the plane mirror.

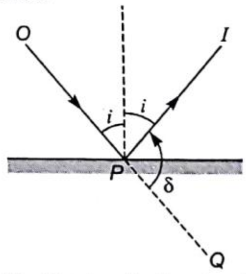


Fig. 17.16 Due to reflection by mirror, the incident ray get deviate from its path.

If i is the angle of incidence, then deviation produced by plane mirror is $\delta = \pi - 2i$.

This expression and concept is not valid only for plane mirrors but is also valid for any arbitrary shaped mirror.

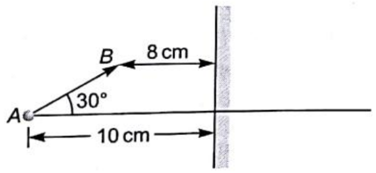
C-BIs

Concept Building Illustrations

Illustration | 3 *The object is at a perpendicular distance of 5 cm from the plane of mirror. Determine the location of image formed by the plane mirror.*

Solution We know that perpendicular distance of object from mirror = perpendicular distance of image from mirror, so, the required distance is 5 cm, perpendicular to the plane of mirror on opposite side of the object.

Illustration | 4 *For the extended object shown, draw the image formed by the plane mirror.*



Solution Simply you locate the image of points A and B and then join these image points to get the image formed by the plane mirror. The image of ends A and B can be drawn by using the described concepts in the chapter. Then join these image points A' and B' to get the complete image of object AB.

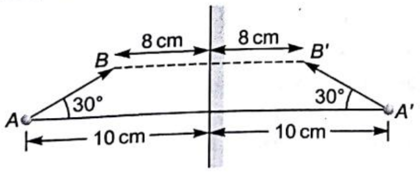
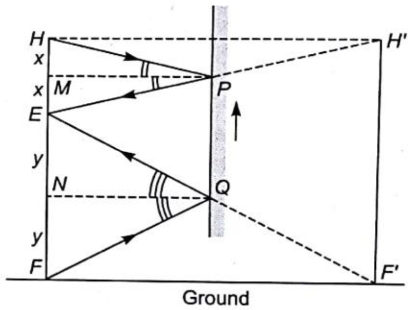


Illustration | 5 *A person of height H wants to see his complete image in a plane mirror. Determine the minimum height of the plane mirror which can serve the purpose.*

Solution To see the image of any object by an observer it is required that reflected rays reach the eyes of the observer. To see the complete image of himself means the person must be able to see the hairs as well as his toes in the mirror. It means the incident rays from the top of the head H after reflection from the mirror must enter into the eyes of person and same should happen with incident rays from the feet.



The situation is shown in figure here we have taken any arbitrary sized mirror, later on we will remove the not required part. Ray diagram for the situation is shown clearly in the figure.

From geometry,

$$HM = ME = x$$

and

$$EN = NF = y$$

$$H = HE + EF$$

$$= 2HM + 2EN$$

$$= 2(x + y)$$

$$\Rightarrow \quad x + y = \frac{H}{2}$$

Required length of mirror

$$= PQ = MN = x + y = \frac{H}{2}.$$

Spherical Mirrors

We can have curved mirrors on which when light is incident would be reflected back just like from plane mirrors. A special type of curved mirror is spherical mirror. Spherical mirrors are those in which the surface of the mirror is a part of the sphere. Spherical mirrors are constructed by taking a portion out from a hollow glass sphere and then by making one surface of this cut out portion reflecting by silvering it.

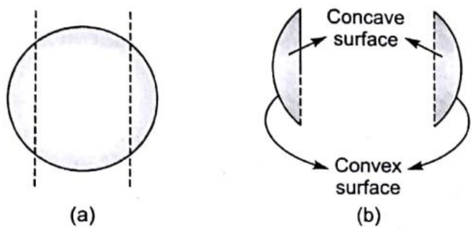


Fig. 17.17

We have two types of spherical mirrors :
(a) Concave mirrors and
(b) Convex mirrors.

In a concave mirror, the reflection of light takes place at the concave surface while in a convex mirror the reflection of light takes place at the convex surface.

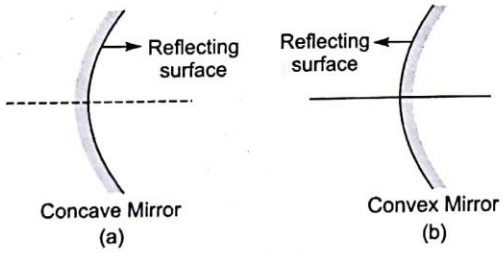


Fig. 17.18

Just like for plane mirrors, the light is incident on the reflecting surface of spherical mirror and from there it is reflected back. If a parallel beam of light is incident on a plane mirror, then we know that the reflected light would have been also a parallel beam, but for spherical mirrors this won't be the situation.

Some important terms used for spherical mirrors :

Centre of curvature (C) The centre of the sphere of which the mirror is a part is termed as centre of curvature.

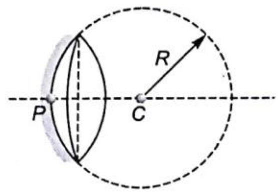


Fig. 17.19 Concave Mirror

Radius of curvature (R) The radius of the sphere of which the mirror is a part is termed as the radius of curvature.

Pole (P) The centre point of the surface of the mirror is termed as the pole of mirror.

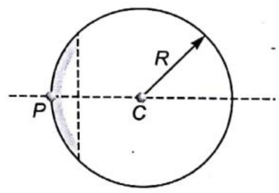


Fig. 17.20 Convex Mirror

Principal axis The line joining the pole and the centre of curvature of the mirror is termed as principal axis.

Aperture The surface area of the mirror that determines the amount of light falling on the mirror is termed as its aperture.

Paraxial rays The rays which are very close to PA and almost parallel to PA are termed as paraxial rays. This just provides a conceptual base to make the theory simplified, same as the ideal string having zero mass in mechanics.

Principal focus (F) and focal length (f) When paraxial rays fall on a spherical mirror, then they converge to or appear to diverge from a point after reflection. This point

where the reflected rays converge or appear to diverge from, is termed as the principal focus of mirror. The separation between focus and pole of the mirror is termed as focal length of the mirror.

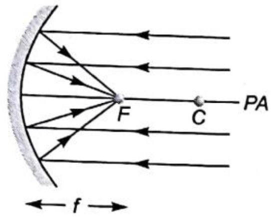


Fig. 17.21 (a) For a concave mirror, focus is real and lying in front of reflecting surface.

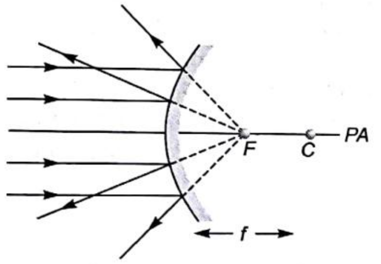


Fig. 17.21 (b) For a convex mirror, focus is virtual and lying behind the reflecting surface.

Concave mirror is also known as a converging mirror because it converges the light rays falling on it, while convex mirror is termed as a diverging mirror as it diverges the light rays falling on it. This concept can be clearly visualized from diagrams in Fig. 17.21.

Assumptions used for spherical mirrors Throughout our discussion of spherical mirrors some assumptions would be made which are as :

Assumption I Mirror would be considered of a small aperture, so that the rays close to pole should be participating in the formation of image.

Assumption II Paraxial rays would be considered.

If we won't take above two assumptions, then there won't be any sharp focus for spherical mirrors.

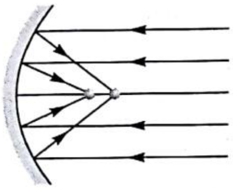


Fig. 17.22 (a) A concave mirror under normal condition *ie*, when no assumption is taken.

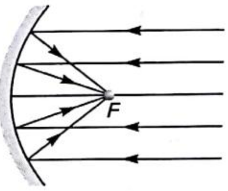


Fig. 17.22 (b) A concave mirror under assumption.

Although the two diagrams in Fig. 17.22 seem to be very similar but the diagram (b) is only magnified for sake of clarity.

Three important rays for spherical mirrors to locate the image.

As such any object emits the light rays in all directions but only two are required to locate the image of any point object. Here we are mentioning three rays with the help of which we can easily locate the image of an object formed by a spherical mirror :

1. A ray coming parallel to principal axis after reflection passes through focus. This ray is in accordance with definition of principal focus.
2. A ray passing through focus becomes parallel to principal axis after reflection.

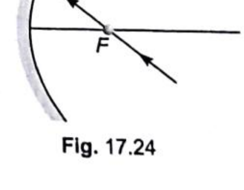


Fig. 17.23 PC is normal to mirror at *P*.

Fig. 17.24

3. A ray passing through the centre of curvature retraces its path after reflection as any line joining C and any point on the surface of mirror is normal to the mirror at that point.

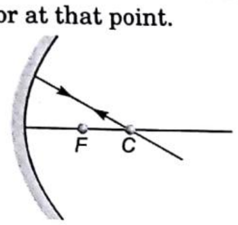


Fig. 17.25

In addition to above three standard rays if we require we can draw ray diagram in accordance with the first law of reflection, same as shown in Fig. 17.26. Here, $\angle OPC = \angle CPI$ as $\angle i = \angle r$.

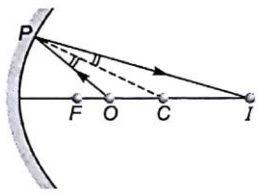


Fig. 17.26

Although, we discussed many points above by using a concave mirror, but these are also valid for convex mirrors.

Relation between Focal Length and Radius of Curvature and the Mirror Formula

Here we shall derive the relation between f and R for a spherical mirror without explaining its derivation. If f is the focal length of mirror and R is its radius of curvature, then $f = \frac{R}{2}$.

To solve the question related to spherical mirrors we also require the mirror formula, which is given as : $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

where v is the image distance along principal axis measured from the pole of mirror.

u is the object distance along the principal axis measured from the pole of mirror.

f is the focal length of mirror.

The above two expressions are valid for both types of spherical mirrors *ie*, for concave mirrors as well as for convex mirrors.

To use the above expressions and moreover in optics in general we have to use some sign convention. Sign convention means while using the formulas we have to substitute the values of various variables with appropriate signs. For example, in mirror formula while substituting values of the known quantity say f and u, we will substitute them with signs. In optics, sign with the value of physical quantity tells that on which side and where the corresponding location would be. In this book we are going to use the most standard sign convention and throughout our discussion of optics we will follow the same.

The Sign Conventions Used in This Book

1. The pole of the mirror (or optical centre of lens) would be taken as origin.
2. The principal axis in case of mirror (or optical axis in case of lens) would be taken as X-axis.
3. A line perpendicular to X axis and passing through origin would be taken as Y-axis.
4. Direction of incident rays would be considered as positive X-axis.
5. Above X-axis would be considered as positive Y-axis.

Here we draw some ray diagrams to illustrate the signs of various parameters.

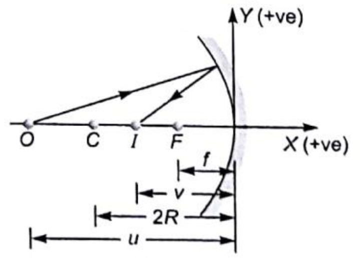


Fig. 17.27

Here all u, v, f and R would be negative and hence if we substitute value of any of the physical quantity in the standard equation, then we have to substitute with a negative sign.

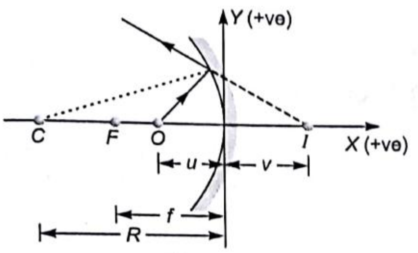


Fig. 17.28

u, f and R are negative and v is positive.

Always keep in mind that while solving questions, the known values have to be substituted with signs while unknown quantities will come out automatically with required signs. You don't have to bother about the location of the unknown physical quantity.

Magnification

In plane mirrors we have seen that size of image is same as that of the size of object, but in case of spherical mirrors the size of image formed by the mirror may be different from the size of object. To compare the size of image and the size of object, a term magnification is used. Magnification is defined as the ratio of size of image to size of object. It is denoted by m.

$$\text{Magnification, } m = \frac{\text{Size of image}}{\text{Size of object}}$$

In general, there are two types of magnifications :

(a) Lateral or transverse magnification

(b) Longitudinal magnification

Lateral magnification When the object is placed perpendicular to principal axis, then the magnification is termed as transverse magnification.

For a spherical mirror, $m = -\frac{v}{u}$

$$m = \frac{A'B'}{AB} = -\frac{v}{u}$$

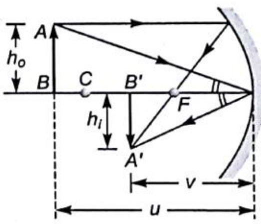


Fig. 17.29

For shown situation,

$A'B' = -h_i$ [As towards negative Y-axis]

$AB = h_o$

So, $\frac{-h_i}{h_o} = -\frac{v}{u}$

$$\frac{h_i}{h_o} = +\frac{v}{u}$$

Longitudinal magnification When the object is placed along the principal axis, then the magnification is termed as longitudinal magnification.

Generally, in this book, we shall deal only with lateral magnification.

C-BIs

Concept Building Illustrations

Illustration | 6 *If the radius of curvature of a spherical mirror is 20 cm, determine its focal length.*

Solution From $f = \frac{R}{2}$

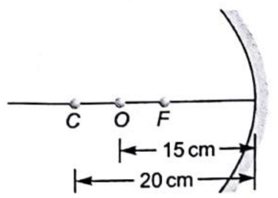
$$= \frac{20}{2}\text{ cm} = 10\text{ cm}.$$

Illustration | 7 *A point object is placed in front of a concave spherical mirror of radius of curvature 20 cm. The object distance from pole is 15 cm, determine the location of image.*

Solution From mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

From given information and according to standard sign convention, we have

$$u = -15\text{ cm},\quad R = -20\text{ cm},\quad v = ?$$



So, $$\frac{1}{v} + \frac{1}{-15} = \frac{1}{-20/2}$$

$\Rightarrow$ $$\frac{1}{v} = \frac{1}{15} - \frac{1}{10} = -\frac{1}{30}$$

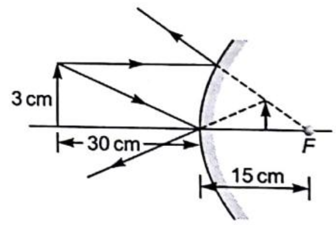
$\Rightarrow$ $$v = -30\text{ cm}$$

ie, image of point object O will be at a distance of 30 cm from the pole of mirror, negative sign tells that image is located on the left side *ie*, in front of mirror.

Illustration | 8 *A 3 cm high object is placed perpendicular to the PA in front of a convex mirror. The distance of the object from the mirror is 30 cm and the focal length of mirror is 15 cm. Determine the size of image formed by mirror.*

Solution From given information

$$u = -30,\quad f = 15\text{ cm}$$



Using mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$\Rightarrow$ $$\frac{1}{v} + \frac{1}{-30} = \frac{1}{15}$$

$\Rightarrow$ $$v = 10\text{ cm}$$

ie, image will form behind the mirror.

From magnification formula,

$$m = -\frac{v}{u} = -\left[\frac{10}{-30}\right] = \frac{1}{3}$$

$$\frac{h_i}{h_o} = \frac{1}{3}$$

$\Rightarrow$ $$h_i = \frac{h_o}{3}$$

$$h_o = 3\text{ cm}$$

So, $$h_i = 1\text{ cm}.$$

Image Formation of a Point Object by Spherical Mirrors

Here in this section we shall give you the details of ray diagram for various positions of point object in front of spherical mirrors.

Convex Mirror

1. When the real object is placed in front of convex mirror (at any location on PA), then the ray diagram would be as shown in Fig. 17.30.

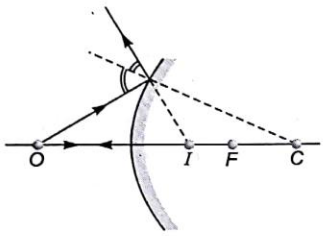


Fig. 17.30

Here image would be virtual and is lying between the focus and pole *ie*, on other side of mirror.

Thus we can conclude that if object is real and is placed in front of mirror, then the image formed would be virtual and lying between the pole and focus.

2. Now consider the case when the object is virtual and is lying between the pole and focus. For this situation, the ray diagram is as shown in Fig. 17.31. Here image would be real and lying in front of the mirror.

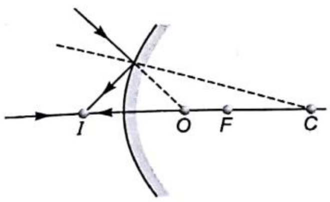


Fig. 17.31

Thus, if a virtual object between the pole and focus is considered, then its image formed by the convex mirror is real in nature and is lying in front of mirror.

3. When object is virtual and is lying beyond focus, then image would be virtual and lying beyond the focus. For one such location the ray diagram is as shown in Fig. 17.32.

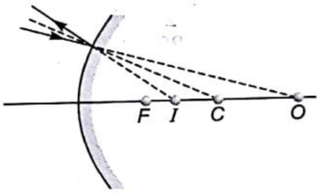


Fig. 17.32

Thus we can say that for a convex mirror if object is real, image would be virtual while if object is virtual, the image can be real or virtual.

Concave Mirror

Now here we are explaining the nature of image for different locations of a point object in front of a concave mirror.

1. **When object is at infinity** Object at infinity means the object is far away from the pole of mirror and in this situation the incident rays from the object are parallel to the principal axis.

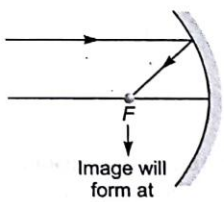


Fig. 17.33

In this case the image forms at focus and is real. Ray diagram is as shown in Fig. 17.33.

2. **When object is in between infinity and C** In this position of object, the image would be real and form between F and C.

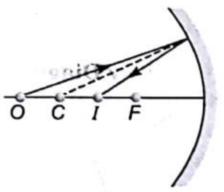


Fig. 17.34

3. **Object is at *C*** Image would be real and forms at *C*.

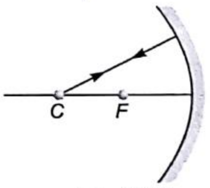


Fig. 17.35

4. **Object is between *C* and *F*** Image would be real and beyond *C*.

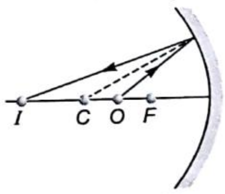


Fig. 17.36

5. **Object is at focus** Image would be at infinity.

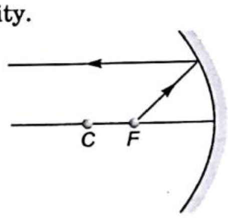


Fig. 17.37

6. **Object is between focus and pole** Image would be virtual and lying behind the mirror.

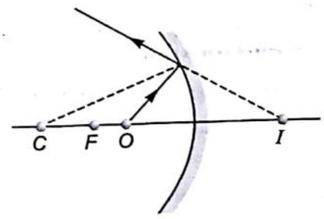


Fig. 17.38

7. **Object is virtual and lying behind the mirror** Image would be real and lying between the focus and pole.

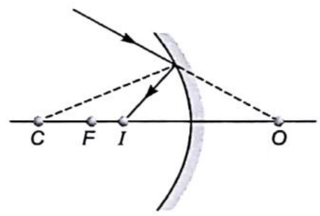


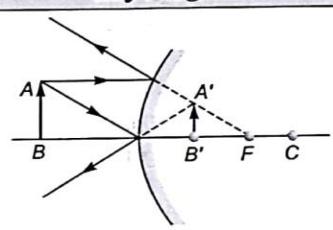
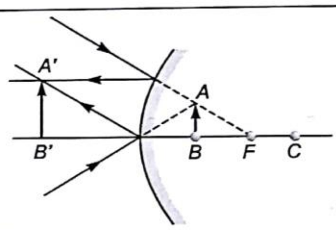
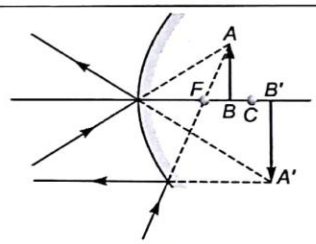
Fig. 17.39

So, it means that for a concave mirror as the real object moves from infinity towards the focus of mirror the image moves from focus to infinity and would be real. As the object moves from focus to pole, the image will be virtual and lying behind the mirror. If the object is virtual, then the image would be real and lying between focus and pole.

Image Formation of an Extended Objects by Spherical Mirrors

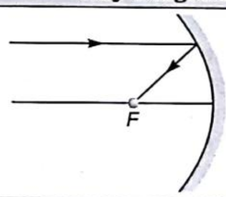
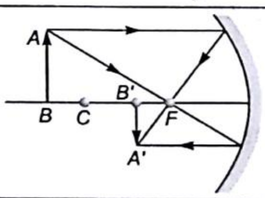
This topic is exactly same as the previous one, with the only difference that here an extended object is considered. First we shall consider the convex mirrors and then we shall take up concave mirrors.

Convex Mirrors

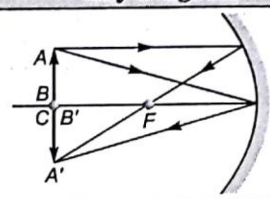
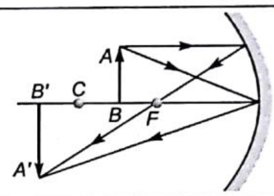
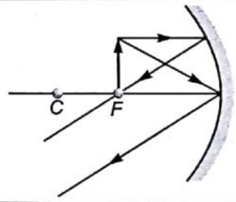
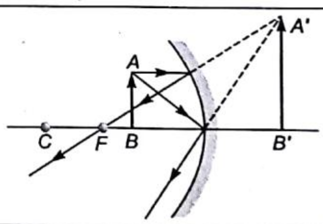
	Nature and location of object	Description of Image	Ray Diagram
1.	Object is real and lying anywhere in front of mirror.	Image would be located between the focus and pole, virtual, erect and diminished.*	
2.	Virtual object and lying between the focus and pole of mirror	Image in front of mirror, real, erect and magnified**	
3.	Virtual object, behind focus	Behind mirror, virtual image, inverted and can be magnified or diminished depending upon the location of object.	

In all ray diagrams, AB represents the object and $A'B'$ represents the image.

Concave Mirrors

	Object Location	Description of Image	Ray Diagram
1.	At infinity, real object	Image at focus, real point image	
2.	Between infinity and C	Between C and F, real, inverted and diminished	

*Diminished means size of image is smaller than the size of object.
**Magnified means size of image is larger than the size of object.

	Object Location	Description of Image	Ray Diagram
3.	At C	At C, real, inverted of same size.	
4.	Between C and F	Beyond C, real, inverted, and magnified	
5.	At F	At infinity, very large	
6.	Between focus and pole	Behind mirror, virtual, erect, and magnified	

Now, we shall first consider some important points about image formation by spherical mirrors :

1. For real extended objects, the image formed by a convex mirror is always virtual, erect and diminished.
2. For a real extended object placed in front of a concave mirror, the nature and location of image is dependent upon the location of object.
3. As a real extended object is moved from infinity to the centre of curvature along the principal axis of concave mirror, the image moves from focus to centre of curvature.
4. For a real extended object only, the concave mirror can give real and inverted image.

Refraction

Have you ever experienced that a coin in a bucket full of water when seen from upwards, seems to be somewhat above than its actual depth *ie*, coin seems to be floating. When a spoon is dipped in water, then it appears to be bent. These are the examples which arise due to refraction. When a light ray travels from one medium to another, then at the boundary

separating the two media, the light ray gets deviated from its path. This phenomenon is termed as refraction. In refraction, the light ray bends towards the normal or away from the normal.

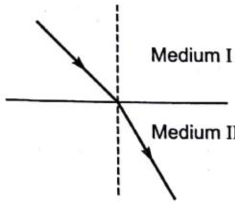


Fig. 17.41 When light goes from one medium to another it deviates from its path.

The boundary separating the two media is also termed as interface, and as the light comes from medium I to medium II, not the entire light passes to medium II. Only a portion of light in medium I passes to medium II and the remaining gets reflected or absorbed, but while discussing refraction we won't bother about reflected or absorbed parts.

As we have seen, bending of light ray from its original path due to change in medium is termed as refraction. Now the questions arises, does the light bend by same amount in different media? Does different light bend by same amount in same medium? Under which conditions the light will bend towards the normal and under which conditions it will bend away from the normal ? Answer to all these questions can be given by laws of refraction, in which a new term refractive index appears, so first let us be clear about the term refractive index.

Refractive Index

As we have already seen the speed of light is different in different media and moreover this fact is responsible for bending of a light ray as it goes from one medium to another *ie*, refraction. We know that speed of light is maximum in vacuum, and is equal to 3×10^8 ms^{-1} and in other media like water, glass etc it would be less than 3×10^8 ms^{-1}.

The ratio of speed of light in vacuum to speed of light in a particular medium is termed as the refractive index (RI) of this medium. It is denoted by μ (*pronounced as meu*), and is a dimensionless quantity. If c be the speed of light in vacuum and v be the speed of light in the medium, then for the medium, the refractive index is,

$$\mu = \frac{c}{v}.$$

As we know c is always greater than v, so $\mu > 1$. The medium having larger μ is said to be denser than the medium having lower μ. For example, among two media A and B having refractive indices 1.33 and 1.5 respectively, A is a rarer medium and B is a denser medium.

Another term related to refractive index is the relative refractive index. Refractive index of medium 1 *wrt* medium 2 is the relative refractive index of two media, and is equal to

$${}_2\mu_1 = \frac{\mu_1}{\mu_2}$$

where ${}_2\mu_1$ represents the refractive index of medium 1 *wrt* medium 2. Here,

μ_1 represents the refractive index of medium 1, and μ_2 represents the refractive index of medium 2.

For different lights, the same medium would have different values of μ as speed of different lights are different in the same medium.

Laws of Refraction

Consider a light ray which goes from medium 1 to medium 2 by crossing the interface AB as shown in Fig. 17.42. The refractive index of media 1 and 2 are μ_1 and μ_2, respectively. The line XY is normal to interface, the light ray PQ is the incident ray and due to refraction taking place at the boundary, the ray deviates to QR, this ray QR is termed as a refracted ray.

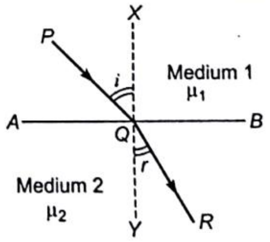


Fig. 17.42

The angle made by the incident ray with normal is termed as the angle of incidence *ie*, $\angle PQX$ and angle between the refracted ray and the normal is the angle of refraction *ie*, $\angle YQR$.

Same as the laws of reflection, there are two laws of refraction :

1st law : The incident ray, the refracted ray and the normal to the interface at the point of incidence are in the same plane.

2nd law : According to this law, ratio of sine of angle of incidence to sine of angle of refraction is equal to the ratio of refractive index of medium 2 to refractive index of medium 1.

ie, $$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} = {}_1\mu_2$$

The 2nd law of refraction is also termed as **Snell's law**.

If incident ray goes along the normal to a boundary separating two media *ie*, $i = 0$, then the ray will enter into 2nd medium without getting deviated, this can be clearly explained with Snell's law. If $i = 0$, then from Snell's law, $\frac{\sin 0}{\sin r} = {}_1\mu_2$, as ${}_1\mu_2 \neq 0$, so $\sin r = 0$ which is possible when $r = 0$. Hence, we can say that if light is striking a boundary normally, then no refraction takes place at the boundary.

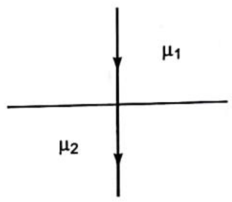


Fig. 17.43 If a light ray is incident normally, then no refraction takes place.

If the light goes from an optically rarer medium to an optically denser medium, then the light ray bends towards the normal as shown in Fig. 17.44. For a denser medium, μ is more. Here $\mu_2 > \mu_1$.

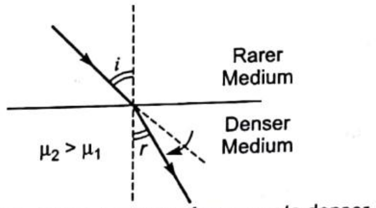


Fig. 17.44 When light goes from rarer to denser medium, then it deviates towards the normal.

From Snell's law, $\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$

As $\mu_2 > \mu_1$, so $\frac{\mu_2}{\mu_1} > 1$

$$\Rightarrow \quad \frac{\sin i}{\sin r} > 1 \Rightarrow \sin i > \sin r$$

$\Rightarrow i > r$ *ie*, $r < i$ which means the light ray bends towards the normal as the light goes from the rarer to denser medium.

If the light goes from an optically denser medium to optically rarer medium, then the light ray bends away from the normal as shown in Fig. 17.45.

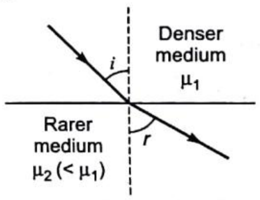


Fig. 17.45 When light goes from denser to rarer medium, then it deviates away from the normal.

From Snell's law, $\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$

As $\mu_2 < \mu_1$, so $\frac{\mu_2}{\mu_1} < 1$

$$\frac{\sin i}{\sin r} < 1 \Rightarrow \sin i < \sin r \Rightarrow i < r$$

ie, when light goes from the denser to rarer medium, the light ray bends away from the normal.

Assumptions : While dealing with refraction at plane and spherical surfaces we shall assume

(a) the normal incidence *ie*, angle of incidence and angle of refraction to be very small.

(b) paraxial rays would be considered here only.

C-BIs

Concept Building Illustrations

Illustration | 9 *If refractive index of water wrt air is 4/3, then determine the refractive index of air wrt water ?*

Solution Let μ_w and μ_a be the refractive indexes of water and air, respectively.

From given information,

$$_a\mu_w = \frac{\mu_w}{\mu_a} = \frac{4}{3}$$

$$_w\mu_a = \frac{\mu_a}{\mu_w} = \frac{1}{_a\mu_w} = \frac{1}{4/3} = \frac{3}{4}$$

Illustration | 10 *If speed of light in a medium is $\frac{2}{3} \times 10^8$ ms^{-1}, then determine the refractive index of medium ?*

Solution $\mu = \frac{c}{v} = \frac{3 \times 10^8}{2/3 \times 10^8} = 4.5$

Illustration | 11 *Refractive index of water wrt air is 4/3 and of kerosene wrt water is 7/8. Then determine the refractive index of kerosene. Refractive index of air is 1.*

Solution $_a\mu_w = \frac{\mu_w}{\mu_a} = \frac{4}{3}$

$$_w\mu_k = \frac{\mu_k}{\mu_w} = \frac{7}{8}$$

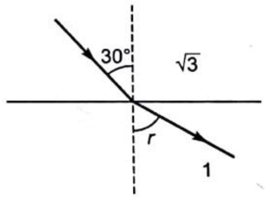
Multiply above two equations,

$$\frac{\mu_w}{\mu_a} \times \frac{\mu_k}{\mu_w} = \frac{4}{3} \times \frac{7}{8} = \frac{7}{6}$$

$$\Rightarrow \quad \mu_a = 1 \text{ so, } \mu_k = \frac{7}{6}$$

Illustration | 12 *A light ray is going from medium to refractive index $\sqrt{3}$ to another medium having refractive index 1. The ray strikes the interface at an angle of 30° to the normal drawn to point of incidence. Determine the angle of refraction.*

Solution The situation is shown clearly in figure.



From Snell's law

$$\frac{\sin 30°}{\sin r} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \sin r = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad r = 60°$$

Refraction at Plane Surfaces

In this section we shall explore the various concepts related to refraction at plane surfaces. We will observe how the image of an object is formed when the rays from the object gets refracted at the surface separating the two media. Here two possibilities are there. (a) The object is in a denser medium, and is viewed from the rarer medium, for example, a coin in the water and you are observing it from air. (b) The object is in rarer medium, and is viewed from a denser medium, for example, fish in water (pond) is observing the objects on shore (outside the water). Let us discuss these two situations on after another.

(A) When the object is in a denser medium and is viewed from a rarer medium

You can easily get many illustrations of this type of refraction in your daily life. You drop a coin in a bucket full of water, then the coin seems to be a little lifted up in the water. If you stand in a swimming pool your legs seem to shorter than actual. To hit a target in water,

you should aim not directly at its appearance position etc. In all these cases, object is in a denser medium and the observer is in a rarer medium.

Consider a point object O located at a depth x from the interface separating two media as shown in Fig. 17.46. The medium in which the object is located is denser one having refractive index μ_D, and the medium from where object has been observed is rarer one having refractive index μ_R. The ray OQ (incident ray) strikes the interface and gets refracted, as the light ray goes from denser medium to rarer medium the ray bends away from the normal as shown. The refracted ray is shown as QY. To locate the image of object O, we will extend the ray QY backwards, as a result, it intersects OP at I which is the image of object O *ie*, to an observer as seen in figure, the object O appears to be at I.

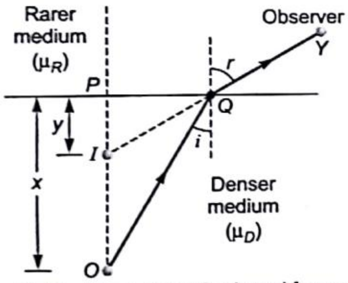


Fig. 17.46 When object is viewed from rarer medium, then it appears to be closer.

If actual depth is, $OP = x$, then IP is its apparent depth, and is given by

$$\text{Apparent depth} = IP = y = \frac{x}{{}_R\mu_D} = \frac{\text{Actual depth}}{{}_R\mu_D}$$

As ${}_R\mu_D > 1$ so $y < x$

ie, Apparent depth < Actual depth *ie*, the object seems to be nearer than the actual depth of the object. The distance by which the object seems to be shifted is termed as the shift, and is given by,

$$\text{shift} = OI = x - y = x - \frac{x}{{}_R\mu_D}$$

$$= x\left[1 - \frac{1}{{}_R\mu_D}\right]$$

Thus when a object is observed from a rarer medium, the apparent depth of the object would be less than the actual depth of the object.

(B) When the object is in a rarer medium and is viewed from a denser medium

Examples of this type are :

The fish observing a flying bird. In this case, we will consider that object is in a rarer medium which is being observed from a denser medium.

Consider a point object O placed in a medium of refractive index μ_R which is being observed from a denser medium having refractive index μ_D as shown in Fig. 17.47. OQ is the incident ray, which after refraction at the boundary goes along QY, *ie*, QY is the refracted ray. When QY is extended backwards, it intersects PO at I which is the image of O as perceived by the observer.

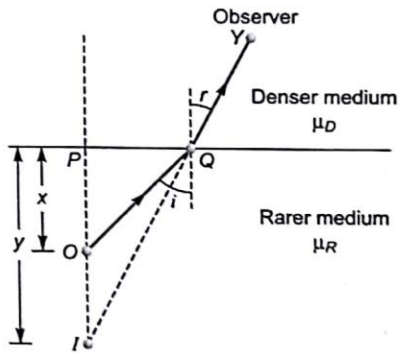


Fig. 17.47 When object is viewed from denser medium, then it appears farther.

If actual depth of object from boundary is, $OP = x$, then the apparent depth IP is given by,

$$IP = y = {}_R\mu_D \times x$$

ie, Apparent depth $= {}_R\mu_D \times$ Actual depth

As ${}_R\mu_D > 1$, so the apparent depth is greater than the actual depth of object. In this case, the shift is equal to $OI = y - x = x({}_R\mu_D - 1)$.

Thus, when an object is viewed from a denser medium it appears to be farther than the actual depth of the object.

Remember our entire expressions are under the above said assumptions.

Refraction Through a Double-Sided Glass Slab

Many times while studying to get refreshed or to change your mood you may indulge in some activities which seem to be silly—like you are observing the things through your transparent paperweight, and then you try to touch the object while seeing it through paperweight and obviously you won't be able to touch it. The explanation for the question that why you are not getting the object in your hand, will be provided in this section.

Consider a rectangular glass slab of thickness t and refractive index μ placed in air as shown in Fig. 17.48. The incident ray OR falls on face AC of the slab, on this face as the light goes from rarer (air) medium to denser medium (glass slab), the light ray bends towards the normal and after this refraction, the image is formed at I_1.

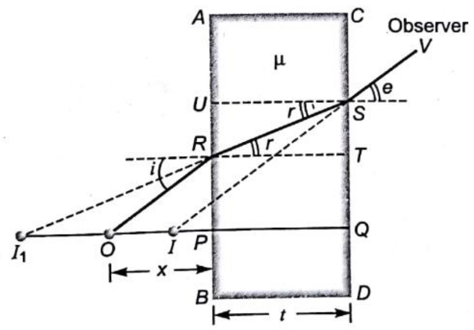


Fig. 17.48

If $OP = x$, then $I_1P = {}_R\mu_D \times x = {}_{air}\mu_{glass} \times x$

$$= \frac{\mu}{1} \times x = \mu x$$

Now this image I_1 will act as object for second refraction at face CD of slab. Object distance (*ie*, distance of I_1) from interface (face CD) is equal to $I_1P + PQ = (\mu x + t)$. This refraction takes place from a denser medium to rarer medium and hence light ray bends away from the normal at S.

The incident ray $I_1 RS$ refracts to SV as shown and this finally refracted ray emerges from the slab and is termed as emergent ray. These emergent rays reaches the eyes of the observer, and the observer will see the image of the object at I *ie*, the location where ray SV when produced backwards will intersect OPQ.

I is the apparent position of object O for observer who is observing the object through the glass slab.

For refraction at surface CD,

Actual depth $= \mu x + t$ and refraction is taking place from a denser to rarer medium. So,

$$\text{Apparent depth } (QI) = \frac{\text{Actual depth}}{\mu}$$

$$QI = \frac{\mu x + t}{\mu} = x + \frac{t}{\mu}$$

Shift produced in the object due to slab

$$= OI = (OP + PQ) - IP$$

$$\text{Shift} = (x + t) - (x + t/\mu)$$

$$\text{Shift} = t(1 - 1/\mu)$$

Applying Snell's law at R,

$$\frac{\sin i}{\sin r} = \frac{\mu}{1}$$

$$\frac{\sin i}{\sin r} = \mu \qquad \text{...(i)}$$

$$\angle TRS = \angle RSU$$

Apply Snell's law at S, $\dfrac{\sin r}{\sin e} = \dfrac{1}{\mu}$

$$\Rightarrow \qquad \frac{\sin e}{\sin r} = \mu \qquad \text{...(ii)}$$

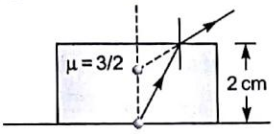
From Eqs. (i) and (ii) we get $i = e$, it means incident and emergent rays are parallel to each other.

C-BIs

Concept Building Illustrations

Illustration | 13 *A glass paper weighs 2 cm thick and having refractive index 3/2 is placed on a printed paper. If the paper is viewed from above the paperweight, at what depth the printed letters do appear ?*

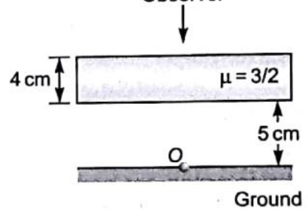
Solution Here the object *ie*, the letters are in contact with the paperweight and the light is emitting from it and on reaching the top surface, gets refracted as shown.



This case is same as the refraction at plane surfaces when the object is in a denser medium and is viewed from a rarer medium.

$$\text{Apparent depth} = \frac{\text{Actual depth}}{{}_R\mu_D} = \frac{2\text{ cm}}{\frac{3/2}{1}} = \frac{4}{3}\text{ cm}$$

Illustration | 14 *A point object O is seen through a glass slab of thickness 4 cm and refractive index 3/2 as shown in figure. Determine the shift in the position of object's appearance due to the slab.*



Solution From double-sided slab theory,

$$\text{Shift} = t\left(1 - \frac{1}{\mu}\right)$$

$$= 4\text{ cm}\left[1 - \frac{1}{3/2}\right]$$

$$= 4\text{ cm} \times \left(1 - \frac{2}{3}\right) = \frac{4}{3}\text{ cm}$$

Total Internal Reflection

When the light goes from a denser medium to a rarer medium, the angle of refraction is greater than the angle of incidence as shown in Fig. 17.49. From Snell's law, $\frac{\sin i}{\sin r} = \frac{\mu_R}{\mu_D}$

or $\sin r = \sin i \times {}_R\mu_D$

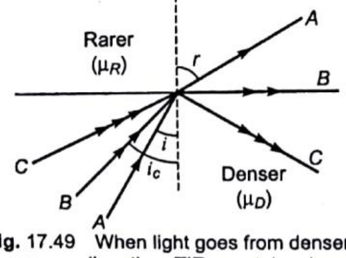


Fig. 17.49 When light goes from denser to rarer medium then TIR may take place.

As the angle of incidence increases, the angle of refraction also increases and for a particular value of angle of incidence the angle of refraction becomes $\pi/2$ as shown in figure by *BB*. This particular angle of incidence at which angle of refraction is $\pi/2$ is known as *critical angle* for the given pair of media. It is denoted by i_c and is given by

$$\frac{\sin i_c}{\sin \pi/2} = \frac{1}{{}_R\mu_D}$$

$$\Rightarrow \quad i_c = \sin^{-1}\left[\frac{1}{{}_R\mu_D}\right]$$

If rarer medium is air $[\mu_{\text{air}} = 1]$, and the denser medium is having refractive index μ, then i_c for the given medium—air pair is $\sin^{-1}\left[\frac{1}{\mu}\right]$.

Now the question arises, what will happen if angle of incidence exceeds the critical angle, would there by any refraction ? In this situation no refraction takes place and the entire light would be sent back to the denser medium, this phenomenon is termed as the total internal reflection. Thus we can conclude that when light goes from a denser medium to a rarer medium, and the angle of incidence is greater than the critical angle, then no light will be refracted and the entire light will be reflected back.

There are a number of applications of TIR—explanation of mirage and looming, high speed communication via fibre optics, twinkling of stars, sparkling diamonds etc.

C-BIs

Concept Building Illustrations

Illustration | 15 *Determine the critical angle for a pair of media having refractive indices* 1.33 *and* 1.5.

Solution Critical angle is given by,

$$i_c = \sin^{-1}\left(\frac{1}{{}_R\mu_D}\right)$$

Here, $\mu_D = \frac{3}{2}$ and $\mu_R = \frac{4}{3}$

Therefore, ${}_R\mu_D = \frac{\mu_D}{\mu_R} = \frac{3\times 3}{2\times 4} = \frac{9}{8}$

Thus, $i_c = \sin^{-1}\left(\frac{1}{9/8}\right) = \sin^{-1}(8/9)$

Lens Theory

We have seen that when light falls on a glass slab with parallel faces, then the emergent ray would be parallel to incident ray even though the ray gets shifted sideways, but have you ever thought what happens when a parallel beam of light passes through a medium bounded by curved surfaces ? Let's consider a very common engagement among the students.

All of you may have used a magnifying glass to burn the paper pieces. Light falls on the glass from sun (this light can be approximately considered as a parallel beam of light), you will observe a lighted spot on the paper—it means the light emerging from glass is no longer a parallel beam, but has converged. As this light is concentrated in a small region of paper, it gets heated up and finally gets burnt.

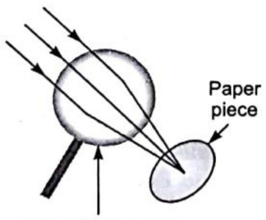


Fig. 17.50

Any transparent medium when bounded by two refracting surfaces out of which at least one is a curved constitutes the lens. If one or both of the surfaces of lens are spherical, then the lens is said to be a spherical lens. In this chapter we shall discuss only thin spherical lenses, here the word thin means that thickness of lens is very small as compared to other distances involved in the consideration. We have generally two types of spherical lenses :

(a) Convex lens A spherical lens which is thicker on the middle than at the edges.

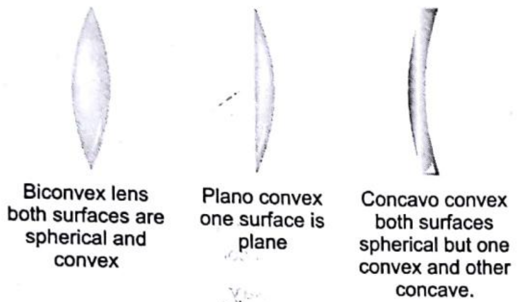


Fig. 17.51

(b) Concave lens A spherical lens which is thicker on the edges than at the middle.

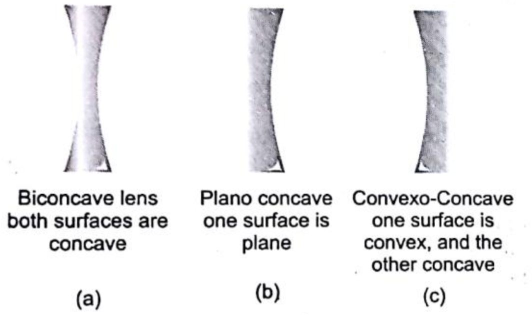


Fig. 17.52

We can construct the spherical lenses as shown in Fig. 17.53.

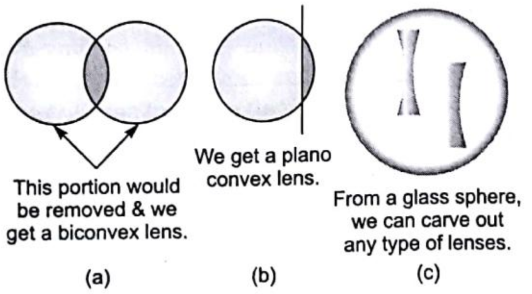


Fig. 17.53

Lens Terminology

1. Optical centre The central point of the lens is termed as its optical centre. It is denoted by O. For thin lenses, the length POQ is negligible, and can be considered as if P, O and Q are coinciding.

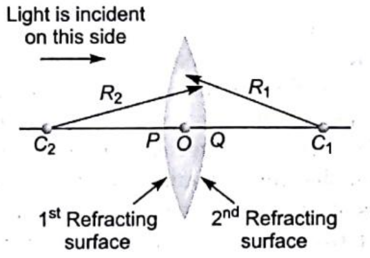


Fig. 17.54

2. Centre of curvatures (C_1 and C_2) A lens is bounded by two spherical surfaces and hence has two centres of curvature. The surface on which the light is incident, is termed as 1st refracting surface and its centre of curvature is denoted by C_1 while the surface from where the light emerges is termed as the 2nd refracting surface, and its centre of curvature is denoted by C_2.

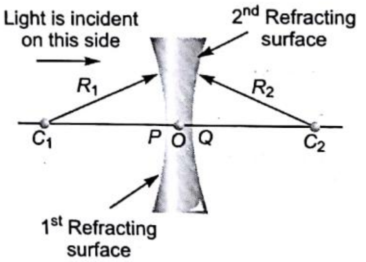


Fig. 17.55

3. Radii of curvature (R_1 and R_2) As for a lens there are two refracting surfaces and two centre of curvatures there would be two radii of curvatures as shown in Fig. 17.55. Radii of curvatures are nothing but the radii of the spheres of which these surfaces were once a part of.

4. Optical axis or Principal axis The line joining C_1 and C_2 passing through O is termed as the optical axis or principal axis of lens.

5. Aperture The area of the lens on which the light falls is termed as its aperture.

6. Focus and Focal length In contrast with spherical mirrors the lenses are having two foci and correspondingly two focal lengths. In general, the focus of an optical instrument

means the point where the reflected/refracted rays converges to or appears to diverge from when a parallel beam of light is incident on optical instrument.

For lenses, 2nd focus is defined same as above *ie*, the image point on optical axis when object is at infinity. Object at infinity means the incident rays are coming parallel from infinity. For both concave and convex lenses, 2nd focus has been shown in Fig. 17.56 (a) and Fig. 17.56 (b).

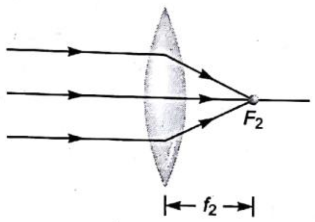


Fig. 17.56 (a) 2nd focus of convex lens is real

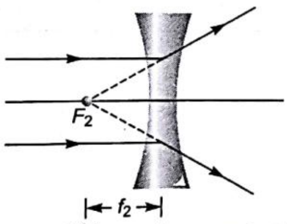


Fig. 17.56 (b) 2nd focus of concave lens is virtual

The distance between the optical centre and 2nd principal focus is termed as the second focal length of lens. For lenses the first principal focus is defined as "the object point on optical axis for which image lies at infinity." And, the distance between optical centre and 1st focus is termed as the first focal length. For both the lenses, 1st principal foci have been shown in Fig. 17.58.

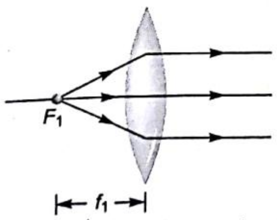


Fig. 17.57 (a) 1st focus of convex lens is real

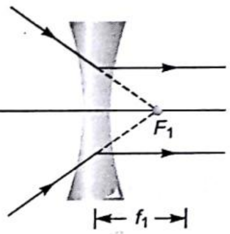


Fig. 17.57 (b) 1st focus of concave lens is virtual

In general, if we say only focus or focal length of lens, then it means that we are talking about 2nd principal focus and 2nd focal length. If medium on both the sides of lens are the same which is the case in general, then both focal lengths are same as well as 1st and 2nd focus are interchangeable, and this is the only case we will discuss in this book.

Refraction by Lenses

As there are two refracting surfaces in a lens, two refraction takes place in a lens. First one, on the surface where the light is incident and the 2nd one from where the light rays emerged, but as we are considering the thin lenses, we can safely approximate that entire instrument (lens) is causing one refraction only. Detailed explanation of this is not required at this level. Here we are showing the things pictorially.

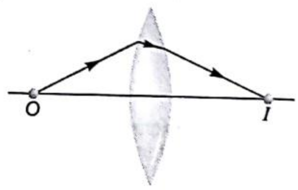


Fig. 17.58 (a) Diagram showing refraction at two refracting surfaces, separately for convex lenses.

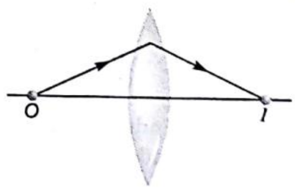


Fig. 17.58 (b) Diagram showing refraction by a convex lens (thin lens)

Similarly, we can draw diagrams for concave lenses.

Generally, the convex lenses converge the rays incident on it, and hence are termed as converging lenses while concave lenses diverge the rays incident on them, and hence known as diverging lenses.

Important Rays' Types in Lens Theory

1. The ray parallel to principal axis after refraction pass through the focus or appear to pass through focus.
2. The ray passing through focus become parallel to optical axis after refraction.
3. The ray passing through the optical centre go undeviated after refraction.

These three rays we shall show in the diagram for both types of lenses.

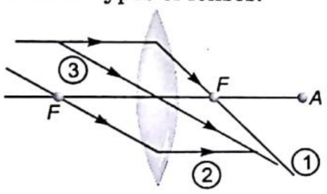


Fig. 17.59 (a) Diagram showing three important rays for a convex lens.

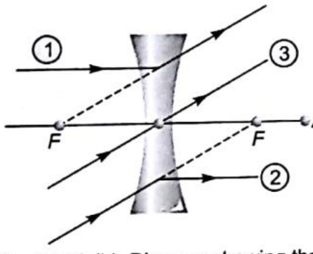


Fig. 17.59 (b) Diagram showing three important rays for concave lens.

Lens Maker's Formula

The focal length of a lens is given by lens Maker's formula which is as follows :

$$\frac{1}{f} = \left(\frac{\mu_L}{\mu_M} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

where μ_L = refractive index of lens material

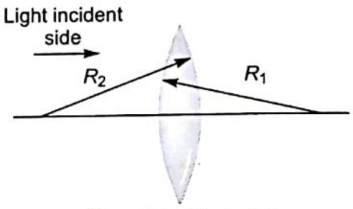
μ_M = refractive index of the surrounding medium

R_1 = radius of curvature of 1st refracting surface

R_2 = radius of curvature of 2nd refracting surface

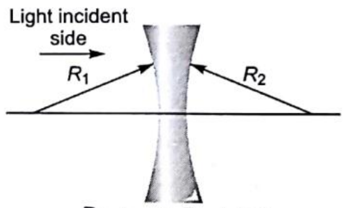
While using the above expressions we have to use the same standard sign convention as we have used for mirrors.

Generally, we have $\mu_L > \mu_M$ *ie*, refractive index of the lens material is greater than the refractive index of the surrounding medium. For convex lense $\frac{1}{R_1} - \frac{1}{R_2} \to +\text{ve}$



$R_1 \to +\text{ve}$, $R_2 \to -\text{ve}$

(a)



$R_1 \to -\text{ve}$, $R_2 \to +\text{ve}$

(b)

Fig. 17.60

For concave lens, $\frac{1}{R_1} - \frac{1}{R_2} \to -\text{ve}$

And, in general, $\frac{\mu_L}{\mu_M} - 1 = +\text{ve}$, so focal length of a convex lens is +ve and for a concave lens it would be −ve.

It is a common fact that lenses having +ve focal length are converging in nature while lenses having −ve focal length are diverging in nature.

The formula relating object distance, image distance and focal length is known as the lens formula.

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \text{ (Lens formula)}$$

Here v = image distance from optical centre

u = object distance from optical centre

f = focal length.

Transverse magnification for lenses is given by, $m = \frac{v}{u}$

C-BIs

Concept Building Illustrations

Illustration | 16 *A biconvex lens having radii of curvature as 15 cm and 7.5 cm is made from a glass having refractive index 1.5. Determine the focal length of this lens when used in air.*

Solution We know, $\frac{1}{f} = \left(\frac{\mu_L}{\mu_M} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Here, $\mu_L = 1.5$, $\mu_M = 1$,

$R_1 = 15$ cm, $R_2 = -7.5$ cm

So, $\frac{1}{f} = \left(\frac{1.5}{1} - 1\right) \times \left(\frac{1}{15} - \frac{1}{-7.5}\right) = 0.5 \times \frac{3}{15}$

$\Rightarrow$ $f = 10$ cm

Illustration | 17 *If above lens is placed in a medium having refractive index 2, then determine the focal length of lens.*

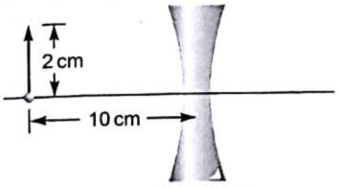
Solution Here, the values of μ_M is changed to 2, all things remaining the same.

So, $\frac{1}{f'} = \left(\frac{1.5}{1} - 1\right) \times \left(\frac{1}{15} - \frac{1}{-7.5}\right)$

$\Rightarrow$ $f' = -20$ cm.

–ve focal length shows that under these conditions the convex lens behaves as a diverging lens.

Illustration | 18 *An extended object of height 2 cm is placed in front of a concave lens as shown in figure. Determine the position of image and also the size of image.*



Solution Using lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Here, $u = -10$ cm

$f = -20$ cm

$$\frac{1}{v} - \frac{1}{-10} = \frac{1}{-20}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{-20} - \frac{1}{10} = -\frac{3}{20}$$

$$\Rightarrow \quad v = -\frac{20}{3} \text{ cm}$$

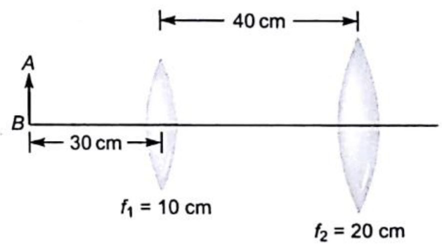
So the image is going to be formed at a distance of 20/3 cm from the optical centre on the same side at which the object is.

Magnification $\Rightarrow m = \frac{v}{u}$

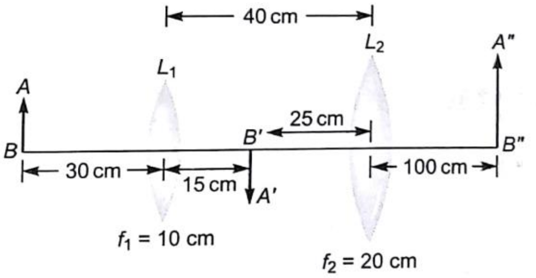
$$\frac{h_i}{h_o} = \frac{-20/3}{-10} = +\frac{2}{3}$$

$$h_i = \frac{2}{3} \times h_o = \frac{4}{3} \text{ cm}$$

Illustration | 19 *Two convex lenses having focal lengths 10 cm and 20 cm are kept at a separation of 40 cm as shown in the figure. The two lenses are having the same principal axis. An object AB of size 1 cm is placed in front of the two lens system as shown in the figure. Determine the location of the final image formed by this two lens system.*



Solution When we have an optical system consisting of more than one optical device, then the image formed by 1st device is acting as object for 2nd and image formed by 2nd is acting as object for 3rd and so on, and in this manner the image formed at the last, is the final image.



For 1st refraction at L_1

$$u_1 = -30 \text{ cm}, \quad f_1 = 10 \text{ cm}$$

Let the image distance be v_1

Then, $\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$

$\Rightarrow \quad \frac{1}{v_1} - \frac{1}{-30} = \frac{1}{10}$

$\Rightarrow \quad \frac{1}{v_1} = \frac{1}{10} - \frac{1}{30} = \frac{2}{30}$

$\Rightarrow \quad v_1 = 15 \text{ cm}$

$$m = \frac{v_1}{u_1} = \frac{15}{-30} = -\frac{1}{2} \text{ cm}$$

Negative magnification tells that the image is inverted and is of size $\frac{1}{2}$ cm as shown. This is the intermediate image formed, this image $A'B'$ acts as object for 2nd refraction at L_2. The distance of $A'B'$ from $L_2 = 40 - 15 = 25$ cm.

For second refraction at L_2,

$u_2 = -25$ cm, $f_2 = 20$ cm. Let the image distance from L_2 be v.

Then, $\frac{1}{v} - \frac{1}{u_2} = \frac{1}{f_2}$

$\Rightarrow \quad \frac{1}{v} - \frac{1}{-25} = \frac{1}{20}$

$\Rightarrow \quad v = 100 \text{ cm}$

Magnification produced by the 2nd lens is

$$m_2 = \frac{v}{u_2} = \frac{100}{-25} = -4$$

ie, image formed by L_2 is 4 times of $A'B'$ and inverted *wrt* $A'B'$ and at a distance of 100 cm from L_2.

The entire situation is as shown in the diagram.

Towards Proficiency Problems
Exercise 1

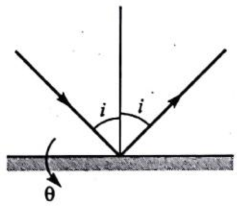
A. Subjective Discussions

1. Light is dual in nature. Comment on this statement ?
2. How we are able to see a table or a chair when they are not emitting their own light ? Why are we not able to see them in the complete darkness ?
3. Virtual images can't be taken on screen. Comment on this.
4. Is it possible to have a real image of a real object formed by a plane mirror ? By a concave mirror ? By a convex mirror ?
5. How can you distinguish between real and virtual images ?
6. When a light ray travels from one medium to another, then at the interface total light would be transmitted. Check out the correctness of given statement.
7. A convex mirror can only form a virtual image for a virtual object. Explain this statement.
8. If a clock is held in front of a mirror, its image is reversed left to right. From the point of view of a person looking into the mirror, does the image of the second hand rotates in the A-C-W direction ? Justify your answer.
9. Which kind of spherical mirror—concave or convex can be used to set up a fire with sunlight ?
10. Can the image formed by a convex mirror be carried to a screen ?
11. A swimmer is under water and looking up at the surface. An another person holds a coin in the air, directly above the swimmer's eyes. To the swimmer, the coin appears to be at a certain height above the water. Is the apparent height of the coin, greater than, less than or the same as its actual height ?
12. Two identical containers, one filled with water ($\mu = 1.33$) and other filled with ethyl alcohol ($\mu = 1.36$) are viewed from directly above. Which container appears to have a greater depth of fluid ? Why.
13. At night when it is dark outside and you are standing in a brightly lit room, it is easy to see your reflection in a window. During the day it is not so easy. Explain your reason.
14. A man is fishing from the dock. (a) If he is using a bow and arrow, should he aim above the fish, or below the fish to strike it exactly ? (b) How would he aim if he were using a laser gun ? Give your reasons.
15. A spherical mirror and a lens are immersed in water. Compared to the way they work in air, which one do you expect will be more affected by water ?

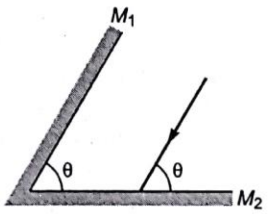
B. Numerical Answer Types

1. If angle of incidence of a light ray falling on a plane mirror be 30°, then what is the angle of reflection ? What is the angle between the incident ray and reflected rays ?
2. If angle of incidence of a light ray falling on a plane mirror increases by 15°, then by what amount the angle between the incident and reflected rays changes ?

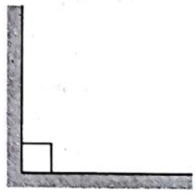
3. A person is standing in the middle of a room and observing the mirror hanging on the facing wall. Determine the minimum height of the mirror, so that person would be able to see the complete image of the wall behind him in the mirror. Take height of wall as H.
4. A light ray is incident on a plane mirror as shown in the figure. If the mirror is rotated by an angle θ in anti-clockwise direction about the point of incidence, then determine the angle by which the reflected ray will rotate.



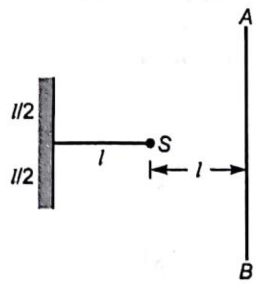
5. Two plane mirrors are kept at an angle θ. A light ray is incident on M_2 as shown, and after undergoing two successive reflections, the reflected ray becomes parallel to M_2. Determine the value of θ.



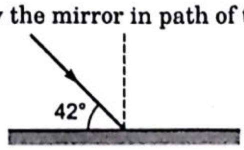
6. Two plane mirrors are kept in contact to each other. Prove that a light ray incident on any mirror at any angle, after two successive reflections, reverses its direction ?



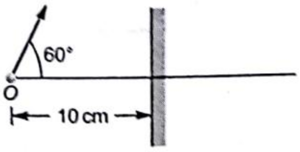
7. Sunlight is falling at an angle of 40° to the vertical. At what angle should a plane mirror be placed in path of sun rays so that the reflected solar beam should point in a vertical downward direction ?
8. Find the length of the portion on line AB such that the image of the point object S is visible from here.



9. Find the deviation produced by the mirror in path of the light ray due to reflection ?



10. A point object is initially at a distance of 10 cm from the plane mirror. Now it starts moving along a direction as shown in the figure with a speed of 1 cm/s. Determine the separation between the object and its image after 5 s of its starting the motion.



11. A spherical mirror is having radius of curvature 20 cm. Determine its focal length if mirror is (a) concave and (b) convex.

12. A 2 cm high object is placed 7.1 cm from a concave mirror whose radius of curvature is 10.2 cm. Find out (a) the location of image, and (b) its size ?

13. An object is placed 6 cm in front of a concave mirror having focal length 10 cm. The object height is 1.2 cm. Determine the location and size of image ?

14. A convex mirror is used to reflect light from an object placed 66 cm in front of the mirror. The focal length of the mirror is 44 cm. Determine the location of the image and the magnification.

15. An object is placed 9 cm in front of a mirror. The image is 3 cm closer to the mirror when the mirror is convex than when it is planar. Determine the focal length of the convex mirror.

16. A point object is placed 20 cm in front of a concave mirror and is within the focal point. When the concave mirror is replaced by a plane mirror, the image moves 15 cm toward the mirror. Determine the focal length of the mirror.

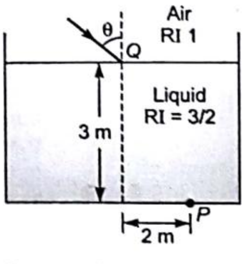
17. A plane mirror and a concave mirror (f = 8 cm) are facing each other and are separated by a distance of 20 cm. An object is placed 10 cm in front of the plane mirror. Assume that the light from the object first falls on the plane mirror and after reflection from the plane mirror falls on the concave mirror. Locate the final image and the nature of image. Also draw a rough ray diagram.

18. A coin is placed 8 cm in front of a concave mirror. The mirror produces a real image that has a diameter 4 times larger than that of coin. Determine the focal length of mirror and image distance from mirror.

19. An object that is 25 cm in front of a convex mirror has an image located 17 cm behind the mirror. How far behind the mirror is the image located when the object is 19 cm in front of the mirror ?

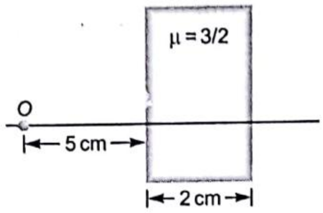
20. A light ray strikes the air-water surface at an angle of incidence of 30°. The refractive index for water is 4/3. Find the angle of refraction when the light ray goes (a) from air to water, and (b) from water to air.

21. At what angle θ should the light ray be incident at Q so that point P would be illuminated by a refracted beam ?



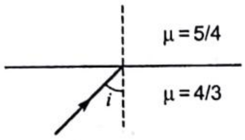
22. A coin is placed at the bottom of a 3 cm deep pond. The refractive index of water is 1.5. At what depth does the coin appear to be, as seen from above the pond ?

23. A point object is seen through a transparent slab of refractive index 3/2 and thickness 2 cm. The object is placed at a distance of 5 cm from front face of the slab. Determine the (a) location of image after 1st refraction (b) location of image after 2nd refraction, and (c) the shift in the appearance of the object.



24. Determine the critical angle for a pair of media having refractive indices 1.5 and 1.8 ?

25. Describe what happens if i is



(a) less than $\sin^{-1}$ (15/16)
(b) greater than $\sin^{-1}$ (15/16)
(c) equal to $\sin^{-1}$ (15/16) for the situation shown in the figure.

26. Two converging lenses (f_1 = 9 cm and f_2 = 6 cm) are separated by 18 cm. The lens on the left has a longer focal length. An object stands 12 cm to the left of the left hand in the combination.
(a) Locate the final image relative to the lens on the right.
(b) Obtain the overall magnification.
(c) Is the final image real or virtual ?
With respect to the original object, is the final image
(d) Upright (erect) or inverted it
(e) Larger or smaller ?

C. Fill in the Blanks

1. Real objects means incident rays are ....
2. An object is visible from all directions due to .... of light.
3. Colour of light depends on its ....
4. A plane mirror converts a left-handed system to ....
5. Concave mirror .... a parallel beam of light incident on it.
6. When an object is 40 cm away from the pole of a concave spherical mirror, the size of the image is equal to that of object. The focal length of the mirror is ....
7. A clock hung on a wall has marks instead of numbers on its dial. On the opposite wall there is a mirror and the image of the mirror in the clock is read, and it indicates the time as 8 : 20. The time in the clock is ....

D. True/False

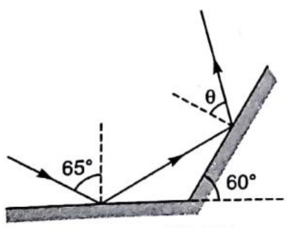
1. Light can be treated as a mechanical wave.
2. When dimensions of the optical instruments encountered by light are very large as compared to wavelength of light, then the wave nature of light can be discarded.
3. We are able to see most of the non-luminas objects because of diffuse reflections.
4. Law of reflection is valid only for the plane smooth surfaces.
5. A plane mirror can form real images.
6. Image formed by plane mirror is always of same size as the object is.
7. If angle of incidence is greater than the critical angle, then no part of the light will be transmitted.

High Skill Questions

Exercise 2

A. Only One Option Correct

1. For the situation shown in figure. the value of θ is



(a) 65° (b) 60°
(c) 55° (d) 50°

2. Two diverging light rays, originating from the same point, have an angle of 10° between them. After the rays reflect from a plane mirror, the angle between the reflected rays is
(a) 10° (b) 20°
(c) 35° (d) 15°

3. A 2 cm high object is placed at a distance of 15 cm in front of a convex mirror having focal length 5 cm. The image is at a distance of ...... from the mirror.
(a) 2.25 cm (b) 3.75 cm
(c) 4 cm (d) 4.75 cm

4. The magnification produced by mirror in above question is
(a) 0.25 (b) –0.25
(c) 0.5 (d) –0.5

5. For a real extended object the image formed by a concave mirror can be
(a) real (b) virtual
(c) erect (d) All of these

6. As a real object moves from infinity to C, its image formed by a concave mirror
(a) would be real and moving from F to C
(b) would be real and moving from C to F
(c) would be virtual and moving from F to C
(d) would be real and always at C

7. Refractive index of medium 1 *wrt* 2 is 3. If refractive index of medium 1 is 4, then the refractive index of medium 2 is
(a) 3/4 (b) 4/3
(c) 8/3 (d) 2

8. A converging beam of solar rays is incident on a concave spherical mirror whose radius of curvature is 0.8 m. Determine the position of the point on the principal axis of the mirror, where the reflected rays intersect, if the

extensions of the incident rays intersect the principal axis 40 cm from the mirror's pole ?

(a) 10 cm from pole
(b) 30 cm from pole
(c) 50 cm from pole
(d) 20 cm from pole

9. A student uses a converging lens of focal length 10 cm to produce an enlarged virtual image of a scale marking. The suitable distance between the scale marking and the lens could be

(a) 5 cm
(b) 10 cm
(c) 15 cm
(d) Information insufficient

10. A virtual image is one

(a) towards which light rays converge but don't pass through
(b) from which light rays diverge but don't pass through
(c) from which light rays diverge as they pass through
(d) towards which light rays converge and pass through

11. A convex lens made of a material of refractive index 1.5 is placed in a medium having refractive index 1.6, then it will behave as

(a) converging lens
(b) diverging lens
(c) plane glass plate
(d) Information insufficient

12. An object is placed 1 m in front of a plane mirror. An observer stands 3 m behind the object. For what distance must an observer focus his eyes in order to see the image of the object ?

(a) 3 m
(b) 4 m
(c) 5 m
(d) 8 m

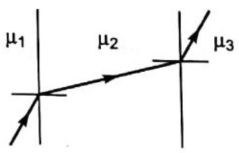
13. A thin diverging lens of focal length 20 cm and a converging mirror of focal length 10 cm are placed 5 cm apart, coaxially. Where shall an object be placed so that the object and its real image coincide ?

(a) 60 cm away from lens
(b) 20 cm away from lens
(c) 40 cm away from lens
(d) 30 cm away from lens

14. If a convergent beam of light passes through a diverging lens, the result

(a) may be a converging beam
(b) may be a diverging beam
(c) may be a parallel beam
(d) All of the above

15. The figure below shows the path of a portion of a light ray as it passes through three different materials. Mark out the correct relation among refractive indices of three materials.



(a) $\mu_1 > \mu_2 > \mu_3$
(b) $\mu_2 > \mu_1 > \mu_3$
(c) $\mu_1 < \mu_3 < \mu_2$
(d) $\mu_1 < \mu_2 < \mu_3$

B. More Than One Options Correct

1. Mark the correct statement(s)

(a) Speed of light is greatest in vacuum
(b) Speed of different lights are same in vacuum
(c) Speed of different lights are different in same medium other then vacuum
(d) None of the above

2. Mark the correct statement(s)

(a) Light is dual in nature
(b) Light is a form of energy
(c) Light is responsible for our visual sensation
(d) Light is a non-mechanical wave

3. When light goes from one medium to another then, which of the properties of light will not change ?

(a) speed
(b) wavelength
(c) frequency
(d) colour

4. Mark out the correct statement(s) for the image formed by a plane mirror.
   (a) It can be real.
   (b) It can be inverted.
   (c) It must be virtual and erect.
   (d) It must be virtual and erect for real-extended objects.
5. For a real-extended object, the image formed by a convex mirror will be
   (a) real (b) virtual
   (c) erect (d) diminished
6. If a light ray goes from a medium having higher refractive index to a medium having lower refractive index, then ray will [Assume angle of incidence to be not zero]
   (a) bend towards the normal
   (b) not bend
   (c) bend away from the normal
   (d) None of the above
7. If a light ray goes from a medium having higher refractive index to a medium having a lower refractive index medium, then the ray can
   (a) be reflected back
   (b) be refracted
   (c) bend away from the normal
   (d) go undeviated
8. Which of the following can/must have +ve focal length ?
   (a) Concave mirror
   (b) Convex mirror
   (c) Concave lens
   (d) Convex lens
9. Convex lens
   (a) always behaves as a diverging lens
   (b) generally behaves as a diverging lens
   (c) may behave as a converging lens
   (d) may behave as a parallel double-sided slab

C. Assertion & Reason

Directions (Q. Nos. 1 to 5) *Some questions (Assertion-Reason type) are given below. Each question contains* ***Statement I (Assertion)*** *and* ***Statement II (Reason)****. Each question has 4 choices (a), (b), (c) and (d) out of which* ***ONLY ONE*** *is correct. So, select the correct choice.*

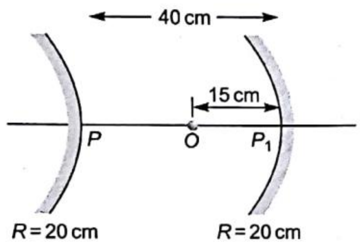
Choices are
(a) **Statement I** is True, **Statement II** is True; **Statement II** is a correct explanation for **Statement I**
(b) **Statement I** is True, **Statement II** is True; **Statement II** is **NOT** a correct explanation for **Statement I**
(c) **Statement I** is True, **Statement II** is False
(d) **Statement I** is False, **Statement II** is True

1. **Statement I** As light goes from one medium to another, the speed of light changes.
   Statement II Speed of light is a property of the medium.
2. **Statement I** As light goes from one medium to another the colour of light doesn't change.
   Statement II Colour of the light depends upon the frequency, and frequency of light is the property of source.
3. **Statement I** A plane mirror can form inverted image of real-extended object.
   Statement II A plane mirror always forms an image whose size is same as that of the size of object.
4. **Statement I** A convex lens forms real image of a real-extended object.
   Statement II A convex lens is an example of spherical lens and is converging in nature.
5. **Statement I** Total internal reflection can take place only when light goes from a denser to rarer medium.
   Statement II The ray bends away from the normal if it goes from a denser to rarer medium.

D. Comprehend the Passage Questions

Passage I

Two mirrors—one concave and the other convex are placed at a separation of 40 cm as shown in the figure. The principal axis and radii of



curvature of both the mirrors are the same. A point object is placed at a distance of 15 cm from concave mirror. Consider 1st reflection by concave, then by convex, and then again by concave.

Based on above information answer the following questions :

1. The location of image after 1st reflection is
(a) 30 cm to left of P_1 (b) 30 cm left to P_2
(c) 30 cm right to P_1 (d) 30 cm right to P_2
2. The location of image after 2nd reflection
(a) 5 cm to left of P_1
(b) 5 cm to left of P_2
(c) 5 cm to right of P_1
(d) 5 cm to right of P_2
3. The final image is located at
(a) $\frac{90}{7}$ cm to left of P_1
(b) $\frac{90}{7}$ cm to left of P_2
(c) $\frac{90}{7}$ cm to right of P_1
(d) $\frac{90}{7}$ cm to right of P_2
4. If instead of a point object an extended object of height 2 cm is considered, then height of the image after 1st reflection is
(a) 2 cm (b) 1 cm
(c) 4 cm (d) None of these
5. In Q. 4, height of the image after 2nd reflection is
(a) 2 cm (b) 1 cm
(c) 4 cm (d) None of these
6. In Q. 4, height of the image after 3rd reflection is
(a) $\frac{4}{7}$ cm
(b) $\frac{3}{7}$ cm
(d) $\frac{5}{7}$ cm
(d) None of the above

E. Match the Columns

1. Match the entries of column I with the entries of column II.

	Column I		Column II
(A)	Plane mirror	(P)	Virtual image
(B)	Concave mirror	(Q)	Real image
(C)	Convex lens	(R)	Erect image
(D)	Convex mirror	(S)	Inverted image

2. Match the entries of column I with entries of column II.

	Column I		Column II
(A)	Mirror	(P)	Reflection
(B)	Lens	(Q)	Refraction
(C)	Light going from denser to rarer medium	(R)	TIR
(D)	Plane glass slab (with air as surrounding need)	(S)	Image formation

Answers

Towards Proficiency Problems
Exercise 1

B. Numerical Answer Types

1. 30°, 120° **2.** 30° **3.** $\frac{H}{3}$ **4.** 2θ in clock-wise direction
5. 60° **7.** 70° **8.** $3l$ **9.** 84° **10.** 15 cm
11. (a) −10 cm, (b) 10 cm **12.** (a) 18.1 cm from pole, (b) 5.12 cm
13. 15 cm behind the mirror, virtual image, 3 cm **14.** 26.4 cm behind the mirror, $\frac{6}{15}$
15. 18 cm **16.** $-\frac{140}{3}$ cm **17.** Real, at $\frac{120}{11}$ cm in front of the concave mirror
18. − 6.4 cm, 32 cm **19.** 14 cm **20.** (a) $r_1 = \sin^{-1}\left(\frac{3}{8}\right)$, (b) $r_2 = \sin^{-1}\left(\frac{2}{3}\right)$
21. $\sin^{-1}\left(\frac{3}{\sqrt{13}}\right)$ **22.** 2 cm **23.** (a) 7.5 cm, (b) 1.8 cm, (c) 5.7 cm **24.** $\sin^{-1}\left(\frac{5}{6}\right)$
26. (a) $\frac{9}{2}$ cm on right side of lens, (b) $-\frac{3}{4}$, (c) Real, (d) Upright, (e) Smaller

C. Fill in the Blanks

1. Diverging **2.** Scattering **3.** Frequency **4.** RHS
5. Converges **6.** 20 cm **7.** 4 : 40

D. True/False

1. F **2.** T **3.** T **4.** F **5.** T **6.** T **7.** T

High Skill Questions
Exercise 2

A. Only One Option Correct

1. (c) **2.** (a) **3.** (b) **4.** (a) **5.** (d) **6.** (a) **7.** (b) **8.** (d) **9.** (a) **10.** (b)
11. (b) **12.** (c) **13.** (a) **14.** (d) **15.** (b)

B. More Than One Options Correct

1. (a, b, c) **2.** (a, b, c, d) **3.** (c, d) **4.** (a, b) **5.** (b, c, d)
6. (c) **7.** (a, b, c, d) **8.** (b, c, d) **9.** (b, c, d)

C. Assertion & Reason

1. (a) **2.** (a) **3.** (b) **4.** (b) **5.** (b)

D. Comprehend the Passage Questions

1. (a) **2.** (b) **3.** (a) **4.** (c) **5.** (a) **6.** (a)

E. Match the Columns

1. A → P, Q, R, S; B → P, Q, R, S; C → P, Q, R, S; D → P, Q, R
2. A → P, S; B → Q, S; C → Q, R, S; D → Q, S

Explanations

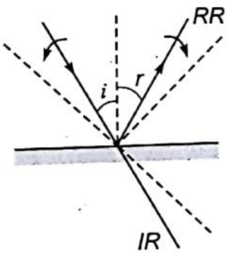
Towards Proficiency Problems
Exercise 1

Numerical Answer Types

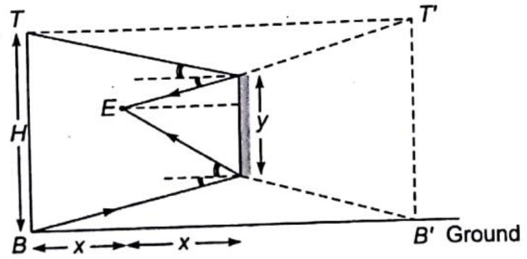
1. $\angle r = \angle i = 30°$.

   Angle between incident and reflected rays is 120°.

2. Initial angle between IR and RR is $\pi - 2i$, when i increases by 15°, the angle between IR and RR is decreased by 30°.

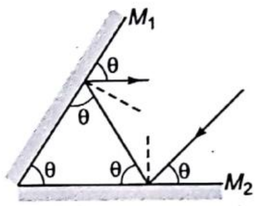


3. The ray diagram of the situation is as shown in the figure.



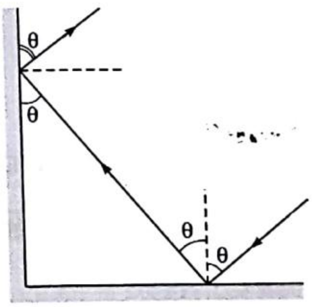
After using geometrical concepts we shall get the height of mirror.

5. The ray diagram is as shown in the figure and from geometrical concepts and the laws of reflection, we can mark the angles as shown. Now, sum of angles of Δ is 180°.

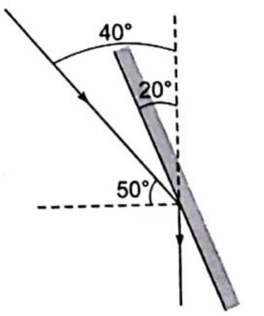


$$\theta + \theta + \theta = 180°$$
$$\Rightarrow \quad 3\theta = 180°$$
$$\Rightarrow \quad \theta = 60°.$$

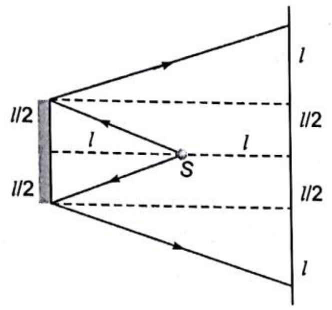
6.



7.

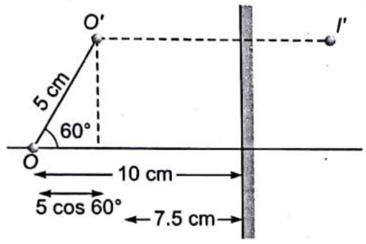


8.



9. $\delta = \pi - 2i = 180° - 2(48°) = 84°$

10. At $t = 5$ s, the situation is as shown in figure.



$$O'I' = 15 \text{ cm}$$

12. $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \quad \frac{1}{v} - \frac{1}{7.1} = \frac{1}{-5.1}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{7.1} - \frac{1}{5.1}$$

$$\Rightarrow \quad v = -18.1 \text{ cm}$$

$$\frac{h_i}{h_o} = -\left(\frac{-18.1}{-7.1}\right) = -2.56$$

$$\Rightarrow \quad h_i = 2.56 \times 2 \text{ cm} = 5.12 \text{ cm}$$

15. For a plane mirror, image is at 9 cm behind the mirror, so for a convex mirror, the image distance is 6 cm from the mirror.

So, $\frac{1}{6} + \frac{1}{-9} = \frac{1}{f}$

$$\Rightarrow \quad f = 18 \text{ cm}$$

16. Here, for a concave mirror,

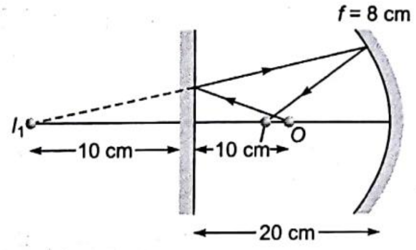
$$u = -20 \text{ cm}, \quad v = +35 \text{ cm}$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{35} - \frac{1}{20} = \frac{1}{f}$$

$$\Rightarrow \quad f = -\frac{140}{3} \text{ cm}$$

17. For concave mirror, I_1 is treated as the object.



$$\frac{1}{v} - \frac{1}{30} = \frac{1}{-8}$$

$$\Rightarrow \quad v = -\frac{120}{11} \text{ cm}$$

20. (a) $\frac{\sin 30°}{\sin r_1} = \frac{\frac{4}{3}}{1}$

$$\Rightarrow \quad \sin r_1 = \frac{3}{4} \times \frac{1}{2}$$

$$\Rightarrow \quad r_1 = \sin^{-1}\left(\frac{3}{8}\right)$$

(b) $\frac{\sin 30°}{\sin r_2} = \frac{1}{\frac{4}{3}}$

$$\Rightarrow \quad \sin r_2 = \frac{4}{3} \times \frac{1}{2}$$

$$\Rightarrow \quad r_2 = \sin^{-1}\left(\frac{2}{3}\right)$$

21. $\frac{\sin \theta}{\sin r} = \frac{\frac{3}{2}}{1}$

$$\Rightarrow \quad \sin \theta = \frac{3}{2} \times \frac{2}{\sqrt{13}}$$

$$= \frac{3}{\sqrt{13}}$$

$$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{3}{\sqrt{13}}\right)$$

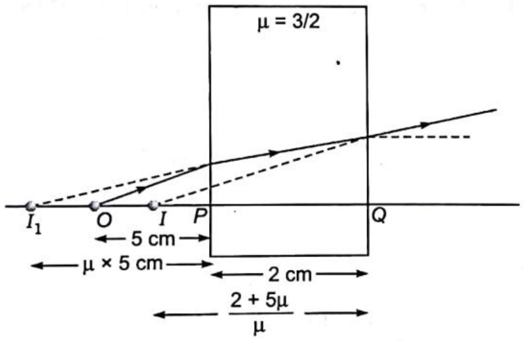
22. Apparent depth $= \frac{3 \text{ cm}}{1.5}$

$$= 2 \text{ cm}$$

23. (a) $PI_1 = 7.5$ cm

(b) $PI = 1.8$ cm

(c) $I_1I = 5.7$ cm



24. Critical angle, $i_c = \sin^{-1}\left(\frac{1}{_R\mu_0}\right)$

$$= \sin^{-1}\left[\frac{1.5}{1.8}\right]$$

$$= \sin^{-1}\left(\frac{5}{6}\right)$$

25. Critical angle for the given situation is,

$$\frac{\sin i_c}{\sin \frac{\pi}{2}} = \frac{\frac{5}{4}}{\frac{4}{3}} = \frac{15}{16}$$

$$\Rightarrow \qquad i_c = \sin^{-1}\left(\frac{15}{16}\right)$$

(a) If $i < i_c$, then refraction takes place and angle of refraction is given by

$$\frac{\sin i}{\sin r} = \frac{5/4}{4/3}$$

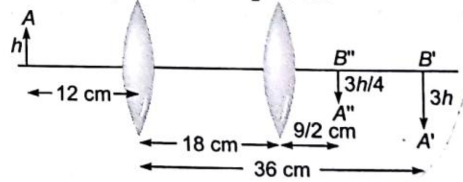
$$\Rightarrow \qquad \sin r = \frac{16}{15} \times \sin i.$$

(b) If $i > i_c$, then TIR takes place.

(c) If $i = i_c$, then the light ray will go along the boundary *ie*, just going to graze at an angle of $\frac{\pi}{2}$.

26. For Ist refraction,

$f_1 = 9$ cm, and $f_2 = 6$ cm



$$\frac{1}{v_1} - \frac{1}{-12} = \frac{1}{9}$$

$$\Rightarrow \qquad \frac{1}{v_1} = \frac{1}{9} - \frac{1}{12} \Rightarrow v_1 = 36 \text{ cm}$$

For IInd refraction,

$$u = 18 \text{ cm}, \quad v_2 = ?$$

$$\frac{1}{v_2} - \frac{1}{18} = \frac{1}{6} \Rightarrow \frac{1}{v_2} = \frac{1}{6} + \frac{1}{18}$$

$$\Rightarrow \qquad v_2 = \frac{9}{2} = 4.5 \text{ cm}$$

Basic Mathematics For Physics

The First Steps' Learning

- Angles
- Measurement of Angles
- Trigonometrical Ratios
- General Trigonometric Identities
- Signs of Trigonometry Ratios
- Range of Trigonometric Function
- Pythagorus Theorem
- Compound Angles
- Logarithm
- Exponential
- The Coordinate System
- Straight Line
- The Circle
- Scaler Addition
- Vector Addition
- Vectors in Two Dimensions
- Vectors in Three Dimensions
- Function
- Differentiation
- Integration

Already in the first chapter we had explained about the importance of mathematics in physics. Without having basic knowledge of mathematics it is very difficult to deal with physics. To command over physics you must have sound knowledge of basic mathematics and to fulfill this requirement here we are mentioning few of the mathematics topic in brief.

We are going to review the basic concepts of

1. *Trigonometry* 2. *Algebra*
3. *Coordinate Geometry* 4. *Vectors*
5. *Differentiation* 6. *Integration*

Here, we are providing you the basic concepts and formulae of these topics along with the solved illustrative questions and exercises.

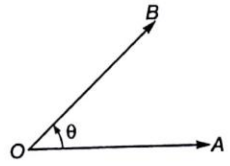
A. Trigonometry

'Trigonometry' is derived from the Greek Words 'Trignon' and 'Metron' which mean 'measuring the sides and angles of a triangle or simply the 'measurement of a triangle'.

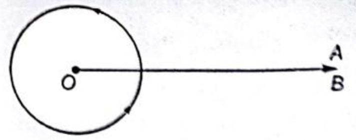
In the beginning, it only considered the measurement of sides, angles, area,perimeter etc. But now-a-days its application is in numerous areas, for example it is used by ship captains for navigation to determine the height of tides in the ocean, to determine the height of a mountain, to determine the speed of an air craft. The use and application of this branch of mathematics in science (Physics) is pivotal.

Angles

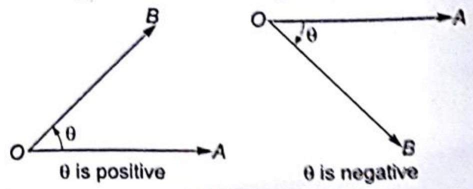
Suppose any straight line or ray OA is rotating about fixed point O, then angle θ is generated. Here the original ray *ie*, OA is called the initial line, point O is origin and ray OB *is terminal line or generating line. The measure of angle is amount of rotation performed to get the generating line from initial line.*



After one complete rotation, the initial and generating lines will co-incide together.



As a general convention, if direction of rotation is anticlockwise, then angle is considered to be positive while if direction of rotation is clockwise, then the angle is considered negative.



θ is positive θ is negative

Measurement of Angles

The general unit of measuring an angle is the right angle which is quarter of one complete rotation. But it is inconvenient to convert all angles into right angles, so three methods for measuring an angle has been devised. Here we mentioning them one by one.

(i) Sexagesimal Measure/English System

Here, one right angle is divided into 90 equal parts called degree (^{0})1 right angle = 90°.

A degree is divided in 60 equal parts called minutes (') 1° = 60'

A minute is divided into 60 equal parts called seconds (") 1' = 60"

For example, 45 degrees 30 minutes 30 seconds can be written in short as 45° 30' 30" :

(ii) Centesimal Measure/French System

Here, one right angle is divided into 100 equal parts called grades (') 1 right angle = 100^g.

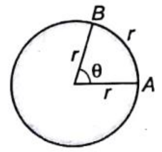
A grade is divided into 100 equal parts called minutes (') $1^g = 100'$

A minute is divided into 100 equal parts called seconds (") 1' = 100"

eg, 45 grades 30 minutes 30 seconds can be written in short as $45^g\,30'30''$

Minutes and Seconds of above two system are different.

(iii) Radian Measure Circular System



θ is the angle subtended by an arc $\overline{AB}$ on centre of circle of radius *r*.

$$\theta^c = \frac{\overline{AB}}{r}$$

$$1^c = \frac{r}{r}$$

$$1 \text{ right angle} = \left(\frac{\pi}{2}\right)^c$$

Relation between English, French and Radian system is

$$\frac{D}{9} = \frac{G}{10} = \frac{20R}{\pi}$$

where D is the number of degrees.
G is number of grades.
R is number of radians.

C-BIs

Concept Building Illustrations

Illustration | 1 *If the ray revolves two complete rotations, then how much angle does it covers ?*

(i) In English System
(ii) In French System
(iii) In Radian System

Solution In one rotation it covers 4 right angles so in two revolution it covers 8 right angles.

(i) 1 right angle = 90°
 8 right angle = 8 × 90 = 720°

(ii) 1 right angle = 100^g
 8 right angle = 8 × 100 = 800^g

(iii) $1 \text{ right angle} = \left(\frac{\pi}{2}\right)^c$

$$8 \text{ right angle} = 8 \times \frac{\pi}{2} = (4\pi)^c$$

Illustration | 2 *Convert* $(6\pi)^c$ *in degree system*

Solution $\left(\frac{\pi}{2}\right)^c = 90°$

$(\pi)^c = 2 \times 90 = 180^c$

$(6\pi)^c = 6 \times 180° = 1080°$

Example | 3. *Convert* 45° 30' *in radian system*

Solution $45°\ 30' = 45° + \left(\frac{1}{2}\right)^{\circ} = \frac{91°}{2}$

$1° = \left(\frac{\pi}{180}\right)^c$

$\left(\frac{91}{2}\right)^0 = \left(\frac{91}{2} \times \frac{\pi}{180}\right) = \left(\frac{91\pi}{360}\right)^c$

Example | 4. *The minute hand of watch is 2 cm long, how far will its tip move in 40 min ? (use* $\pi = 3.14$)

Solution In 60 min hand of watch completes one rotation.

$$40 \text{ min} = \frac{2\pi}{60} \times 40 = \left(\frac{4\pi}{3}\right)^c$$

Radius of watch = 2 cm

$$\theta = \frac{l}{r}$$

$$l = r\theta = 2 \times \frac{4\pi}{3}$$

$$= \frac{2 \times 4 \times 3.14}{3} = 8.32 \text{ cm}$$

Example | 5. *A wheel makes 180 revolutions in one minute. How much radians does it turn in one second?*

Solution In 60 s - 180 revolutions

1 s - 180/60 = 3 revolution

1 revolution - 2π rad

3 revolutions - (6π) rad

Concept Check 1

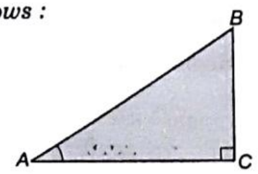
Exercise

1. Convert into radian
   (a) 67° 30' (b) 45° 30' 30"
   (c) $50^g 50'$ (d) $75^g 50' 50''$
2. Convert into centesimal
   (a) 45° 30' (b) 30° 30' 30"
   (c) $(5\pi)^c$ (d) $\left(\frac{3\pi}{2}\right)^c$
3. Convert into sexagesimal
   (a) $50^g 50'$ (b) $30^g 50' 50''$
   (c) $\left(\frac{5\pi}{2}\right)^c$ (d) $\left(\frac{66\pi}{5}\right)^c$
4. The sum of two angles is 80^g and their difference is 18°. Find the angles.
5. In two different circles arcs of same length subtend angles 60° and 75° at the centre. Find the ratio of their radii.

Trigonometrical Ratios

If ΔABC is any right angled triangle, right angled at C, then *wrt* $\angle A$, *BC is perpendicular, AC is the base and AB the hypotenuse.*

Then *wrt* $\angle A$, *the six trigonometric ratios are as follows :*



1. $\sin A = \frac{BC}{AB} = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$
2. $\cos A = \frac{AC}{AB} = \frac{\text{Base}}{\text{Hypotenuse}}$
3. $\tan A = \frac{BC}{AC}$
   $= \frac{\sin A}{\cos A} = \frac{\text{Perpendicular}}{\text{Base}}$

4. $\operatorname{cosec} A = \dfrac{AB}{BC} = \dfrac{1}{\sin A}$

5. $\sec A = \dfrac{AB}{AC} = \dfrac{1}{\cos A}$

6. $\cot A = \dfrac{AC}{BC} = \dfrac{\cos A}{\sin A}$

From here we can easily get

$$\cot A = \frac{1}{\tan A},$$
$$\operatorname{cosec} A = \frac{1}{\sin A},$$
$$\sec A = \frac{1}{\cos A}$$

General Trigonometric Identities

1. $\sin^2 A + \cos^2 A = 1$

$$\left(\frac{BC}{AB}\right)^2 + \left(\frac{AC}{AB}\right)^2 = \frac{BC^2 + AC^2}{AB^2}$$
$$= \frac{AB^2}{AB^2} \text{ (by pythagorus theorem}$$
$$AB^2 = AC^2 + BC^2)$$

2. $1 + \tan^2 A = \sec^2 A$

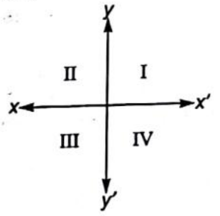
$$1 + \left(\frac{AB}{AC}\right)^2 = \frac{(AC)^2 + BC^2}{AC^2}$$
$$= \left(\frac{AB}{AC}\right)^2$$
$$= \sec^2 A$$

3. $1 + \cot^2 A = \operatorname{cosec}^2 A$

$$1 + \left(\frac{AC}{BC}\right)^2 = \frac{BC^2 + AC^2}{BC^2}$$
$$= \frac{AB^2}{BC^2} = \left(\frac{AB}{BC}\right)^2 = \operatorname{cosec}^2 A$$

Signs of Trigonometry Ratios

Any two dimensional space can be divided into 4 quadrants with the help of two mutual perpendicular lines which are termed as axes as shown in figure :



In First Quadrant, all trigonometric ratios are positive.

In 2nd Quadrant, sin and cosec are positive.

In 3rd Quadrant, tan and cot are positive and In 4th Quadrant, cos and sec are positive.

To remember this we use 'ASTC' :

A (all)	S (student)	T (take)	C (coffee)
Ist Quadrant All positive	IInd Quadrant sin and cosec + ve	IIIrd Quadrant tan and cot + ve	IVth Quadrant cos and sec

Range of Trigonometric Function

As hypotenuse is the greatest side of any right triangle, so we can say that both the ratios, perpendicular to hypotenuse and base to hypotenuse are less than 1 in magnitude. So sin and cosine of an angle is always less than 1, sec and cosec of an angle, is always greater than 1, tan and cot can have any value

$$\therefore \quad -1 \le \sin\theta \le 1$$
$$-1 < \cos\theta \le 1$$
$$-\infty < \tan\theta < \infty$$

$$-\infty < \cot\theta < \infty$$
$$-\infty < \sec\theta \le -1 \text{ and } 1 < \sec\theta < \infty$$
$$-\infty < \operatorname{cosec}\theta \le -1 \text{ and } 1 \le \operatorname{cosec}\theta < \infty$$

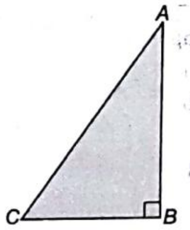
As infinite doesn't exist *ie., the values when denomination is zero doesn't exist, so we can say that*
1. If $\sin\theta$ is zero, then $\cot\theta$ and $\operatorname{cosec}\theta$ don't exist.
2. If $\cos\theta$ is zero, then $\tan\theta$ and $\sec\theta$ don't exist.

Pythagorus Theorem

In the right-angled triangle,

$$(\text{Base})^2 + (\text{Perpendicular})^2 = (\text{Hypotenuse})^2$$
$$(BC)^2 + (AB)^2 = (AC)^2$$

Generally used pythagorean triplets are (3, 4, 5); (5, 12, 13)



C-BIs

Concept Building Illustrations

Illustration | 1 *In right-angled ΔABC if $AB = 3$, $BC = 4$ and $CA = 5$, then find* $\sin A$ *and* $\cot C$.

Solution As the triangle is a right-angled triangle, so

$$\sin A = \frac{BC}{AC} = \frac{4}{5}$$

For angle C, AB will act as perpendicular and BC as base, so

$$\cot C = \frac{BC}{AB} = \frac{4}{3}$$

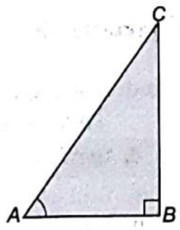


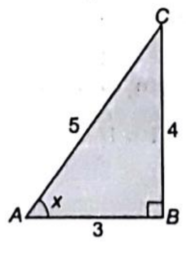
Illustration | 2 *If* $\cos x = -\frac{3}{5}$ *and x lies in 2nd quadrant, find the other five trigonometric functions.*

Solution As magnitude of $\cos x$ is $\frac{3}{5}$, so

$$\cos x = \frac{AB}{AC} = \frac{3}{5}$$

If $AB = 3$, then $AC = 5$ by Pythagorus theorem

$$AB^2 + BC^2 = AC^2$$
$$BC^2 = 5^2 - 3^2 = 16$$



$$BC = 4.$$

As x is in 2nd Quadrant, so only sine and cosec are positive.

$$\sin x = +\frac{BC}{AC} = +\frac{4}{5}, \quad \tan x = -\frac{BC}{AB} = -\frac{4}{3}$$
$$\cot x = -\frac{AB}{BC} = -\frac{3}{4}, \quad \sec x = -\frac{AC}{AB} = -\frac{5}{3}$$
$$\operatorname{cosec} x = \frac{AC}{BC} = \frac{5}{4}$$

Illustration | 3 *In any right-angled triangle if tan of an angle is 1.2, find sin and cosine of that angle.*

Solution $\tan A = 1.2 = \frac{12}{10} = \frac{6}{5}$

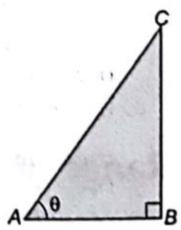
$$\tan A = \frac{BC}{AB} = \frac{6}{5}$$

$\therefore$ If $BC = 6$, then $AB = 5$.

$$AC^2 = AB^2 + BC^2$$

(by Pythagorus)

$$AC^2 = 5^2 + 6^2$$
$$AC = \sqrt{25 + 36} = \sqrt{61} = 7.8$$



So
$$\sin A = \frac{BC}{AC} = \frac{6}{7.8}$$
$$\cos A = \frac{AB}{AC} = \frac{5}{7.8}$$

Illustration | 4 *A ladder 8.5 m long is placed with its foot at a distance of 4 m from wall of a house and just touches a window. Find the height of window and sine of angle which the ladder makes with the wall.*

Solution Let AC be the ladder and AB be the wall, also suppose $AB = x$.

$x^2 = 8.5^2 - 4^2$

(by Pythagorus theorem)

$x^2 = (8.5 + 4)(8.5 - 4)$

$= 56.25$

$x = 7.5,$

and $\sin A = \dfrac{BC}{AC} = \dfrac{4}{8.5}$

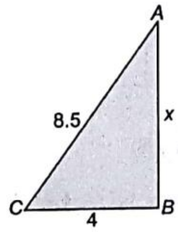
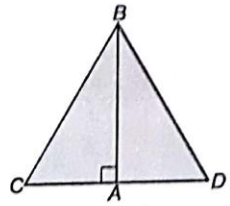


Illustration | 5 *In any triangle ABC the side $CB = 20$, $\angle BAC = 90°$, $AB = 12$, $AC = 16$. If BD is drawn perpendicular to BC and meets CA produced to D, find BD and AD.*



Solution As $\angle CBD = 90°$,

From ΔCBD, $\tan C = \dfrac{BD}{BC}$ ...(i)

From ΔABC, $\tan C = \dfrac{AB}{AC} = \dfrac{12}{16}$ ...(ii)

Now, from Eqs. (i) and (ii)

$$\frac{BD}{BC} = \frac{12}{16}$$

$$\Rightarrow \quad \frac{BD}{20} = \frac{12}{16} \quad BD = 15$$

$$CD^2 = BC^2 + BD^2$$

(From Pythagorus Theorem).

$$(AC + AD)^2 = BC^2 + BD^2$$

$$\Rightarrow (16 + AD)^2 = 20^2 + 15^2$$

$$\Rightarrow \quad AD = 9$$

Concept Check 2

Exercise

1. In ΔABC if $AB = 5$, $BC = 12$ and $AC = 13$, then find all trigonometric ratios for $\angle A$ and $\angle B$.
2. If x lies in 3rd Quadrant such that $\sin x = -\dfrac{4}{7}$, then find all other trigonometric ratios for x.
3. If $\tan x = -\dfrac{5}{12}$, then find all other trigonometric ratios for x
4. Draw the right angled triangle ABC if $\angle C = 90°$ $AC = 2.8$ cm, $\tan A = 7$. Find $\sin A$ and $\tan B$ also.
5. A stick of height 13 m is resting against a vertical wall. If one end of stick is at a distance of 5 m from the wall, find all trigonometric ratios of angles which the ladder makes with floor.
6. In ΔDEF if $ED = 35$, $EF = 37$ and $DF = 12$, find all trigonometric ratio for angle E.
7. A ladder is 14.5 m long. How far must its foot be placed from a wall, so that the ladder may just reach the top of wall which is 10.5 m from ground. Find all trigonometric ratios of angles between the ladder and wall.
8. Express all trigonometric ratios of angle A in terms of $\sin A$.
9. If $\sin A + \cos A = 0$, then find A.
10. If $\sec x = \dfrac{13}{5}$, find the value of $\dfrac{2\sin x - 3\cos x}{4\sin x - 9\cos x}$.

Trigonometric Ratios of some Angles

$\theta =$	$0°$	$30°$	$45°$	$60°$	$90°$	$180°$	$270°$	$360°$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0	$-\infty$	0
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞	-1	$-\infty$	0
$\operatorname{cosec}\theta$	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	$-\infty$	-1	∞
$\cot\theta$	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	$-\infty$	0	∞

Compound Angles

Conversion of Trigonometric Ratios into Various Quadrants

θ	$(-\theta)$	$\frac{\pi}{2}-\theta$	$\frac{\pi}{2}+\theta$	$\pi-\theta$	$\pi+\theta$	$\frac{3\pi}{2}-\theta$	$\frac{3\pi}{2}+\theta$	$2\pi-\theta$	$2\pi+\theta$
$\sin\theta$	$-\sin\theta$	$\cos\theta$	$\cos\theta$	$\sin\theta$	$-\sin\theta$	$-\cos\theta$	$-\cos\theta$	$-\sin\theta$	$\sin\theta$
$\cos\theta$	$\cos\theta$	$\sin\theta$	$-\sin\theta$	$-\cos\theta$	$-\cos\theta$	$-\sin\theta$	$\sin\theta$	$\cos\theta$	$\cos\theta$
$\tan\theta$	$-\tan\theta$	$\cot\theta$	$-\cot\theta$	$-\tan\theta$	$\tan\theta$	$\cot\theta$	$-\cot\theta$	$-\tan\theta$	$\tan\theta$

Here, we mention a few of trigonometric formulae for your quick reference.

1. $\sin(A+B) = \sin A\cos B + \cos A\sin B$
2. $\sin(A-B) = \sin A\cos B - \cos A\sin B$
3. $\cos(A+B) = \cos A\cos B - \sin A\sin B$
4. $\cos(A-B) = \cos A\cos B + \sin A\sin B$
5. $\tan(A+B) = \dfrac{\tan A + \tan B}{1-\tan A\tan B}$
6. $\tan(A-B) = \dfrac{\tan A - \tan B}{1+\tan A\tan B}$
7. $\sin(2A) = \sin(A+A)$
   $= \sin A\cos A + \cos A\sin A$
   $= 2\sin A\cos A$
8. $\cos(2A) = \cos(A+B)$
   $= \cos A\cos A - \sin A\sin A$
   $= \cos^2 A - \sin^2 A$
   $= 2\cos^2 A - 1 = 1 - 2\sin^2 A$
9. $\tan 2A = \tan(A+B)$
   $= \dfrac{\tan A + \tan B}{1-\tan A\tan B}$
   $= \dfrac{2\tan A}{1-\tan^2 A}$
10. $\sin 3A = 3\sin A - 4\sin^3 A$
    $\cos 3A = 4\cos^3 A - 3\cos A$
    $\tan 3A = \dfrac{3\tan A}{1-3\tan^2 A}$

C-BIs

Concept Building Illustrations

Illustration | 1 *Find the value of*

$$2\cot 45° + \cos^3 60 - 2\sin^4 60 + \frac{3}{4}\tan^2 30°$$

Solution $\cot 45° = 1$ and $\cos 60° = \frac{1}{2}$

$\sin 60° = \frac{\sqrt{3}}{2}$ and $\tan 30° = \frac{1}{\sqrt{3}}$

So, given expression would be,

$$2 \times 1 + \left(\frac{1}{2}\right)^3 - 2\left(\frac{\sqrt{3}}{2}\right)^4 + \frac{3}{4}\left(\frac{1}{\sqrt{3}}\right)^2$$

$$= 2 + \frac{1}{8} - \frac{9}{8} + \frac{1}{4} = \frac{16+1-9+2}{8} = \frac{10}{8} = \frac{5}{4}$$

Illustration | 2 *Find the value of*

$$2\sin 30° \cos 30° \cot 60°$$

Solution The given expression is same as

$\sin 60° \cot 60°$ (as $\sin 2\theta = 2\sin\theta\cos\theta$)

$$\sin 60 \times \frac{\cos 60°}{\sin 60°} = \frac{1}{2}$$

Illustration | 3 *Find the value of*

$$\tan^2 45° \sin 60° \tan 30° \tan^2 60°$$

Solution $\tan^2 45 \sin 60 \cot(90° - 30°) \tan^2 60$

$$= \tan^2 45 \sin 60 \frac{1}{\tan 60} \times \tan^2 60$$

$$= \tan^2 45 \sin 60 \tan 60 = (1)^2\left(\frac{\sqrt{3}}{2}\right)(\sqrt{3}) \text{ is } \frac{3}{2}$$

Illustration | 4 If $\tan^2 45° - \cos^2 60°$

$= x\sin 45° \cos 45° \tan 60°$.

Find the value of x.

Solution $\tan^2 45° - \cos^2 60° = \frac{x}{2}\sin 90° \tan 60°$

$$\left(\text{as } \sin\theta\cos\theta = \frac{\sin 2\theta}{2}\right)$$

$$(1)^2 - \left(\frac{1}{2}\right)^2 = \frac{x}{2} \times 1 \times \sqrt{3}$$

$$\frac{3}{4} = x\frac{\sqrt{3}}{2} \text{ so } x = \frac{\sqrt{3}}{2}$$

Illustration | 5 *Find the value of A if* $\cos 2A = \sin 3A$.

Solution $\sin(90° - 2A) = \sin 3A$

$$\left(\because \cos\left(\frac{\pi}{2} - \theta\right) = \cos\theta\right)$$

$$90° - 2A = 3A$$

$$5A = 90°$$

$$A = \frac{90}{5} = 18°.$$

Illustration | 6 *Prove that*

$\sec A \sec(90° - A) = \tan A + \tan(90° - A)$.

Solution LHS $\sec A \operatorname{cosec} A = \frac{1}{\sin A \cos A}$

$$= \frac{\sin^2 A + \cos^2 A}{\sin A \cos A}$$

$$= \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$$

$$= \tan A + \cot A$$

$$= \tan A + \tan(90° - A) = \text{RHS}$$

Illustration | 7 *In ΔABC angle C is 50° and angle A is complement of 10°. Find the angle B. (Complement of angle θ is angle $90° - \theta$).*

Solution $\angle C = 50°$, $\angle A = 90° - 10° = 80°$

$$\angle A + \angle B + \angle C = 180°$$

$$\angle A + 80° + 50° = 180°$$

$$\angle A = 180° - 130° = 50°$$

Example | 8 If $x\sin(90° - A)\cot(90° - A)$

$= \cos(90° - A)$.

Find the value of x.

Solution

$$x\sin(90° - A)\cot(90° - A) = \cos(90° - A)$$

$$x\cos A \tan A = \sin A$$

$$x\cos A \times \frac{\sin A}{\cos A} = \sin A$$

$$\Rightarrow \quad x = 1$$

Concept Check 3

Exercise

1. Find the complement of
   (a) $67° 30'$ (b) $50° 50'$ (c) $\frac{\pi}{10}$
2. In a ΔABC, angle A is complement of $50°$ and angle B is complement of 30. Find angle C.
3. In a ΔABC, if angle A is complement of angle B. Find angle C.
4. Find the value of angle A. If
   (i) $\tan A = \cot A$ (ii) $\cot A = \tan 2A$
   (iii) $\cos A = \cos 3A$
5. Prove the following identities :
   (i) $\sin A \tan(90° - A) \sec(90° - A) = \cot A$
   (ii) $\sin A \cos\left(\frac{\pi}{2} - A\right) + \cos A \sin\left(\frac{\pi}{2} - A\right) = 1$
8. Find the value of
   (a) $\sin(15°)$ (b) $\sin(75°)$
   (c) $\sin(115°)$ (d) $\sin\left(22\frac{1°}{2}\right)$
   (e) $\sin(720°)$ (e) $\cot(15°)$

B. Algebra

Here, in this part we are revising your concepts and formulae of Algebra which you have studied in your previous classes.

First, we shall define you some conceptuals used in Algebra :

Constant A quantity whose value don't change with time are termed as constants.

For example $2x^2 + x, 5x, ax + b$

In above equation $2, 5, a, b, c$ are constants.

Variable Quantities whose value will change with time are termed as variables.

Example x, y, z in above equation.

Expression Combination of terms having constant and variables is termed as an expression.

Equation When any expression is equated to zero, then it is said to be equation.

Example $2x^2 + 3x = 0, ax + b = 0$

Roots of an Equation The value of variable which when substituted in equation satisfies the equation.

Example

(i) $x - 3 = 0$

If we put $x = 3$, then this equation becomes $3 - 3 = 0$

$\therefore$ LHS = RHS

So, $x = 3$ is the root of this equation.

(ii) $2x - 5 = 0$

$$2x = 5$$
$$x = \frac{5}{2}$$

(iii) $ax + b = 0$

$$ax = -b$$
$$x = -\frac{b}{a}$$

Degree of Equation The highest power of variable in any equation is said to be its degree.

Example $ax^3 + bx \Rightarrow$ Degree = 3
$x^2 + 2x \Rightarrow$ Degree = 2

(i) Degree is always a whole number
(ii) As the degree of an equation increases, the number of roots also increases.
(iii) In general, the number of roots of any equation will be equal to its degree.

Quadratic Equation Any equation having degree two is known as, quadratic equation. Any quadratic equation has only two roots.

General form of a quadratic equation is

$$ax^2 + bx + c = 0$$

And roots of this equation are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where, quantity inside square root in known as Discriminant denoted by D:

$$D = b^2 - 4ac$$

On the basis of the discriminant, roots of a quadratic equation are divided into following ways :

1. If $D > 0$, then the roots of the quadratic equation are real and distinct and given by
$$\frac{-b+\sqrt{D}}{2a}, \text{ and } \frac{-b-\sqrt{D}}{2a}.$$
2. If $D = 0$, then the roots of the quadratic equation are real and identical *ie*, the quadratic equation has two equal roots given by $\frac{-b}{2a}$.
3. If $D < 0$, then both the roots of the quadratic equation are imaginary.

[Square root of a negative real number is known as an imaginary number.
The smallest imaginary number is $\sqrt{-1}$ known as iota, and is denotes by i]

Formation of a Quadratic Equation

Let the roots of the quadratic equation $ax^2 + bx + c = 0$ be α and β.

$$\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a}$$

$$\beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

Sum of roots :

$$\alpha + \beta = \frac{-b+\sqrt{b^2-4ac}}{2a} + \frac{-b-\sqrt{b^2-4ac}}{2a}$$

$$= -\frac{b}{a}$$

Product of roots :

$$\alpha\beta = \left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)\left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right)$$

$$= \frac{(-b)^2 - (\sqrt{b^2-4ac})^2}{4a^2}$$

$$= \frac{b^2 - b^2 + 4ac}{4a^2}$$

$$\alpha\beta = \frac{c}{a}$$

The quadratic equation can be written as

$$a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) = 0$$

$$x^2 - \left(\frac{-b}{a}\right)x + \frac{c}{a} = 0$$

$$x^2 - (\alpha+\beta)x + d\beta = 0$$

$$\Rightarrow \quad x^2 - (\text{sum of Roots})\,x + [\text{product of roots}] = 0$$

C-BIs

Concept Building Illustrations

Example | 1 *Find the degree of*

(i) $2x^2 + 3x$ (ii) $ax^3 + b$ (iii) $ax^2 + by^3$

Solution

(i) $2x^2 + 3x$

As the highest power of variable *ie*, x is 2, so the degree = 2

(ii) $ax^3 + b$

Here, the highest power of x is 3.
So, the degree = 3

(iii) $ax^2 + by^3$

In this expression, there are two variables *ie*, x and y and the highest power of the variables is 3.
So, degree = 3.

Example | 2 *Find the roots of*

(i) $3x - 5 = 0$ (ii) $5y - 4 = 0$
(iii) $3x = 0$

Solution

(i) $3x - 5 = 0$

$3x = 5$

$x = \frac{5}{3}$ is the required root.

(ii) $5y - 4 = 0$

$5y = 4$

$y = \frac{4}{5}$ is the required root

(iii) $3x = 0$

$x = \frac{0}{3}$

$x = 0$ is the root of equation.

Example | 3 *Find roots of following equations :*

(i) $x^2 - x - 12 = 0$

(ii) $4x^2 - 8x + 1 = 0$

(iii) $5x^2 - 4x + 9 = 0$

Solution

(i) $x^2 - x - 12 = 0$

$a = 1, b = -1, c = -12$

Roots are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{1 \pm \sqrt{1 - 4 \times 1 \times -12}}{2}$$

$$= \frac{1 \pm \sqrt{49}}{2} = \frac{1 \pm 7}{2}$$

$\therefore$ Roots $\frac{1+7}{2}$ and $\frac{1-7}{2}$

4 and -3

(ii) $4x^2 - 8x + 1 = 0$

$a = 4, b = -8; c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{8 \pm \sqrt{64 - 4 \times 4 \times 1}}{2 \times 4}$$

$$= \frac{8 \pm \sqrt{48}}{8}$$

$$x = \frac{8 \pm 4\sqrt{3}}{8} = 0$$

Roots are

$$x = \frac{2 \pm \sqrt{3}}{2} \text{ and } \frac{2 - \sqrt{3}}{2}$$

(iii) $5x^2 - 4x + 9 = 0$

$a = 5, b = -4, c = 9$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{4 \pm \sqrt{16 - 4 \times 5 \times 9}}{2 \times 5}$$

$$= \frac{4 \pm \sqrt{-164}}{10}$$

$$= \frac{4 \pm 2i\sqrt{41}}{10}$$

$$x = \frac{2 \pm i\sqrt{41}}{5}$$

$\therefore$ Roots are $\frac{2 + i\sqrt{41}}{5}$ and $\frac{2 - i\sqrt{41}}{5}$

Example | 4 *Find the quadratic equation if, its roots are*

(i) $2 + \sqrt{3}$ *and* $2 - \sqrt{3}$

(ii) -1 *and* 3

Solution

(i) As roots are $\alpha = 2 + \sqrt{3}, \beta = 2 - \sqrt{3}$

$\therefore$ Sum of roots

$\alpha + \beta = 2 + \sqrt{3} + 2 - \sqrt{3} = 4$

Product of roots

$\alpha\beta = (2 + \sqrt{3})(2 - \sqrt{3}) = 2^2 - \sqrt{3}^2$

$= 4 - 3 = 1$

$\therefore$ The required equation is

$x^2 - 4x + 1 = 0$

(ii) $\alpha = -1, \beta = 3$

$\alpha + \beta = -1 + 3 = 2$

$\alpha\beta = -1 \cdot 3 = -3$

$\therefore$ The required equation is

$x^2 - 2x - 3 = 0$

Example | 4 *If* α *and* β *are roots of* $2x^2 - 3x - 6 = 0$*, then find the equation whose roots are* $\alpha + 3$ *and* $\beta + 3$.

Solution As α and β are roots of $2x^2 - 3x - 6 = 0$

Here, $a = 2, b = -3, c = -6$

$\alpha + \beta = -\frac{b}{a}$ is $\alpha + \beta = \frac{3}{2}$

$\alpha\beta = \frac{c}{a} \Rightarrow \alpha\beta = -\frac{6}{2} = -3$

As roots of the equation to be found are $\alpha+3, \beta+3$,

Sum of roots is $\alpha+3+\beta+3$

$$=(\alpha+\beta)+6$$

$$=\frac{3}{2}+6=\frac{15}{2}$$

Product of roots is $(\alpha+3)(\beta+3)$

$$=\alpha\beta+3\alpha+3\beta+9$$

$$=\alpha\beta+3(\alpha+\beta)+9$$

$$=-3+3\cdot\frac{3}{2}+9$$

$$=-3+\frac{9}{2}+9=\frac{21}{2}$$

$\therefore$ The required quadratic equation is

$$x^2-\frac{15x}{2}+\frac{21}{2}=0$$

Concept Check 4

Exercise

1. Find the degree of
(a) $3x+1$ (b) bx^2+cx
(c) $4x^4-1$

2. Find the root of
(a) $ax-b=0$ (b) $x-5=0$
(c) $(a-2)x+5=0$

3. Find the roots of
(a) $x^2-5x+6=0$ (b) $2x^2-4x+5=0$
(c) $x^2-2\sqrt{2}x+4=0$

4. Find the quadratic equation whose roots are
(a) $p+\sqrt{q}$ and $p-\sqrt{q}$
(b) 2 and -2

5. If α and β are roots of $x^2-2x+4=0$, then find the equation whose roots are
(a) $\alpha-1$ and $\beta-1$
(b) α^2+1 and β^2+1

Logarithm

If $a^x=N$, where $a>0$ and $a\neq 1$, then $x=\log_a N$,

where a is known as base of log. If the base of log is e, then it, is known as the natural logarithm, while if the base is 10 then it is known as an ordinary logarithm.

> We can assume the base also according to our convenience

Properties of Logarithm

1. $\log_e x = 2.303\log_{10} x$
2. $\log_a(xy)=\log_a x+\log_a y$
3. $\log_a\left(\frac{x}{y}\right)=\log_a x-\log_a y$
4. $\log_a x^n = n\log_a x$
5. $\log_{a^n} x=\frac{1}{n}\log_a x$
6. $\log_a b=\frac{\log_c b}{\log_c a}$
7. $\log 1=0$ for any base
8. If $\log_a x>\log_a y$, then $x>y$ if $a>1$
   $x<y$ if $0<a<1$
9. $\log_a b=c$, then $b=a^c$.

Exponential

It is just reverse of Logarithm, if we take power of any number, then it is said to be exponential.

Example $a^x, e^x, 2^x$ etc

Properties of Exponential

1. $x^0 = 1$
2. $x^{-n} = \frac{1}{x^n}$
3. $x^a \cdot x^b = x^{a+b}$
4. $\frac{x^a}{x^b} = x^{a-b}$
5. $(xy)^n = x^n y^n$
6. $\left(\frac{x}{y}\right)^n = x^n \cdot y^{-n}$
7. $(x^n)^m = x^{nm}$
8. $\left(\frac{x}{y}\right)^{1/n} = \frac{x^{1/n}}{y^{1/n}}$
9. $(x^{1/n})^n = x$
10. $x^{1/m} = \sqrt[m]{x}$
11. $x^{-n/m} = \frac{1}{\sqrt[m]{x^n}}$

Important Formulae

1. $(a+b)(a-b) = a^2 - b^2$
2. $(a+b)^2 = a^2 + b^2 + 2ab$
3. $(a-b)^2 = a^2 + b^2 - 2ab$
4. $(a+b)^2 - (a-b)^2 = 4ab$
5. $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$
6. $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$
7. $a^3 + b^3 = (a+b)[a^2 + b^2 - ab]$
8. $a^3 - b^3 = (a-b)[a^2 + b^2 + ab]$
9. $1 + 2 + \ldots\ldots + n =$ sum of n natural numbers $= \frac{n(n+1)}{2}$
10. Sum of squares of n natural numbers $= 1^2 + 2^2 + 3^2 + \ldots\ldots + x^2 = \frac{n(n+1)(2n+1)}{6}$

C-BIs

Concept Building Illustrations

Example | 1 *Evaluate*

(a) $2^x \cdot 2^{-y} 4^z$ (b) $3^x 6^6 8^z$

(c) $30^2 8^2 5^3$ (d) $\frac{7^x}{49^{\frac{y}{2}}}$

(e) $7^{\frac{-3}{2}}$ (f) $7^{-\frac{3}{2}}$

Solution

(a) $a^x \cdot a^y = a^{x+y}$

$2^x \cdot 2^{-y} \cdot 4^z = 2^x \cdot 2^{-y} \cdot 2^{2z} = 2^{x-y+2z}$

(b) $3^x \cdot 6^6 \cdot 8^z = 3^x (2 \cdot 3)^y (2^3)^z$

$= 3^x \cdot 2^y \cdot 3^y \cdot 2^{3z} = 3^{(x+y)} \cdot 2^{y+3z}$

(c) $30^2 8^2 5^3 = (2 \cdot 3 \cdot 5)^2 (2^3)^2 5^3$

$= 2^2 \cdot 3^2 5^2 \cdot 2^6 5^3 = 2^8 \cdot 3^2 \cdot 5^5$

(d) $\frac{7^x}{49^{\frac{y}{2}}} = \frac{7^x}{\left(49^{\frac{1}{2}}\right)^y}$

$= \frac{7^x}{(\sqrt{49})^x} = \frac{7x}{7^y} = 7^{x-y}$

(e) $\frac{5^{1/2}}{20^4} = \frac{5^{1/2}}{(2 \cdot 2 \cdot 5)^4}$

$= \frac{5^{1/2}}{(2^2)^4 \cdot 5^4} = \frac{5^{1/2-4}}{2^8} = \frac{5^{-7/2}}{28} = \frac{1}{5^{7/2} \cdot 2^8}$

(f) $7^{-3/2} = \frac{1}{7^{3/2}}$

$= \frac{1}{(7^3)^{1/2}} = \frac{1}{7\sqrt{7}}$

Example | 2 *Evaluate*

(a) $3^1 \cdot 3^2 \cdot 3^3 ...... 3^{10}$

Solution (a) $3^1 \cdot 3^2 ..... 3^{10} = 3^{(1+2+3...10)}$

As $1+2+3 ...... + n = \dfrac{n(n+1)}{2}$

Put $n = 10$ we get $= \dfrac{10 \times 11}{2} = 55$

$\therefore \quad 3^{(1+2+3+....+10)} = 3^{55}$

Example | 3 *Evaluate*

(a) $\log 2 + \log 6 + 3 \log 5$

(b) $\log 5 - \log 10 + \log 10^3$

(c) $\log_8 2 + \log_2 4 + 5 \log_2 2$

(d) $\log_3 5 + \log_{27} 9 + \log_9 4$

Solution

(a) $\log 2 + \log 6 + \log 5^3$

As $\log a + \log b = \log (ab)$

$\therefore \quad \log 2 + \log 6 + \log 5^3 = \log [2 \times 6 \times 5^3]$

$= \log [2 \times 6 \times 125]$

$= \log (1500)$

(b) $\log 5 - \log 10 + \log 10^3$

$= \log 5 - 1 + 3 \log 10$ (As $\log 10 = 1$)

$= \log 5 - 1 + 3$

$= 2 + \log 5$

(c) $\log_8 2 + \log_2 4 + 5 \log_2 2$

$= \log_{2^3} 2 + \log_2 2^2 + 5 \log_2 2$

$= \dfrac{1}{3} \log_2 2 + 2 \log_2 2 + 5 \log_2 2$

$\left(\text{As } \log_{a^n} b = \dfrac{1}{n} \log_a b \text{ and } \log_a b^m = m \log_a b\right)$

$= \dfrac{1}{3} + 2 + 5$ (As $\log_2 2 = 1$)

$= \dfrac{21+1}{3} = \dfrac{22}{3}$

(d) $\log_3 5 + \log_{27} 9 + \log_9 4$

$= \log_3 5 + \log_{3^3} 3^2 + \log_{3^2} 4$

$= \log_3 5 + \dfrac{2}{3} \log_3 3 + \dfrac{1}{2} \log_3 4$

$= \log_3 5 + \log_3 (3)^{2/3} + \log_3 4^{1/2}$

$= \log_3 5 \cdot 3^{2/3} \cdot 4^{1/2} = \log_3 5 \cdot 9^{1/3} \cdot 2$

$= \log_3 10 \cdot 9^{1/3} = \log_3 10 + \log_3 9^{1/3}$

$= \log_3 10 + \log_3 3^{2/3}$

$= \log_3 10 + \dfrac{2}{3}$

Example | 4 *If $\log_4 x > 2$, then find the value of x $\tan_4 x > 2$.*

Solution

$\log_4 x > 2 \log_4 4 \, (\log_4 4 = 1)$

$\log_4 x > \log_4 4^2$

$\log_4 x > \log_4 16$

$\therefore \quad x > 16$ (As base > 1)

Example | 5 *If α and β are roots of $x^2 - x + 1 = 0$, then find the value of*

(a) $\alpha^2 + \beta^2$ (b) $\alpha^2 - \beta^2$

(c) $\alpha^3 + \beta^3$

Solution If α and β are roots of $x^2 - x + 1 = 0$, then sum of roots

$\alpha + \beta = 1$ and product of roots $\alpha\beta = 1$

(a) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= 1^2 - 2 \times 1$ is $1 - 2 = 1$

(b) $\alpha^2 - \beta^2 = (\alpha - \beta)(\alpha + \beta)$ (As $\alpha + \beta = 1$)

$= (\alpha - \beta)$

$= \sqrt{(\alpha - \beta)^2}$

$= \sqrt{(\alpha^2 + \beta^2) - 2\alpha\beta}$

$= \sqrt{(\alpha + \beta)^2 - 2\alpha\beta - 2\alpha\beta}$

$= \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$

$= \sqrt{1^2 - 4 \times 1}$

$= \sqrt{-3} = i\sqrt{3}$

(c) $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$= 1^3 - 3 \times 1 \times 1 = 1 - 3 = -2$

Example | 6 *If α and β are of $x^2 - 2x + 4 = 0$, then find quadratic equation whose roots are α^3 and β^3.*

Solution If α and β are roots of $x^2 - 2x + 4 = 0$

$\alpha + \beta = 2$

$\alpha\beta = 4$

$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$= 2^3 - 3 \times 4 \times 2$

$= 8 - 24 = -16$

$\alpha^3\beta^3 = (\alpha\beta)^3 = 4^3 = 64$

$\therefore$ Equation whose roots are α^3 and β^3, are

$$x^2-(\alpha^3+\beta^3)x+(\alpha^3\beta^3)=0$$
$$x^2-(-16)x+64=0$$
$$x^2+16x+64=0$$

Example | 7 *Evaluate*

$$5^2+6^2+\ldots+15^2$$

Solution

$$5^2+6^2+\ldots+15^2=(1^2+2^2+\ldots+15^2)-(1^2+2^2\ldots+4^2)$$
$$=\sum_{n=1}^{15}n^2-\sum_{m=1}^{4}m^2$$
$$=\left[\frac{n(n+1)(2n+1)}{6}\right]-\left[\frac{m(m+1)(2m+1)}{6}\right]$$
$$=\frac{15\times16\times21}{6}-\frac{4\times5\times9}{6}=840-30=810$$

C. Coordinate Geometry

The branch of Mathematics which deals with points, straight lines and curves which are drawn on any plane, constitutes the subject matter of Coordinate Geometry.

The Coordinate System

It is a type of frame by which we can determine the location of any object.

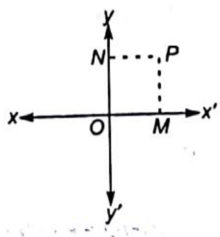
Point

Location of any object is specified *w.r.t.* some particular frame. If we change the frame, the position of that object will also change.

For example let a student A is studying at his study chair and is assumed to be a point, then if we want to determine the position of A *w.r.t.* some corner of study room and *w.r.t* his dining room centre, then both positions are different as the the two frames are different.

The Cartesian Co-ordinate System

On the plane two perpendicular fixed straight lines XOX' and YOY' could be taken known as the coordinate axes, and the point O where they intersect is known as origin.



In the figure shown XOX' is x-axis and YOY' is y-axis with O as origin.

Let P be any point on this plane from where PM and PN are two perpendiculars drawn on x- and y-axes, respectively.

Here, $PM=NO=y$ (Known as ordinate)

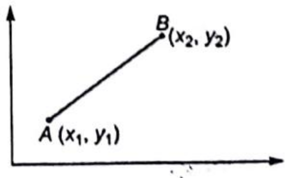
$PN=OM=x$ (Known as abscissa)

On this plane each point is represented as pair of coordinates *ie*, $P(x, y)=P$ (abscissa, ordinate) coordinate of origin is (0, 0).

Distance Formula

This formula is used to find the distance between two points when their coordinates are given. Let the two points be A and B whose coordinates are $A(x_1, y_1)$ and $B(x_2, y_2)$, then the distance between these two points is

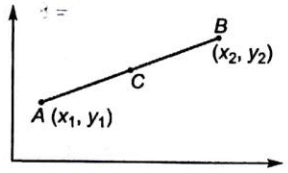
$$|AB|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$



Section Formula

This formula is used to find the coordinates of a point which divides the given line segment into a given ratio.

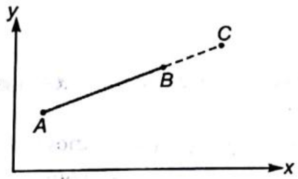
(i) **Internal Division** If C be any point which divides the line-segment joining AB in $m:n$ ratio *ie*,



$\frac{AC}{CB} = \frac{m}{n}$, then the coordinates of C are given by

$$x = \frac{mx_2 + nx_1}{m+n}, \text{ and } y = \frac{my_2 + ny_1}{m+n}.$$

(ii) **External Division** If C be any point on line AB after producing AB, C is external to AB such that

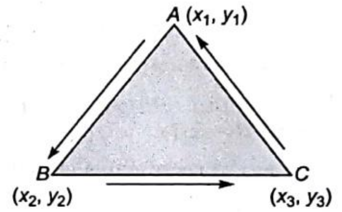


$\frac{AC}{CB} = \frac{m}{n}$, then the coordinates of C are given by

$$x = \frac{mx_2 - nx_1}{m-n}, \text{ and } y = \frac{my_2 - ny_1}{m-n}.$$

Area of Triangle

When the coordinates of the vertices of a triangle are given, then its area can be determined by using this formula. Let the coordinates of the vertices of ΔABC are $A(x_1, y_1)$ $B(x_2, y_2)$, and $C(x_3, y_3)$, then area is



$$\Delta \left| \frac{1}{2} [x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)] \right|$$

If three points are collinear (points lying on the same line), then the area of triangle formed by these three points will be zero.

C-BIs

Concept Building Illustrations

Example | 1 *Find the length of line joining the points* $(1, -1)$ *and* $(-1, 1)$.

Solution $x_1 = 1, y_1 = -1, x_2 = -1, y_2 = 1$

$$\text{Distance} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$
$$= \sqrt{(1 + (1))^2 + (-1 - 1)^2}$$
$$= \sqrt{2^2 + 2^2} = 2\sqrt{2} \text{ units}$$

Example | 2 *Show that the points* $(1, -1)$ $(-1, -1)$ *and* $(-\sqrt{3}, \sqrt{3})$ *are angular points of an equilateral triangle.*

Solution Let the three vertices be $A(1, 1)$, $B(-1, -1)$ and $C(-\sqrt{3}, \sqrt{3})$, then

$$|AB| = \sqrt{(1-(-1))^2 + (1-(-1))^2}$$
$$= \sqrt{2^2 + 2^2} = 2\sqrt{2}$$
$$|BC| = \sqrt{(-\sqrt{3} - (-1))^2 + (\sqrt{3} - (-1))^2}$$
$$= \sqrt{(1 + \sqrt{3})^2 + (1 - \sqrt{3})^2} = 2\sqrt{2}$$
$$|CA| = \sqrt{(1 - (-\sqrt{3}))^2 + (1 - \sqrt{3})^2} = 2\sqrt{2}$$

As three sides of the triangle are equal, so the triangle equilateral.

Example | 3 *If the point $P(h, k)$ is equidistant from two points $A(3, 4)$ and $B(1, -2)$, then find the relation between h and k.*

Solution As $PA = PB$

So, by the distance formulae

$$\sqrt{(h-3)^2+(k-4)^2} = \sqrt{(h-1)^2+(k+2)^2}$$

On squaring both the sides

$$(h-3)^2+(k-4)^2 = (h-1)^2+(k+2)^2$$

$$\Rightarrow \quad h^2-6h+9+k^2-8k+16 = h^2-2h+1+k^2+4k+4$$

$$\Rightarrow \quad 4h+12k-20=0$$

$$\Rightarrow \quad h+3k-5=0$$

Example | 4 *Find mid-point of line joining $A(3, 1)$ and $B(-5, 7)$.*

Solution Let the coordinates of mid-points be $P(x, y)$

Then, $x = \dfrac{3-5}{2} = -1$, and $y = \dfrac{1+7}{2} = 4$.

$\therefore$ Mid-point is $P(-1, 4)$

Example | 5 *Find the point which divides the line joining $A(2, 3)$ and $B(5, -3)$ in 1:2 ratio*

(i) Internally *(ii) Externally*

Solution

(i) Let the point which divide in 1 : 2 ratio internally be $P(x, y)$

$$x = \frac{1\times 5+2\times 2}{1+2} = \frac{5+4}{3} = 3$$

$$y = \frac{1\times(-3)+2\times 3}{1+2} = \frac{-3+6}{3} = 1$$

$\therefore$ $P(3, 1)$

(ii) Let $P(h, k)$ be the point which divides it in 1 : 2 ratio externally :

$$h = \frac{1\times 5-2\times 2}{1-2} = \frac{5-4}{-1} = -1$$

$$k = \frac{1\times(-3)-2\times 3}{1-2} = \frac{-3-6}{-1} = 9$$

$\therefore$ $P(-1, 9)$

Example | 6 *Find the area of triangle whose angular points are $A(2, 1)$, $B(4, 3)$ and $C(2, 5)$.*

Solution Area

$$= \left|\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]\right|$$

$$= \left|\frac{1}{2}[2(3-5)+4(5-1)+2(1-3)]\right|$$

$$= \left|\frac{1}{2}[2\times -2+4\times 4+2\times -2]\right|$$

$$= \left|\frac{1}{2}[-4+16-4]\right|$$

$$= \left|\frac{1}{2}\times 8\right| = 4 \text{ sq. units}$$

Concept Check 5

Exercise

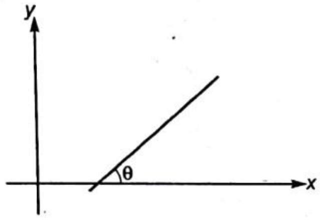
1. Find the length of the line joining the points
   (a) $(a, -a)$ and $(-b, b)$
   (b) $(3, 4)$ and $(-1, 1)$
2. Show that the four points $(0, -1)$, $(-2, 3)$, $(6, -7)$ and $(8, 3)$ are angular points of a rectangle.
3. Find the point which is equidistant from $(0, 0)$, $(32, 10)$ and $(42, 0)$
4. Show that the point $(7, 10)$ is equidistant from three points $(-10, -9)$, $(32, 5)$, and $(18, 33)$.
5. Find the point which divides the line joining the points $(5, -3)$ and $(3, -5)$ in 3 : 5 ratio
   (i) internally (ii) externally
6. Find area of triangle whose angular points are $(4, -5)$, $(5, -6)$ and $(3, 1)$.

Straight Line

On any plane when two points are joined by the shortest distance, then a straight line is produced.

Slope

With every line a term is associated known as slope. A line makes certain angle with X-axis. Slope is defined as the tangent of the angle which the line makes with positive direction of x-axis. Slope is denoted by m, $m = \tan\theta$



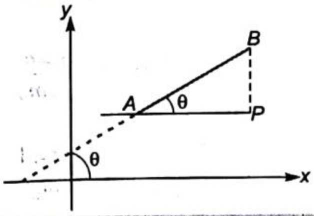
Slope of Line Joining two Points A and B

Consider a straight line formed by joining two points A and B whose coordinates are given by $A(x_1, y_1)$ and $B(x_2, y_2)$ as shown in figure below. From the diagram it is clear that,

$$BP = y_2 - y_1$$

and $$AP = x_2 - x_1$$

$$\therefore \quad \text{slope} = m = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$$



1. If the three points A, B and C are collinear then slope of AB = slope of BC

   A B C

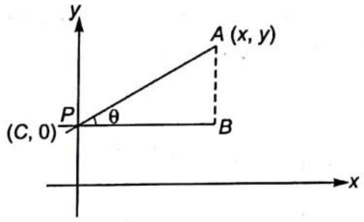
2. Slope of line parallel to x-axis is 0

   $m = \tan\theta = \tan(0) = 0$
3. Slope of line parallel to y-axis is ∞.

   $m = \tan\theta = \tan 90° = \infty$

Equation of a Straight Line

Consider any line AP which cuts y-axis at point P as shown in figure.



$$\tan\theta = \frac{AB}{PB} = \frac{y - c}{x}$$

As, line makes $\angle\theta$ with x-axis, so $m = \tan\theta$.

$$\therefore \qquad m = \frac{y - c}{x}$$

So, $y = mx + c$ is the equation of straight line whose slope is m and y-intercept is c.

1. If the line passes through origin, then $c = 0$.
2. Equation of line passing through the point (x_1, y_1), and having slope m is

   $$(y - y_1) = m(x - x_1)$$

Angle between Two Lines

If two lines are given whose equations are $y = m_1 x + c_1$ and $y = m_2 x + c_2$, then the angle between the two lines is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

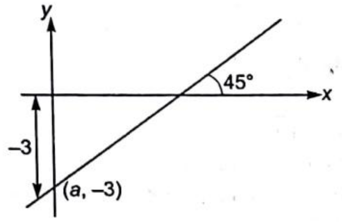
Two lines are parallel if the slopes of two lines are equal *ie*, $m_1 = m_2$ and lines are perpendicular if the product of slopes of two lines is equal to -1 *ie*,

$$m_1 m_2 = -1$$

C-BIs

Concept Building Illustrations

Example | 1 *Find the equation of straight line which makes angle of 45° with x-axis and cuts an axis of y at a distance of −3 units from origin.*



Solution $m = \tan 45°$

$$m = 1,$$

and $$c = -3,$$

So, the equation of the straight line would be

$$y = 1x - 3$$
$$y = x - 3$$

Example | 2 *Find the slope and y-intercept of the line* $3x + 2y = 7$.

Solution $2y = -3x + 7$

$$y = -\frac{3}{2}x + \frac{7}{2}$$

Now, compare this equation with $y = mx + c$.

Slope $= m = -\frac{3}{2}$, and y-intercept $= c = \frac{7}{2}$.

Example | 3 *Find the equation of straight line which passes through* $(\sqrt{3}, 2)$, *and which makes an angle of 60° with x-axis.*

Solution Slope $m = \tan 60°$

$$m = \sqrt{3}$$

Equation of the line is given by,

$$y - y_1 = m(x - x_1)$$
$$y - 2 = \sqrt{3}\,(x - \sqrt{3})$$
$$y - 2 = x\sqrt{3} - 3$$
$$x\sqrt{3} - y = -2 + 3$$
$$x\sqrt{3} - y = 1$$

Example | 4 *Find the slope of line joint* (3, − 2) *and* (− 1, 4).

Solution Slope $= m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{4-(-2)}{-1-3}$$
$$= \frac{4+2}{-1-3}$$
$$= \frac{6}{-4}$$
$$m = -\frac{3}{2}$$

Example | 5 *If angle between two lines is* $\frac{\pi}{4}$ *and slope one of the lines is* $\frac{1}{2}$. *Find slope of another line.*

Solution Let slope of two lines be m_1 and m_2

then $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

$$m_1 = \frac{1}{2}, \text{ and } \theta = \frac{\pi}{4} \text{ so } \tan\theta = 1$$

$$1 = \left|\frac{\frac{1}{2} - m_2}{1 + \frac{m_2}{2}}\right| \quad 1 = \left|\frac{1 - 2m_2}{2 + m_2}\right|$$

$$|1 - 2m_2| = |2 - m_1|$$
$$1 - 2m_2 = 2 + m_2$$

and $$1 - 2m_2 = -(2 + m_2)$$

$\Rightarrow$ $$3m_2 = -1$$

and $$m_2 = 3$$

$\Rightarrow$ $$m_2 = -\frac{1}{3}$$

and $$m_2 = 3$$

Example | 6 *Show that middle point of the joint of* (3, 4) *and* (5, 1) *is on line* $x - 2y + 1 = 0$

Solution Let $A(3, 4)$ and $B(5, 1)$, then mid point of AB is given by

$$P\left(\frac{3+5}{2}, \frac{4+1}{2}\right) \equiv P\left(4, \frac{5}{2}\right)$$

If point P lies on line, then it satisfies the equation of line

$$x - 2y + 1 = 0$$

$$4 - 2 \cdot \frac{5}{2} + 1 = 0 \text{ is } 5 - 5 = 0$$

$\therefore$ P lies on line AB.

Concept Check 6

Exercise

1. Find the slope of line passing through $(3, -2)$ and $(3, 4)$.
2. Find the equation of line joining points $(3, -2)$ and $(-1, 4)$.
3. Find the equation of line which makes an angle of 30° with positive direction of x-axis and which also passes through $(3, 2)$.
4. Find the equation of straight line which is perpendicular to $2x - 3y + 1 = 0$, and passes through $(5, 2)$.
5. Find the slope of line which passes through origin and the mid-point of the line segment joining the points $A(0, 4)$ and $B(8, 0)$.
6. In what ratio the line $x - y - 2 = 0$ cuts the line joining $(3, -1)$ and $(8, 9)$.
7. Find slope and intercepts on co-ordinate axes on line $3x - 4y + 10 = 0$?
8. Find the angle between the lines $y - \sqrt{3}x + 5 = 0$ and $\sqrt{3}y - x + 2 = 0$.

The Circle

A point moves in such a way that its distance from a fixed point is always constant, then the locus of the moving point is known as circle. The fixed point is called centre of circle. The constant distance is called the radius of circle.

As $\quad CP = r$

So, $\quad \sqrt{(x - x_1)^2 + (y - y_1)^2} = r$

$$(x - x_1)^2 + (y - y_1)^2 = r^2$$

This is the equation of circle.

If centre is origin, then $x_1 = y_1 = 0$

In this situation, the equation of the circle is $x^2 + y^2 = r^2$.

General Equation of Circle

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$(x^2 + 2gx) + (y^2 + 2fy) = -c$$

Make a perfect square of both the brackets by adding g^2 and f^2 in 1st and 2nd brackets, respectively.

$$(x^2 + 2gx + g^2) + (y^2 + 2fy + f^2) = g^2 + f^2 - c$$

$$(x + g)^2 + (y + f)^2 = (\sqrt{g^2 + f^2 - c})^2$$

is an equation of circle. Now compare it with $(x - x_1)^2 + (y - y_1)^2 = r^2$ which gives

$$x_1 = -g, y_1 = -f, r = \sqrt{g^2 + f^2 - C}$$

So, the centre is $C(-g, -f)$, and the radius is

$$r = \sqrt{g^2 + f^2 - C}$$

If $\sqrt{g^2 + f^2 - C} = 0$ *ie*, radius = 0, then the circle is known as point circle or circle whose area is zero.

Area of Circle

If the radius of a circle is r, then its area is

$$\Delta = (\pi r)^2 \text{ sq. units}$$

So we can say that area is independent of centre of circle.

Perimeter of Circle

It is defined as the length of the circumference of circle.

It the radius of the circle is r, then the perimeter is $(2\pi r)$ units.

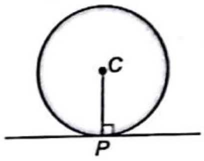
Concyclic Points/Cyclic Quadrilateral

If all the four vertices of, quadrilateral lie on circumference of a circle, then the quadrilateral is said to be a cyclic quadrilateral and all the four vertices or four points are known as the concyclic points.

In a cyclic quadrilateral, the sum of opposite angle of the quadrilateral is always 180°.

Tangent to a Circle

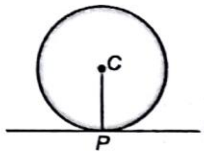
The line which touches the circle at only one point is called a tangent to circle.



Tangent of perpendicular from the centre of circle to the tangent is always equal to radius *ie*, $CP =$ radius.

Normal

The line which is perpendicular to tangent at point of contact is normal to the circle. A normal always passes through the centre of circle.



C-BIs

Concept Building Illustrations

Example | 1 *Find equation of circle whose centre is $(-4, -3)$, and whose radius is 5 units.*

Solution $C(x_1, y_1) \equiv C(-4, -3)$ and radius

$$\Rightarrow \quad r = 5$$

As equation of the circle is

$$(x - x_1)^2 + (y - y_1)^2 = r^2$$
$$(x - (-4))^2 + (y - (-3))^2 = 5^2$$
$$(x + 4)^2 + (y + 3)^2 = 25$$
$$x^2 + y^2 + 8x + 6y = 0$$

Example | 2 *Find the centre and radius of circle whose equation is*

$$x^2 + y^2 + 4x + 6y - 1 = 0$$

Solution $x^2 + y^2 + 4x + 6y - 1 = 0$

Compare this equation with general equation of circle

ie, $\quad x^2 + y^2 + 2gx + 2fy + c = 0$

So, $\quad g = 2, f = 4, c = -1$

Centre is $C(-g, -f) \equiv C(-2, -3)$

Radius is $r = \sqrt{g^2 + f^2 - c}$

$$= \sqrt{2^2 + 3^2 - (-1)}$$
$$r = \sqrt{4 + 9 + 1}$$
$$= \sqrt{14} \text{ units}$$

Example | 4 *Find the equation of circle which passes through the points $(a, 0)$, $(-a, 0)$ and $(0, b)$.*

Solution Let the circle which passes through the given points be

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

As the circle passes through these points, so these points will satisfy the equation of circle.

$$a^2 + 2ga + c = 0 \quad \text{...(i)}$$
$$a^2 - 2ga + c = 0 \quad \text{...(ii)}$$
$$b^2 + 2fb + c = 0 \quad \text{...(iii)}$$

On adding Eqs. (i) and (ii)

$$2a^2 + 2c = 0$$
$$c = -a^2 \quad \text{...(iv)}$$

Putting value of c in Eq. (i),

$$a^2 + 2ga - a^2 = 0 \text{ is } g = 0 \quad \text{...(v)}$$

Putting value of c in Eq. (iii),

$$b^2 + 2fb - a^2 = 0$$
$$f = a^2 - b^2 \quad \text{...(vi)}$$

Putting the values of g, f and c from Eqs. (iv), (v) and (vi) in equation of circle,

$$x^2 + y^2 + 2(0)x + 2\frac{(a^2 - b^2)}{2b}y - a^2 = 0$$

$$x^2 + y^2 + \frac{(a^2 - b^2)}{b}y - a^2 = 0$$

Example | 6 *Find the equation of circle whose two normals are $x + y - 1 = 0$ and $x - y + 2 = 0$, and whose radius is 5 units.*

Solution As normal to any circle passes through centre of circle, so the point of intersection of two normals will be centre of circle.

$$x + y = 1 \quad \text{...(i)}$$
$$x - y = -2 \quad \text{...(ii)}$$

On solving Eqs. (i) and (ii),

$$x = -\frac{1}{2}, y = \frac{3}{2}$$

$\therefore$ The centre is $C\left(-\frac{1}{2}, \frac{3}{2}\right)$

Radius is $r = 5$

So, equation of circle is

$$\left(x + \frac{1}{2}\right)^2 + \left(y - \frac{3}{2}\right)^2 = 5^2$$

$$x^2 + y^2 + x - 3y + \frac{1}{4} + \frac{9}{4} - 25 = 0$$

$$x^2 + y^2 + x - 3y - \frac{45}{2} = 0$$

Concept Check 7

Exercise

1. Find the equation of circle whose centre is $(-3, 0)$, and the radius is 3 units.
2. Find the centres and radii of following circles :
   (a) $x^2 + y^2 - 5x - 2y + 25 = 0$
   (b) $3x^2 + 3y^2 - 6x + 5y - 3 = 0$
3. Find the equation of circle whose centre lies on positive direction of x-axis and which passes through $(1, 2)$ and $(3, -1)$.
4. Find the equation of circle which passes through $(1, 3)$, $(2, 1)$, and $(-1, 1)$.
5. Find the equation of circle whose centres lie on line $x + 2y = 3$, and one normal is $2x - 3y = 5$, and whose radius is 3 units.

D. Vector

A man starts walking from point A and reaches point B, then it covers some distance *ie*, distance between points A and B.

A B

Then, he walks back from point B to point A. In both motion he covers the same distance but both the motions are different due to one new terminology *ie*, direction.

Scalar Quantity

A physical quantity which is completely specified by its magnitude only is called a scalar quantity.

For example, the distance which the man covers between two points A and B is a scalar physical quantity.

Vector Quantity

A physical quantity which is specified by its magnitude as well as direction is called a vector quantity.

Representation of a Vector

Geometrically, a vector is represented by a directed line segment. The arrow head will represent the direction of motion, *ie*, in this figure, the direction is from point A to point B. The length of line represents the magnitude of vector.

Mathematically, a vector is represented by an arrow placed over the symbol of physical quantity like $\vec{\mathbf{a}}, \vec{\mathbf{b}}\ \overrightarrow{\mathbf{AB}}, \vec{\mathbf{F}}$ etc. Here, in above situation, vector AB is represented by $\overrightarrow{\mathbf{AB}}$. Here A is initial point and B is terminal point. So, we can say that $\overrightarrow{\mathbf{AB}}$ and $\overrightarrow{\mathbf{BA}}$ both are different. These two vectors are having equal magnitudes, but are in opposite directions. In some books vector physical quantities are differentiated from others by making them in bold faces.

Magnitude of a Vector

If a man starts from point A and has to reach point B, then his motion will represented by vector $\overrightarrow{\mathbf{AB}}$. The magnitude of this vector is the distance between the two points *ie*, initial and terminal points. Magnitude of a vector can be represented mathematically by modulus sign, (the vector itself in two vertical lines).

$$\text{Magnitude} = |\overrightarrow{\mathbf{AB}}| = |\vec{\mathbf{a}}|$$

Magnitude of a vector is always a positive real number.

Types of Vectors

1. **Null Vector** A vector whose magnitude is zero is known as a null vector or zero vector.

Or If initial point and terminal points of any vector will coincide, then the vector is said to be a null vector.

Direction of the null vector can't be determined. The null vector is represented by $\vec{\mathbf{0}}$.

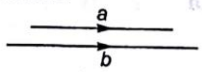
2. **Unit Vector** A vector whose magnitude is one unit is called unit vector.
Unit vector is represented by a cap over the symbol of vector. For example $\hat{\mathbf{a}}$ is an unit vector in the direction of $\vec{\mathbf{a}}$.
If we divide the vector by its magnitude, then we get the unit vector in the direction of the given vector.
The unit vector in the direction of $\overrightarrow{\mathbf{AB}}$ is

$$\hat{\mathbf{AB}} = \frac{\overrightarrow{\mathbf{AB}}}{|\overrightarrow{\mathbf{AB}}|}$$

Magnitude of unit vector is

$$|\hat{\mathbf{AB}}| = \frac{|\overrightarrow{\mathbf{AB}}|}{|\overrightarrow{\mathbf{AB}}|} = 1$$

3. **Equal Vectors** Two vectors are said to be equal if they have the same magnitude, and are in same direction. $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are equal if
(a) $|\vec{\mathbf{a}}| = |\vec{\mathbf{b}}|$ and
(b) $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ have same direction.
4. **Like Vectors** If two parallel vectors have same direction then they are said to be like vectors.

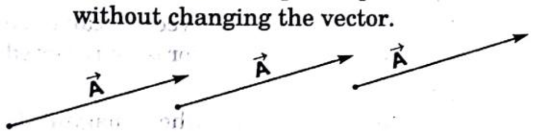


5. **Negative Vectors** A vector $\vec{\mathbf{B}}$ is said to be negative of vector $\vec{\mathbf{A}}$, if its magnitude is same as that of $\vec{\mathbf{A}}$ but direction is opposite to that of $\vec{\mathbf{A}}$.

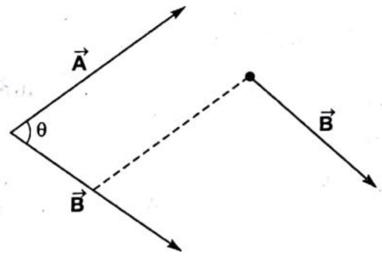
6. **Coplanar Vectors** If two or more vectors lie in the same plane, these vectors are said to be coplanar.

Properties of Vector

1. A vector can be displaced parallel to itself without changing the vector.

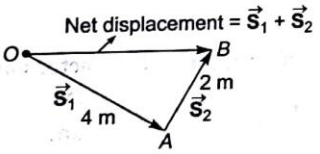


2. If a vector has been rotated by an angle other than 2π, then the vector changes in direction although the magnitude remains same.
3. To determine the angle between two vector $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, we have to place the two vectors in such a way that their tails coincide and in this position the smaller of the two angles subtended by them is termed as the angle between two vectors.



Scaler Addition

A particle makes a displacement OA along a straight line and then makes a displacement AB along a straight line as shown in the figure.



The distance travelled during OA and displacement during OA are the same *ie*, 4 m.

Problem What is the total distance travelled from O to B?

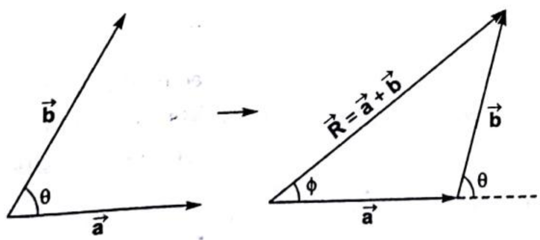
Inference Scalers are added algebraically :

Net displacement $\neq 4 + 2$

Hence, the vectors are added by some other way.

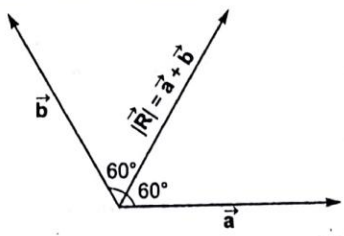
Vector Addition

1. Geometrical Way
Triangular Law of Vector Addition

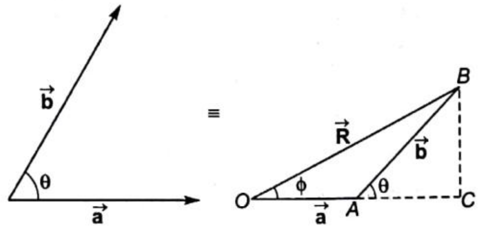


Draw the given vectors in such a way that the tail of one matches with the head of the other. Complete the triangle by drawing the third side. Mark out the arrow which points from initial tail to final head. This will give us the resultant vector. It is only valid for adding two vectors.

If two vectors of equal magnitudes are added, then their resultant lies along the angle bisector.



2. Analytical method ABC is a right-angled triangle



In ΔABC,

$$\cos\theta = \frac{AC}{AB}$$

$$\Rightarrow \quad \cos\theta = \frac{AC}{b}$$

$$\Rightarrow \quad AC = b\cos\theta$$

$$\sin\theta = \frac{BC}{AB}$$

$$\Rightarrow \quad \sin\theta = \frac{BC}{b}$$

$$\Rightarrow \quad BC = b\sin\theta$$

In $\Delta\, OBC$

$$(BC)^2 + (OC)^2 = (OB)^2$$

$$b^2\sin^2\theta + (a + b\cos\theta)^2 = R^2$$

$$b^2\sin^2\theta + a^2 + b^2\cos^2\theta + 2ab\cos\theta = R^2$$

$$a^2 + b^2(\sin^2\theta + \cos^2\theta) + 2ab\cos\theta = R^2$$

$$R^2 = a^2 + b^2 + 2ab\cos\theta$$

$$R = \sqrt{a^2 + b^2 + 2ab\cos\theta}$$

$$\tan\phi = \frac{BC}{OC}$$

$$\tan\phi = \frac{b\sin\theta}{a + b\cos\theta}$$

Special Cases :

1. $\theta = 0°$

$$R = \sqrt{a^2 + b^2 + 2ab\cos 0°}$$

$$R = \sqrt{a^2 + b^2 + 2ab}$$

$$R = a + b$$

$$|\vec{\mathbf{R}}| = |\vec{\mathbf{a}}| + |\vec{\mathbf{b}}|$$

Two vectors are added like scalars if angle between them is 0, OR

Two vectors are added like scalars when they are in same direction.

2. $\theta = 90°$ or $\frac{\pi}{2}$

$$R = \sqrt{a^2 + b^2 + 2ab\cos 90°}$$

$$R = \sqrt{a^2 + b^2}$$

Range of Resultant

$$R = \sqrt{a^2 + b^2 + 2ab\cos\theta}$$

$$-1 \le \cos\theta \le 1$$

$$R_{max} = \sqrt{a^2 + b^2 + 2ab} = a + b$$

$$|\vec{\mathbf{R}}|_{max} = |\vec{\mathbf{a}}| + |\vec{\mathbf{b}}|$$

$$R_{min} = \sqrt{a^2 + b^2 - 2ab} = |a - b|$$

or $$|\vec{\mathbf{R}}|_{max} = ||\vec{\mathbf{a}}| - |\vec{\mathbf{b}}||$$

$$R_{\ max} \le R \le R_{max}$$

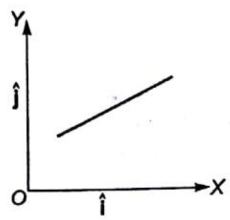
$$|a - b| \le R \le a + b$$

Vectors in Two Dimensions

Generally, any vector in two dimensions will be represented as $\vec{\mathbf{r}} = a_x\,\hat{\mathbf{i}} + a_y\,\hat{\mathbf{j}}$.

where, $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ are unit vectors, along x- and y-axes.

Where, a_x and a_y are components along both the axes.



Vectors in Three Dimensions

As a vector is represented in two dimensions, similarly, a vector can be represented in three dimensions also in the similar manner.

$$\vec{\mathbf{r}} = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}} + z\,\hat{\mathbf{k}}$$

where $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$ are unit vectors, along x , y and z-axes, respectively, to whereas x, y, z are components along axes.

If vector is given in component form, if for example $\vec{\mathbf{a}} = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{i}} + z\,\hat{\mathbf{k}}$, then its magnitude is $|\vec{\mathbf{a}}| = \sqrt{x^2 + y^2 + z^2}$, and the unit vector in the direction of $\vec{\mathbf{a}}$ is,

$$\hat{\mathbf{a}} = \frac{\vec{\mathbf{a}}}{|\vec{\mathbf{a}}|} = \frac{x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}} + z\,\hat{\mathbf{k}}}{\sqrt{x^2 + y^2 + z^2}}.$$

Addition of Two Vectors

If the two vectors are

$$\vec{\mathbf{r}}_1 = a_1\,\hat{\mathbf{i}} + b_1\,\hat{\mathbf{j}} + c_1\,\hat{\mathbf{k}}$$

$$\vec{\mathbf{r}}_2 = a_2\,\hat{\mathbf{i}} + b_2\,\hat{\mathbf{j}} + c_2\,\hat{\mathbf{k}},$$

then

$$\vec{\mathbf{r}}_1 + \vec{\mathbf{r}}_2 = (a_1\,\hat{\mathbf{i}} + b_1\,\hat{\mathbf{j}} + c_1\,\hat{\mathbf{k}}) + (a_2\,\hat{\mathbf{i}} + b_2\,\hat{\mathbf{j}} + c_2\,\hat{\mathbf{k}})$$

$$= (a_1 + a_2)\,\hat{\mathbf{i}} + (b_1 + b_2)\,\hat{\mathbf{j}} + (c_1 + c_2)\,\hat{\mathbf{k}}$$

Properties of Vector Addition

1. Addition of two vector quantity is also a vector quantity.
2. Addition of a vector quantity and a scalar quantity is not possible.
3. Zero vector is an additive identity, as there is no effect an any vector, if zero vector is added to it.

$$\vec{\mathbf{0}} + \vec{\mathbf{a}} = \vec{\mathbf{a}} + \vec{\mathbf{0}} = \vec{\mathbf{a}}$$

4. If $\vec{\mathbf{a}} + \vec{\mathbf{b}} = \vec{\mathbf{b}} + \vec{\mathbf{c}}$ then $\vec{\mathbf{a}} = \vec{\mathbf{c}}$.

Subtraction of Vectors

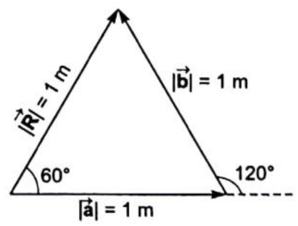
If the two vectors are $\vec{\mathbf{r}}_1 = a_1\,\hat{\mathbf{i}} + b_1\,\hat{\mathbf{j}} + c_1\,\hat{\mathbf{k}}$

$$\vec{\mathbf{r}}_2 = a_2\,\hat{\mathbf{i}} + b_2\,\hat{\mathbf{j}} + c_2\,\hat{\mathbf{k}},$$

then

$$\vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2 = (a_1\,\hat{\mathbf{i}} + b_1\,\hat{\mathbf{j}} + c_1\,\hat{\mathbf{k}}) - (a_2\,\hat{\mathbf{i}} + b_2\,\hat{\mathbf{j}} + c_2\,\hat{\mathbf{k}})$$

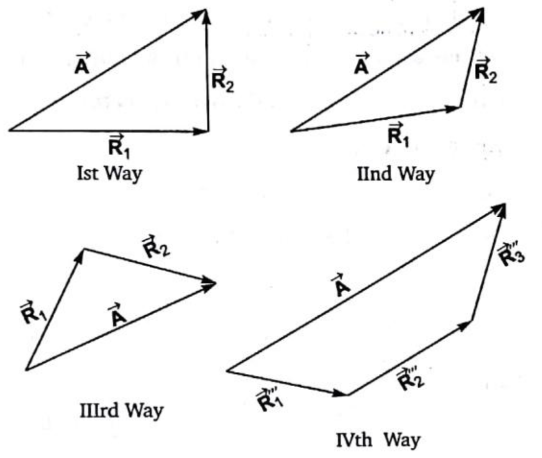
$$= (a_1 - a_2)\,\hat{\mathbf{i}} + (b_1 - b_2)\,\hat{\mathbf{j}} + (c_1 - c_2)\,\hat{\mathbf{k}}$$



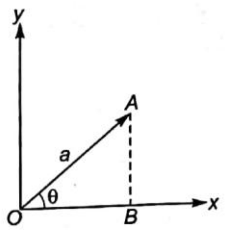
Resolution of Vectors

The process of splitting a vector into two or more components so that when these components are added vectorially, we get the original vector back, is termed as resolution of vectors. A given vector can be resolved into any number of component vectors in any number of ways.

For example, here is illustrated how a vector $\vec{\mathbf{A}}$ can be resolved in various ways ?



There are infinite ways in which a vector can be resolved, but the most useful one is to resolve a vector in two or three components along two or three mutual perpendicular directions. Here, we are considering only the case is which we resolve a vector into two components along two mutual perpendicular directions.



Consider a vector $\overrightarrow{\mathbf{OA}}$ whose magnitude is say 'a' units and is making an angle θ with positive direction of x-axis as shown. Then from trigonometry,

$$OB = OA\cos\theta = a\cos\theta$$
$$AB = OA\sin\theta = a\sin\theta$$

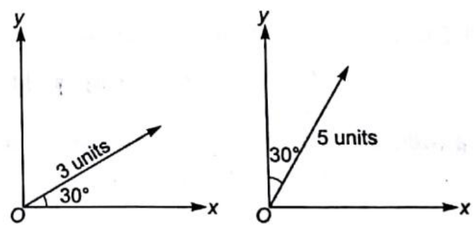
From the triangle law of vector addition, $\overrightarrow{\mathbf{OB}} + \overrightarrow{\mathbf{BA}} = \overrightarrow{\mathbf{OA}}$, so $\overrightarrow{\mathbf{OB}}$ and $\overrightarrow{\mathbf{OA}}$ are components of $\overrightarrow{\mathbf{OA}}$ along X and Y directions, respectively.

$$\overrightarrow{\mathbf{OB}} = a\cos\theta\,\hat{\mathbf{i}},$$

and $$\overrightarrow{\mathbf{BA}} = a\sin\theta\,\hat{\mathbf{j}}$$

Consider the following example

Resolve the shown vector along X and Y axes and find X and Y components of given vector.



Solution (a) $R_x = 3\cos 30°$

$R_y = 3\sin 30°$

(b) $R_x = 5\sin 30°$

$R_y = 5\cos 30°$.

Multiplication of Vectors

Just like addition of vectors, multiplication of vectors is also very different from multiplication of scalars. Multiplication of a vector can be done is three ways :

1. Multiplication of a vector with a scalar
2. Scalar product or dot product of two vectors.
3. Vector product or cross product of two vectors.

Here, we discuss only first two types of multiplication.

1. Multiplication of a Vector with a Scalar

If a vector $\vec{\mathbf{A}}$ is given as $\vec{\mathbf{A}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$ and we have to multiply it with a scalar number λ, then the new vector $\vec{\mathbf{B}} = \lambda\,\vec{\mathbf{A}}$ would be

$$\vec{\mathbf{B}} = \lambda x\,\hat{\mathbf{i}} + \lambda y\,\hat{\mathbf{j}} + \lambda z\,\hat{\mathbf{k}}.$$

$\vec{\mathbf{B}}$ is parallel to $\vec{\mathbf{A}}$, and is having the magnitude λ times the magnitude of $\vec{\mathbf{A}}$.

2. Dot Product or Scalar Product of Two Vectors

The dot product of two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ which are making an angle of θ is represented by, $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = |\vec{\mathbf{A}}| \times |\vec{\mathbf{B}}| \cos\theta$.

This dot product is a scalar.

(a) If $\theta < \frac{\pi}{2}$, then $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$ is +ve.

$\theta = \frac{\pi}{2}$, then $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$ is 0.

$\theta > \frac{\pi}{2}$, then $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$ is −ve.

(b) Dot product of two perpendicular vectors is zero.

(c) Dot product of a vector with itself is equal to square of the magnitude of vector.

$$\vec{\mathbf{a}} \cdot \vec{\mathbf{a}} = |\vec{\mathbf{a}}||\vec{\mathbf{a}}|\cos 0 = |\vec{\mathbf{a}}|^2$$

(d) $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = \vec{\mathbf{b}} \cdot \vec{\mathbf{a}}$

(e) $\hat{\mathbf{i}} \cdot \hat{\mathbf{i}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{k}} = 1$

(f) $\hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{i}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{k}} = \hat{\mathbf{i}} \cdot \hat{\mathbf{k}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{i}} = 0$

(g) It two vectors $\vec{\mathbf{A}} = a_1\,\hat{\mathbf{i}} + b_1\,\hat{\mathbf{j}} + c_1\,\hat{\mathbf{k}}$ and $\vec{\mathbf{B}} = a_2\,\hat{\mathbf{i}} + b_2\,\hat{\mathbf{j}} + c_2\,\hat{\mathbf{k}}$ are given in component form, then

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = a_1 a_2 + b_1 b_2 + c_1 c_2\ .$$

C-BIs

Concept Building Illustrations

Example | 1 *$\vec{\mathbf{a}} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$, and $\vec{\mathbf{b}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$, then find vector $\vec{\mathbf{a}} + \vec{\mathbf{b}}$.*

Solution $\vec{\mathbf{a}} + \vec{\mathbf{b}} = (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) + (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}})$

$= \hat{\mathbf{i}}(2+1) + \hat{\mathbf{j}}(-3+2) + \hat{\mathbf{k}}(4-1)$

$\vec{\mathbf{a}} + \vec{\mathbf{b}} = 3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

Example | 2 *If $\vec{\mathbf{a}} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$, and $\vec{\mathbf{b}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$. Find vector $\vec{\mathbf{a}} - \vec{\mathbf{b}}$.*

Solution $\vec{\mathbf{a}} - \vec{\mathbf{b}} = (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) - (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}})$

$= \hat{\mathbf{i}}(2-1) + \hat{\mathbf{j}}(-3-2) + \hat{\mathbf{k}}(4+1)$

$\vec{\mathbf{a}} - \vec{\mathbf{b}} = \hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$

Example | 3 *Of $\vec{\mathbf{a}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}, \vec{\mathbf{b}} = 2\hat{\mathbf{i}} - \hat{\mathbf{k}}$, and $\vec{\mathbf{c}} = 4\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$. Find vector $\vec{\mathbf{a}} + 2\vec{\mathbf{b}} - 3\vec{\mathbf{c}}$.*

Solution $\vec{\mathbf{a}} + 2\vec{\mathbf{b}} - 3\vec{\mathbf{c}} = (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) + 2(2\hat{\mathbf{i}} - \hat{\mathbf{k}}) - 3(4\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$

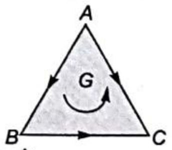
$= (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) + (4\hat{\mathbf{i}} - 2\hat{\mathbf{k}}) - (12\hat{\mathbf{j}} + 9\hat{\mathbf{k}})$

$= \hat{\mathbf{i}}(1+4) + \hat{\mathbf{j}}(1-12) + \hat{\mathbf{k}}(1-2-9)$

$\vec{\mathbf{a}} + 2\vec{\mathbf{b}} - 3\vec{\mathbf{c}} = 5\hat{\mathbf{i}} - 11\hat{\mathbf{j}} - 10\hat{\mathbf{k}}$

Example | 4 *In any ΔABC, if $\overrightarrow{\mathbf{AB}} = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\overrightarrow{\mathbf{BC}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$, then find $\overrightarrow{\mathbf{AC}}$.*

Solution By triangle rule



$\overrightarrow{\mathbf{AB}} + \overrightarrow{\mathbf{BC}} - \overrightarrow{\mathbf{AC}} = 0$

$\overrightarrow{\mathbf{AC}} = \overrightarrow{\mathbf{AB}} + \overrightarrow{\mathbf{BC}}$

$\overrightarrow{\mathbf{AC}} = (2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - \hat{\mathbf{k}}) + (\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}})$

$\overrightarrow{\mathbf{AC}} = 3\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$

Example | 5 *If position vectors of A and B are $3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$ and $2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$, then find $\overrightarrow{\mathbf{AB}}$ and $|\overrightarrow{\mathbf{AB}}|$.*

Solution Position vector of A,

$$\overrightarrow{OA} = 3\hat{i} + 4\hat{j} - 5\hat{k}$$

Position vector of B,

$$\overrightarrow{OB} = 2\hat{i} - 3\hat{j} + 4\hat{k}$$

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$

$$= (2\hat{i} - 3\hat{j} + 4\hat{k}) - (3\hat{i} + 4\hat{j} - 5\hat{k})$$

$$\overrightarrow{AB} = -\hat{i} - 7\hat{j} + 9\hat{k}$$

$$|\overrightarrow{AB}| = \sqrt{(-1)^2 + (-7)^2 + (9)^2}$$

$$= \sqrt{1 + 49 + 81}$$

$$|\overrightarrow{AB}| = \sqrt{131}$$

Example | 6 *Magnitude of vectors $\vec{a} = \lambda\hat{i} + 2\hat{j} - 3\hat{k}$, and $\vec{b} = \sqrt{\lambda}\,\hat{i} + \sqrt{13}\,\hat{j}$ are equal, then find the value of λ.*

Solution As $|\vec{a}| = |\vec{b}|$

$$|\lambda\hat{i} + 2\hat{j} - 3\hat{k}| = |\sqrt{\lambda}\,\hat{i} + \sqrt{13}\,\hat{j}|$$

$$\sqrt{\lambda^2 + 2^2 + (-3)^2} = \sqrt{\lambda + 13}$$

On squaring, $\lambda^2 + 4 + 9 = \lambda + 13$

$\lambda^2 = \lambda$ or $\lambda^2 - \lambda = 0$ or $\lambda(\lambda - 1) = 0$

$\therefore \quad \lambda = 0$ or $\lambda = 1$

Example | 7 *Find whether the two vectors $\vec{a} = 2\hat{i} - 5\hat{j} + 2\hat{k}$ and $\vec{b} = -10\hat{i} + 25\hat{j} - 10\hat{k}$ are anti-parallel or not.*

Solution As $\vec{b} = -10\hat{i} + 25\hat{j} - 10\hat{k}$

$$\vec{b} = -5(2\hat{i} - 5\hat{j} + 2\hat{k})$$

$$\vec{b} = -5\vec{a}$$

$\therefore$ $\vec{a}$ and $\vec{b}$ are anti-parallel vectors.

Example | 8 *Find the vector which is parallel to $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$, and whose magnitude is 10 units.*

Solution Let $\vec{b}$ be a vector parallel to $\vec{a}$.

Then, $\quad \vec{b} = \lambda\vec{a}$

As magnitude of $|\vec{b}| = 10$,

$$|\vec{b}| = |\lambda\vec{a}| = 10 = \lambda|\vec{a}| = 10$$

$$\Rightarrow \quad \lambda|2\hat{i} - \hat{j} + 2\hat{k}| = 10$$

$$\Rightarrow \quad \lambda\sqrt{2^2 + (-1)^2 + 2^2} = 10$$

$$\lambda \times 3 = 10$$

$$\therefore \quad \lambda = \frac{10}{3}$$

$$\therefore \quad \vec{b} = \frac{10}{3}(2\hat{i} - \hat{j} + 2\hat{k})$$

Example | 9 *If the vectors are $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$, then find $\vec{a}\cdot\vec{b}$ and also find the angle between there two vectors.*

Solution As $\vec{a} = \hat{i} + \hat{j} + \hat{k}$

$$\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$$

$$|\vec{a}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

$$|\vec{b}| = \sqrt{2^2 + (-1)^2 + 3^2} = \sqrt{14}$$

$$\vec{a}\cdot\vec{b} = (\hat{i} + \hat{j} + \hat{k})(2\hat{i} - \hat{j} + 3\hat{k})$$

$$= 1\cdot 2 + 1\cdot(-1) + 1\cdot 3 = 2 - 1 + 3 = 4$$

As $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{4}{\sqrt{3}\cdot\sqrt{14}}$$

$$\cos\theta = \frac{4}{\sqrt{42}} \Rightarrow \theta = \cos^{-1}\left(\frac{4}{\sqrt{42}}\right)$$

Concept Check 8

Exercise

1. If the two vectors are $a = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\vec{\mathbf{b}} = 3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + \hat{\mathbf{k}}$, then find $\vec{\mathbf{a}} + \vec{\mathbf{b}}$.
2. If position vector of vertices of triangle are $2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$ and $3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$, then prove that triangle is a right-angled triangle.
3. If the vectors are $\vec{\mathbf{a}} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$, $\vec{\mathbf{b}} = 2\hat{\mathbf{i}} - 5\hat{\mathbf{j}}$, $\vec{\mathbf{c}} = 3\hat{\mathbf{i}} - \hat{\mathbf{k}}$, then find
   (a) $\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}$
   (b) $\vec{\mathbf{a}} - 2\vec{\mathbf{b}}$
   (c) $3\vec{\mathbf{b}} - \vec{\mathbf{c}}$
   (d) $2\vec{\mathbf{a}} - 3\vec{\mathbf{b}} + 4\vec{\mathbf{c}}$
4. In any quadrilateral *ABCD*, if $\overrightarrow{\mathbf{AB}} = \hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}$, $\overrightarrow{\mathbf{BC}} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$, $\overrightarrow{\mathbf{CD}} = 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$, then find vector $\overrightarrow{\mathbf{AD}}$, $\overrightarrow{\mathbf{AC}}$, $\overrightarrow{\mathbf{BD}}$.
5. If the two vectors $\vec{\mathbf{a}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 7\hat{\mathbf{k}}$ and $\vec{\mathbf{b}} = \frac{3}{2}\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \frac{21}{2}\hat{\mathbf{k}}$, then state that whether the two vectors are parallel or not.
6. Find the vector which is parallel to vector $\vec{\mathbf{a}} = 3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$ and whose magnitude is 20 units.
7. If $\vec{\mathbf{a}} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $\vec{\mathbf{b}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$, then find $|\vec{\mathbf{a}} + \vec{\mathbf{b}}|$.
8. If θ be the angle inclined between the vectors
   $\vec{\mathbf{a}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $\vec{\mathbf{b}} = 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$,
   then find $\sin\theta$.
9. If $|\vec{\mathbf{a}} + \vec{\mathbf{b}}| = |\vec{\mathbf{a}} - \vec{\mathbf{b}}|$, then find the angle between the them vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$.
10. If $\vec{\mathbf{a}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$, $\vec{\mathbf{b}} = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\vec{\mathbf{c}} = 2\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + \hat{\mathbf{k}}$, then find $(\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})\vec{\mathbf{c}}$.

E. Differential Calculus

Function

If there are two physical quantity such that one physical quantity depends on other quantity, then a function has to be defined.

Example | 1 Area of circle $A = \pi r^2$

As the radius will change then due to this change in radius, area of circle also changes. Here r is independent physical quantity while A is dependent physical quantity which depends on 1st quantity *ie*, r.

Here exist a function denoted by $A = f(r)$.

If any function, $y = f(x)$ exist then it means that as the value of x will change then value of y will also change.

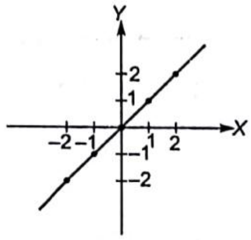
The change in physical quantity is represented by symbol Δ (abbreviated as delta). For *eg*, if speed of a particle changes from v_1 to v_2 then change in speed of this particle is Δv, which is equal to $v_2 - v_1$, *ie*, $\Delta u = v_2 - v_1$. Always remember that change in physical quantity is always find value minus initial. If the physical quantity is decreasing with time then the change is physical qty is – ve otherwise + ve.

Graph of a Function

To plot the graph of any function, first we have to find different values of y for corresponding different value of x and then plot these points (x, y) on coordinate axis and at last join these points to form a smooth curve.

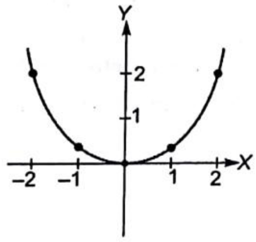
Here, for your quick reference we are providing you some basic graphs.

1. For $y = x$



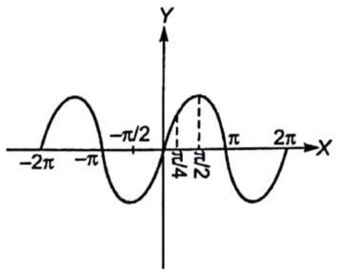
x	1	0	2	−1	−2	3
y	1	0	2	−1	−2	3

2. For $y = x^2$



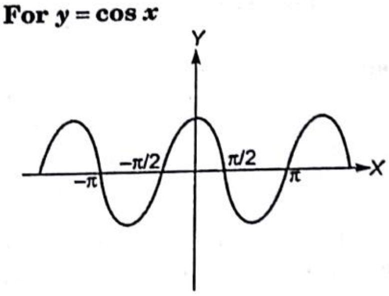
x	0	1	−1	2	−2
y	0	1	1	4	4

3. For $y = \sin x$



x	0	$\pi/4$	$\pi/2$	π	$-\pi$	$-\pi/2$	$-\pi/4$
y	0	$1/\sqrt{2}$	1	0	0	−1	$-1/\sqrt{2}$

4. For $y = \cos x$

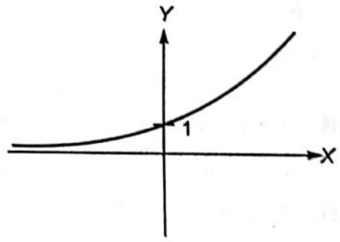


x	0	$\pi/2$	π	$-\pi/2$	$-\pi$	$3\pi/2$
y	1	0	−1	0	−1	0

5. Exponential function

$$y = a^x$$

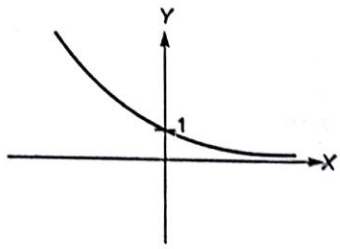
(*i*) *If* $a > 1$ $y = 2^x$



x	0	1	2	−1	−2	3
y	1	2	4	1/2	1/4	8

(*ii*) $0 < a < 1$

$$y = \left(\frac{1}{2}\right)^x$$



x	0	1	2	−1	−2	−3
y	1	1/2	1/4	2	4	8

Differentiation

Differentiation literally means "to break". Let us first look into the mathematical aspect of differentiation. Consider any function $y = f(x)$. In this function x is the independent variable and y is the dependent variable *ie*, y is depending on x, so it is very clear that as x changes, the value of y also changes.

Let x changes by a very small amount say Δx and as a result the corresponding change in y is Δy.

For example, consider a function $y = 3x$, then as x changes from 1.0 to 1.0001 *ie*, $\Delta x = 1.0001 - 1.0 = 0.0001$ the value of y changes from 3.0 to 3.0003 *ie*, $\Delta y = 3.0003 - 3 = 0.0003$.

Although Δy and Δx both are very small but the ratio $\frac{\Delta y}{\Delta x}$ is having a finite considerable value, in above example, it is $\frac{0.0003}{0.0001} = 3$.

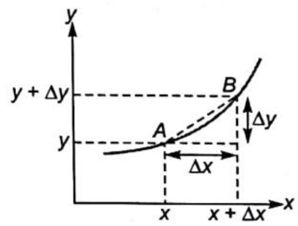
The ratio, change in y to corresponding change in x when Δx is very small or more appropriately we can say when Δx tends to (approaches to) zero is termed as derivative of y *wrt* x. Derivative of y *wrt* $x = \frac{\Delta y}{\Delta x}$ when Δx approaches to 0. It is also termed as differentiation of y *wrt* x or rate of change of y *wrt* x. It is represented by $\frac{dy}{dx}$.

So, $\frac{dy}{dx} = \frac{\Delta y}{\Delta x}$ as Δx approaches zero.

$$= \frac{f(x + \Delta x) - f(x)}{\Delta x},$$

as Δx approaches zero.

Physically we can interpret differentiation as slope of the tangent drawn to the curve at a specified point. Consider $y = f(x)$ whose plot is shown in figure. Two points $A[x, y]$ and $B[x + \Delta x, y + \Delta y]$ are marked on the curve. If we join these two points by a straight line, then $\frac{\Delta y}{\Delta x}$ represents the slope of this line. If we bring the point B closer to A along the curve *ie*, as Δx decreases we are approaching towards A to get the tangent drawn at point A and $\frac{dy}{dx} \cdot \frac{\Delta y}{\Delta x}$ for $\Delta x \to 0$ represent the slope of this tangent.



Some Basic Formulae of Differentiation

1. $\frac{d}{dx}(x^n) = nx^{n-1}$
2. $\frac{d}{dx}(e^x) = e^x$
3. $\frac{d}{dx}(a^x) = a^x \log_e a$
4. $\frac{d}{dx}(\log_e x) = \frac{1}{|x|}$ *(As x can't be negative)*
5. $\frac{d}{dx}(\sin x) = \cos x$
6. $\frac{d}{dx}(\cos x) = -\sin x$
7. $\frac{d}{dx}\tan x = \sec^2 x$
8. $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$
9. $\frac{d}{dx}(\sec x) = \sec x \cdot \tan x$
10. $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cdot \cot x$
11. $\frac{d}{dx}(c) = 0$ (c is constant)
12. $\frac{d}{dx}(cx^n) = c \cdot \frac{d}{dx}(x^n) = cn\,x^{n-1}$

Product Rule

If y is a product of two function f and g which are both functions of x, then the differentiation of y wrt x can be found by using *product rule*.

If $\quad y = f(x) \cdot g(x),$

then $\quad \dfrac{dy}{dx} = f(x) \cdot \dfrac{d}{dx}(g(x)) + g(x) \cdot \dfrac{d}{dx}(f(x))$

= Ist function × Differentiation of IInd
+ IInd function × Differentiation of Ist

Quotient Rule

If any function is in division with another function, then its differentiation is done by using *quotient rule*.

$$y = \frac{f(x)}{g(x)}$$

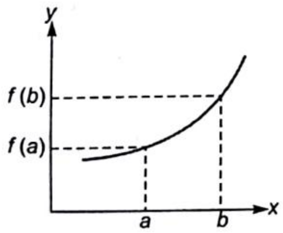
$$\frac{dy}{dx} = \frac{g(x) \cdot \frac{d}{dx}(f(x)) - f(x) \cdot \frac{d}{dx}(g(x))}{[g(x)]^2}$$

Increasing and Decreasing Functions

Increasing Functions

In any function $y = f(x)$, if we increase the value of x and there is corresponding increase in value of y and *vice-versa*, then the function is said to be increasing.

For shown curve x is increasing from a to b so y increases from $f(a)$ to $f(b)$, hence, function is increasing.



If difference between a and b is very small,

let $\quad b - a = \Delta x,$

and $\quad f(b) - f(a) = \Delta y,$

then $\quad \dfrac{f(b) - f(a)}{b - a} = \dfrac{\Delta y}{\Delta x},$

and $\dfrac{\Delta y}{\Delta x}$ is always positive as $b > a$

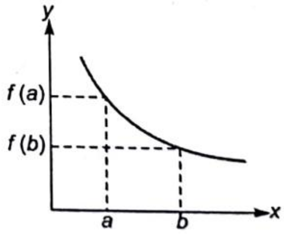
and $\quad f(b) > f(a).$

∴ Function is increasing if differentiation of that function is positive as *ie*, $\dfrac{dy}{dx} > 0.$

Decreasing Functions

If we increase the value of x and there is corresponding decrease in value of y and vice versa, then the function is said to be decreasing.

According to curve if x increases from $a \to b$, then y decreases from $f(a) \to f(b)$.



Or, we can say that function is decreasing if its differentiation is negative *ie*,

$$\frac{dy}{dx} < 0$$

dy/dx as Rate Measure

If a variable quantity x is some function of time *ie*, $x = f(t)$ then small change in time Δt will have corresponding small change in x *ie*, Δx. So, the average rate of change is $\frac{\Delta x}{\Delta t}$.

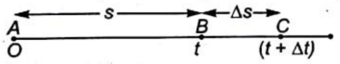
If Δt is very small so that $\Delta t \to 0$, then the rate of change becomes instantaneous and we get the rate of change with respect to time at any instant.

So, it is clear that time rate of change of any variable quantity is derivative of it *wrt* time.

Velocity and Acceleration

Students of science are very familiar with the terms displacement, velocity and acceleration. Velocity of a moving particle is defined as the rate of change of displacement with respect to time.

Acceleration is defined as the rate of change of velocity with respect to time.



A man starts from a fixed point A and reaches to point B which is at distance 's' from A in time t, and then reaches to point C which is at distance of Δs from point B in time Δt then velocity at $B = v = \frac{ds}{dt} = \frac{\Delta s}{\Delta t}$ as $\Delta t \to 0$.

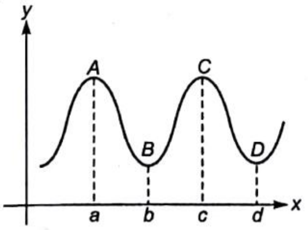
Acceleration at $B = a = \frac{dv}{dt}$ as $\Delta t \to 0$

Maxima and Minima

Maxima and Minima will exist only in those functions whose curves are having crests and troughs.

The point of crest is the point of maxima while the trough is point of minima.

In the above curves, there are two crests *ie*, A and C, so these are two maxima while in the curve there are two trough, so there are two minimas, *ie*, at B and D.



Method to determine the point of Maxima or Minima

For any given function $= f(x)$

(i) Differentiate the given function and get $\frac{dy}{dx}$.

(ii) Put $\frac{dy}{dx} = 0$ and find the values of x. Let one root of this equation be $x = a$.

(iii) Now again differentiate the function

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$$

(iv) Now, put the roots of equation $\frac{dy}{dx} = 0$ *ie*, $x = a$ in double differentiation.

(v) If $\frac{d^2y(a)}{dx^2} < 0$, then $x = a$ is the point of maxima

(vi) If $\frac{d^2y(a)}{dx^2} > 0$, then $x = a$ is a point of minima.

C-BIs

Concept Building Illustrations

Example | 1 *Solve for x and y*

$$(x-1)^2+(y-3)^2=0$$

Solution Sum of two square terms are zero if both terms are simultaneously zero.

So, $(x-1)^2=0$ and $(y-3)^2=0$

$x=1$, and $y=3$

Example | 2 *Solve for x*

$$(x-1)^2+x^2=0$$

Solution Both terms should be simultaneously zero.

So, $(x-1)^2=0$ and $x^2=0$

$x=1$ and $x=0$

If $x=1$, then the second terms, *ie*, x^2 is not zero.

If $x=0$ $x=0$, then the first term *ie*, $(x-1)^2$ is not zero, there is no value of x where both the terms are simultaneously zero.

So, no solution.

Example | 3 *Given a function* $f(x)=x^3-2$

Find $f(1), f(0), f(a)$.

Solution $f(x)=x^3-2$

$$f(1)=1^3-2=-1$$
$$f(0)=0^3-2=-2$$
$$f(a)=a^3-2$$

Example | 4 *Let* $f(x)=\begin{cases}3^{x-1} & -1\le x<0\\ |x-2| & 0\le x<1\\ 3x-1 & 1\le x\le 3\end{cases}$

Find $f(-1), f(0), f(1/2)$ and $f(1)$

Solution According to limits which are given in the question,

(i) $x=-1$ lies in the first interval

$$f(-1)=3^{-1-1}=3^{-2}=\frac{1}{3^2}=\frac{1}{9}$$

(ii) $x=0$ lies in the second interval

$$F(0)=|0-2|=2$$

(iii) $x=\frac{1}{2}$ lies in the second interval

$$f(1/2)=|1/2-2|=\frac{3}{2}$$

(iv) $x=1$ lies in the fourth interval

$$f(1)=3\times 1-1=2$$

Example | 5 *Differentiate the following wrt x.*

(*i*) $y=\sqrt{x}$ (*ii*) $y=\dfrac{x+1}{x-2}$

(*iii*) $y=\sin^2 x$ (*vi*) $y=xe^x$

(*v*) $y=\sqrt{\tan x}$

Solution

(i) $y=x^{1/2}$

$$\frac{dy}{dx}=\frac{1}{2}x^{\frac{1}{2}-1}\left(\text{By } \frac{d}{dx}x^n=nx^{n-1}\right)=\frac{1}{2}x^{-1/2}$$

(ii) $y=\dfrac{x+1}{x-2}$

From quotient rule,

$$\frac{dy}{dx}=\frac{(x-2)\cdot\frac{d}{dx}(x+1)-(x+1)\frac{d}{dx}(x-2)}{(x-2)^2}$$

$$=\frac{(x-2)\cdot 1-(x+1)\cdot 1}{(x-2)^2}$$

$$\frac{dy}{dx}=\frac{-3}{(x-2)^2}$$

(iii) $y=\sin^2 x$

Let $t=\sin x$

$y=t^2$

$\Rightarrow$ $\dfrac{dy}{dt}=2t$ ...(i)

As $t=\sin x$

$\Rightarrow$ $\dfrac{dt}{dx}=\cos x$ ...(ii)

From Eqs. (i) and (ii)

$$\frac{dy}{dx}=\frac{dy}{dt}\cdot\frac{dt}{dx}$$

(vi) $y = xe^x$

By product rule

$$\frac{dy}{dx} = x \cdot \frac{d}{dx}(e^x) + e^x \cdot \frac{d}{dx}(x)$$

$$\frac{dy}{dx} = xe^x + e^x$$

(v) $y = x^2 \sin x$

$$\frac{dy}{dx} = x^2 \frac{d}{dx}(\sin x) + \sin x \cdot \frac{d}{dx}(x^2)$$

$$\frac{dy}{dx} = x^2 \cos x + 2x \sin x$$

Example | 6 *Differentiate* $\cos x$ *wrt* $\sin x$.

Solution Let $u = \cos x$ and $v = \sin x$

So, $\frac{du}{dv}$ is differentiation of $\cos x$ *wrt* $\sin x$.

$$\frac{du}{dx} = -\sin x \text{ and } \frac{dv}{dx} = \cos x$$

$$\frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{-\sin x}{\cos x}$$

$$\frac{du}{dv} = -\tan x$$

Example | 7 *Find* $\frac{d^2y}{dx^2}$ *if*

(*i*) $y = \cos x$

(*ii*) $y = x \sin x$

Solution

(i) $y = \cos x$

$$\frac{dy}{dx} = -\sin x$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = -\cos x$$

(ii) $y = x \sin x$

By product rule,

$$\frac{dy}{dx} = x \cos x + \sin x$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(x \cos x) = \frac{d}{dx}(\sin x)$$

$$= -x \sin x + \cos x + \cos x$$

$$\frac{d^2y}{dx^2} = -x \sin x + \cos 2x$$

Example | 8 *Find the interval for increase or decrease of following functions :*

(*i*) $y = x^3 + 3x$ (*ii*) $y = 2x - 1$

(*iii*) $y = x^2 - 2x$ (*iv*) $y = \log_e x$

Solution

(i) $y = x^3 + 3x$

$$\frac{dy}{dx} = 3x^2 + 3 = 3(x^2 + 1)$$

As $x^2 + 1$ is always positive, so $\frac{dy}{dx} > 0$.

$\therefore$ Function is increasing for all values of x.

(ii) $y = 2x - 1$

$$\frac{dy}{dx} = 2 \text{ (which is always positive)}$$

$$\therefore \quad \frac{dy}{dx} > 0$$

So, function is always increasing.

(iii) $y = x^2 - 2x$

$$\frac{dy}{dx} = 2x - 2 = 2(x - 1)$$

The function is increasing if $\frac{dy}{dx} > 0$.

$$2(x-1) > 0 \Rightarrow x > 1$$

Increasing if $x \in (1, \infty)$.

Similarly, decreasing $x \in (-\infty, 1)$.

(iv) $y = \log_e x$

$$\frac{dy}{dx} = \frac{1}{|x|} \text{ (which is always positive)}$$

$\therefore$ Increasing for all x.

Example | 9 *Let the radius of, circle be increasing at uniform rate of 2 cms^{-1}. Find the rate of increase of area and perimeter when radius is 20 cm.*

Solution Area $A = \pi r^2$

$$\frac{dA}{dt} = \pi \cdot \frac{d(r^2)}{dt}$$

$$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$$

$$= 2\pi \times 20 \times 2$$

$$\frac{dA}{dt} = 80\pi \text{ cm}^2/\text{s}.$$

Perimeter, $P = 2\pi r$

$$\frac{dp}{dt} = 2\pi \frac{dr}{dt}$$

$$= 2\pi \times 20$$

$$\frac{dp}{dt} = 40\pi \text{ cms}^{-1}$$

Example | 10 *If displacement is given by $s = (t^3 + 6t^2 + 5t + 6)$ m, then find velocity and acceleration at $t = 5$ s.*

Solution $s = t^3 + 6t^2 + 5t + 6$

$$\text{Velocity, } v = \frac{ds}{dt}$$

$$v = 3t^2 + 12t + 5$$

At $t = 5$ s, $v = 3.5^2 + 12.5 + 5 = 140$ m

$$\text{Acceleration, } a = \frac{dv}{dt}.$$

$$a = 6t + 12$$

At $t = 5$, $a = 6.5 + 12 = 42$ m

Example | 11 *Find the points of maxima and minima of following functions :*

(*i*) $x^3 - 6x^2 + 9x + 15$

(*ii*) $2x^3 - 15x^2 + 36x + 10$

Solution

(i) $y = x^3 - 6x^2 + 9x + 15$

$$\frac{dy}{dx} = 3x^2 - 12x + 9$$

$$\text{Put } \frac{dy}{dx} = 0 \Rightarrow x^2 - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

$$x = 1, x = 3$$

$$\frac{d^2y}{dx^2} = 6x - 12$$

$$\frac{d^2y}{dx^2} \text{ at } x = 1 \Rightarrow 6 \times 1 - 12 = -6$$

$\therefore$ $x = 1$ is, point of maxima.

$$\frac{d^2y}{dx^2} \text{ at } x = 3 \text{ is } 6 \times 3 - 12 = 6$$

$\therefore$ $x = 3$ is a point of minima.

(ii) $y = 2x^3 - 15x^2 + 36x + 10$

$$\frac{dy}{dx} = 6x^2 - 30x + 36 = 0$$

$$(x-3)(x-2) = 0$$

$$x = 2, 3$$

$$\frac{d^2y}{dx^2} = 12x - 30$$

$$\text{at } x = 2 \Rightarrow \frac{d^2y}{dx^2} = 12 \times 2 - 30 = -6$$

$\therefore$ $x = 2$ is a point of maxima.

$$\text{at } x = 3 \Rightarrow \frac{d^2y}{dx^2} = 12 \times 3 - 30 = +6$$

$\therefore$ $x = 3$ is a point of minima.

Concept Check 9

Exercise

1. Solve for x and y
   (i) $|x| + |y - 1| = 0$
   (ii) $|x^2 - 1| + |y^2 + 1| = 0$
2. Solve for x
   (i) $|x + 1| + \sqrt{x - 1} = 0$
   (ii) $|x|^2 - 5|x| + 6 = 0$
   (iii) $|x|^2 + 5|x| + 6 = 0$
3. If $f(x) = \log_a\left(\frac{1}{x}\right)$. Find $f(a^3)$ and $f(a^{-1/3})$
4. If $f(x) = 2x\sqrt{1 - x^2}$, find $f(\sin x/2)$
5. If $f(x) = \frac{1}{x}$, then statement
   $f(a) - f(b) = f\left(\frac{ab}{a-b}\right)$ is True/False
6. If $f(x) = \begin{cases} |3x - 1| & x > 3 \\ e^{2x+1} & -2 \le x \le 3, \\ x^2 - 2 & x \le -2 \end{cases}$ then
   find $f(-2)$, $f(3)$, $f(-3)$.
7. Differentiate the following
   (i) $y = e^x \tan x + x \log_e x$
   (ii) $y = \frac{2x - 3}{3x - 4}$
   (iii) $y = \frac{xe^x}{\tan x}$
   (iv) $y = (ax^2 + bx + c)^2$
   (ix) $y = x + e^x + \log_e x$
8. Find $\frac{d^2y}{dx^2}$ of following functions.
   (i) $y = x^2 + \cos x$
   (ii) $y = e^x(x + \tan x)$

9. Find the interval of increase and decrease of following functions.
   (i) $y = x^2 + x + 1$
   (ii) $y = 2x^3 + 15x^2 + 36x + 5$
   (iii) $y = x^3 + 3x$
10. A circular wave in a lake expands, so that the circumference increases at the rate of a cms^{-1}. Find the rate of increase in radius.
11. The area of an equilateral triangle is increasing at a constant rate of $\sqrt{3}$ sq cms^{-1}. At what rate is its side increasing if side is 2 cm? Also find the rate of increase of its altitude.
12. If displacement is given by $s = (t^2 + 8\sqrt{t})$ m, than find the velocity and acceleration at $t = 4$.
13. If $s = \frac{t^3}{3} - 16t$, than find acceleration at the time when velocity vanishes.
14. Find the point of Maxima or Minima of the following functions.
   (i) $y = x(x-1)^2$
   (ii) $y = x^2 - 3x^{2/3}$
   (iii) $y = x^2 + x + 1$
   (iv) $y = 2x^3 - 6x^2 - 18x + 7$

F. Integral Calculus

Integration

Integration literally means to integrate *ie*, to join. Integration is the reverse process of differentiation and thus we can say if we differentiate a given function, and after that if we integrate it, then finally we will get the original function.

Let $\quad y = f(x)$

$\Rightarrow \quad \frac{dy}{dx} = \frac{d}{dx}[f(x)] = F(x)$

Thus, integration of $F(x)$ would be $f(x)$

$\Rightarrow \quad dy = F(x)\,dx = d[f(x)]$

Integration is represented symbolically by $\int$.

$$\int F(x)\,dx = \int d[f(x)] = f(x) + C$$

The first term in above equation is read as integral of $F(x)$ *wrt* x as variable. In last term, the 'C' is any arbitrary constant which arises because of the fact that differentiation of a constant is zero and integration is reverse of differentiation.

Let $\frac{d}{dx}[f(x)] = F(x)$, and also

$$\frac{d}{dx}[f(x) + c] = F(x)$$

$\Rightarrow \int F(x)\,dx = f(x)$ and $\int F(x)\,dx = f(x) + C$

From above it is clear that it is necessary to add a constant c while doing the integration. What would be its value, the details of this we can skip right now.

Consider the following example :

$$\frac{d}{dx}(\sin x) = \cos x$$

$\Rightarrow \quad d(\sin x) = \cos x\,dx$

$\Rightarrow \quad \int \cos x\,dx = \sin x + C$

Standard Formulae for Integration

1. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$
2. $\int \frac{1}{x}\,dx = \log_e|x| + C$
3. $\int e^x\,dx = e^x + C$
4. $\int a^x\,dx = \frac{a^x}{\log_e a} + C$
5. $\int \sin x\,dx = -\cos x + C$
6. $\int \cos x\,dx = \sin x + C$
7. $\int \sec^2 x\,dx = \tan x + C$

8. $\int \operatorname{cosec}^2 x\, dx = -\cot x + C$
9. $\int \sec x \tan x\, dx = \sec x + C$
10. $\int \operatorname{cosec} x \cot x\, dx = -\operatorname{cosec} x + C$
11. $\int kf(x)\, dx = k\int f(x)\, dx$
12. $\int \{af_1(x) \pm bf_2(x)\}\, dx = a\int f_1(x)\, dx \pm b\int f_2(x)\, dx$

Definite Integral

Integration is just inverse of differentiation and having constant of integration, while in definite integration there is no constant of integration and this integral is always evaluated between two limits known as lower and upper limits.

1. $\int f(x)\, dx = \phi(x) + C$

If we consider the limits on this integral as a to b, then $\int_a^b f(x)\, dx = [\phi(x) + C]_a^b$

$$= (\phi(b) + C) - (\phi(a) + C)$$

$$\int_a^b f(x)\, dx = \phi(b) - \phi(a)$$

Consider the following example :

Evaluate $\int_2^3 x^2 dx$

$$\Rightarrow \quad \int_2^3 x^2\, dx = \left[\frac{x^3}{3}\right]_2^3$$

$$= \frac{1}{3}[3^3 - 2^3] = \frac{19}{3}$$

C-BIs

Concept Building Illustrations

Example | 1 *Integrate the following functions wrt x*

(i) $\int \frac{ax^3 + bx^2 + cx}{x} dx$

(ii) $\int x^{4/3} dx$

(iii) $\int \frac{4 + 3\sin x}{\cos^2 x} dx$

(iv) $\int \frac{\sin 4x}{\cos 2x} dx$

(v) $\int (\tan x + \cot x)^2 dx$

Solution

(i) $\int \frac{ax^3 + bx^2 + cx}{x} dx$

$$\Rightarrow \quad \int \left(\frac{ax^3}{x} + \frac{bx^2}{x} + \frac{cx}{x}\right) dx$$

$$\Rightarrow \quad \int ax^2 dx + \int bx\, dx + \int c\, dx$$

$$\Rightarrow \quad \frac{ax^3}{3} + \frac{bx^2}{2} + cx + d$$

(ii) $\int x^{4/3} dx$

$$\Rightarrow \quad \frac{x^{\frac{4}{3}+1}}{\frac{4}{3}+1} + C$$

$$\Rightarrow \quad \frac{x^{\frac{7}{3}}}{\frac{7}{3}} + C \Rightarrow \frac{3x^{\frac{7}{3}}}{7} + C$$

(iii) $\int \frac{4 + 3\sin x}{\cos^2 x} dx$

$$\int (4\sec^2 x + 3\tan x \sec x)\, dx$$

$$4\int \sec^2 x\, dx + 3\int \tan x \sec x\, dx$$

$$4\tan x + 3\sec x + C$$

(vi) $\int \frac{\sin 4x}{\cos 2x} dx \Rightarrow \int \frac{2\sin 2x \cos 2x}{\cos 2x} dx$

$$\Rightarrow \quad 2\int \sin 2x\, dx$$

$$= 2 \times -\frac{\cos 2x}{2} + C \Rightarrow -\cos 2x + C$$

(v) $\int (\tan x + \cot x)^2 dx$

$$\int \tan^2 x + \cot^2 x + 2\tan x \cdot \cot x\, dx$$

$$\int (\tan^2 x + \cot^2 x + 2)\, dx$$

$$\int (1 + \tan^2 x) + (1 + \cot^2 x)\, dx$$

$$\int (\sec^2 x + \operatorname{cosec}^2 x)\, dx$$

$$\int \sec^2 x\, dx + \int \operatorname{cosec}^2 x\, dx$$

$$\Rightarrow \quad \tan x - \cot x + c$$

Example | 2 *Evaluate*

(*i*) $\int_0^2 x^2 dx$

(*ii*) $\int_{-2}^2 x^3 dx$

(*iii*) $\int_{\pi/6}^{\pi/4} (\tan x + \cot x)^2 dx$

(*iv*) $\int_1^{10} \left(x + \frac{1}{x}\right) dx$

Solution

(i) $\int_0^2 x^2 dx$

$$\Rightarrow \left[\frac{x^3}{3} + c\right]_0^2 = \left(\frac{2^3}{3} + c\right) - \left(\frac{0}{3} + c\right) = \frac{8}{3}$$

(ii) $\int_{-2}^2 x^3 dx$

$$\left[\frac{x^4}{4}\right]_{-2}^2 = \left(\frac{2}{4}\right)^4 - \left(\frac{(-2)^4}{4}\right) = \frac{2^4}{4} - \frac{2^4}{4} = 0$$

(iii) $\int_{\pi/6}^{\pi/4} (\tan x + \cot x)^2 dx$

$$\int_{\pi/6}^{\pi/4} \tan^2 x + \cot^2 x + 2 \tan x \cdot \cot x \, dx$$

$$\int_{\pi/6}^{\pi/4} \tan^2 x + \cot^2 x + 2 \, dx$$

$$\int_{\pi/6}^{\pi/4} \tan^2 x + \cot^2 x + 2 \, dx$$

$$\int_{\pi/6}^{\pi/4} (1 + \tan^2 x) + (1 + \cot^2 x) dx$$

$$\int_{\pi/6}^{\pi/4} (\sec^2 x + \text{cosec}^2 x) dx$$

$$[\tan x - \cot x]_{\pi/6}^{\pi/4}$$

$$\Rightarrow \left(\tan\frac{\pi}{4} - \cot\frac{\pi}{4}\right) - \left(\tan\frac{\pi}{6} - \cot\frac{\pi}{6}\right)$$

$$\Rightarrow (1 - 1) - \left(\frac{1}{\sqrt{3}} - \sqrt{3}\right) = \sqrt{3} - \frac{1}{\sqrt{3}}$$

(iv) $\int_1^{10} \left(x + \frac{1}{x}\right) dx$

$$[x^2 + \log_2 x]_1^{10}$$

$$\Rightarrow (10^2 + \log_e 10) - (1^2 + \log_e 1)$$

$$\Rightarrow 100 + \log_e 10 - 1$$

$$\Rightarrow 99 + \log_e 10$$

Concept Check 10

Exercise

1. Integrate the following function

(i) $\int \left(a\sqrt{x} + \frac{b}{\sqrt{x}}\right)^2 dx$

(ii) $\int \frac{(ax + b)^3}{x} dx$

(iii) $\int \frac{2\sin^2 x + 3\cos^2 x}{\sin^2 x \cdot \cos^2 x} dx$

(iv) $\int \frac{\sin^3 x + 5\cos^3 x}{\sin^2 x \cdot \cos^2 x} dx$

2. Evaluate

$$\int_0^2 \left(\frac{2}{3}x + 1\right) dx$$

Workout 1

Section I

A. Subjective Questions

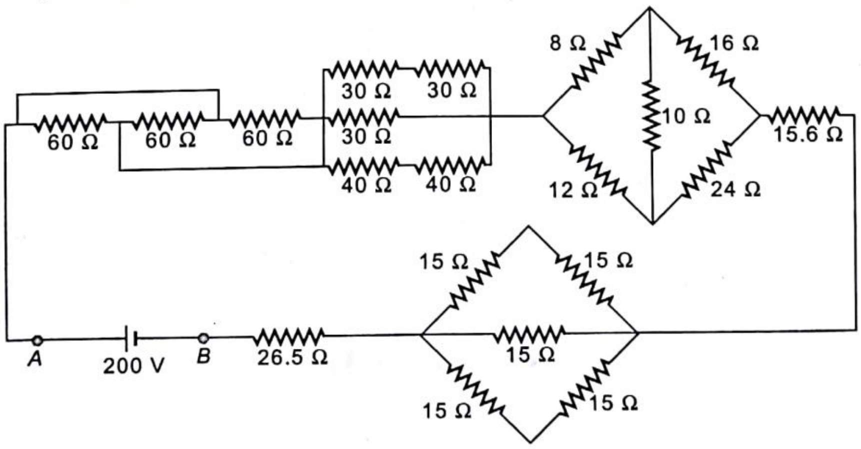
1. Two copper spheres of radii $2R$ and $4R$ kept in contact with each other exert a force of 24 N on each other. If now the radii of both the spheres are changed to $3R$ and $6R$, then find the force which they exert on each other if they are still in contact.

2. A ray of light is incident normally on one face of an equiangular prism. If the emergent ray makes an angle of 30° with the incident ray then find the critical angle and velocity of light in the prism. If the second face of the prism is now silvered, then find the angle of deviation.

3. A ball is dropped from a tower of height 80 m. After it covers half of the distance the force of gravity ceases to act. Then with what velocity it will strike the ground ? Also find the total time the ball takes to reach the ground. ($g = 10 \text{ ms}^{-2}$).

4. A bungalow uses 2 AC's each of power 800 W for 5 hrs a day, 6 tube lights each of power 40 W for 5 hrs a day, 2 fans each of power 150 W for 6 hrs a day. If the electric bill costs 5 Rs per unit then find the electric bill for the month of February 2010.

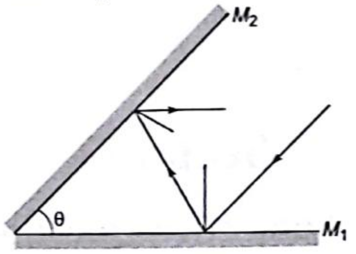
5. In the given figure, find the total resistance between the points A and B.



6. Two friends are standing at a point A, a distance of x metre from a cliff. The first friend fires a shot and moves towards the cliff simultaneously with a certain speed and hears an echo at a distance of y metre. The second friend also fires a shot but simultaneously moves away from the cliff at the same speed as his friend, and hears an echo at a distance z metre from the cliff. Show that $\dfrac{x-y}{x+y} = \dfrac{z-x}{z+x}$.

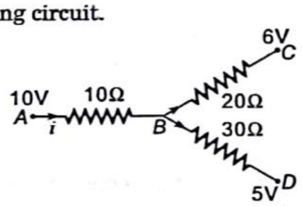
7. A body weighs 200 N on the earth's surface. What will be its weight on a celestial body whose mass is 16 times that of the earth and diameter is double that of the earth?

8. Two plane mirrors are inclined at an angle of θ with each other as shown in the figure. Now a ray of light is incident making an angle of $50°$ with the surface of M_1, and after final reflection from the 2nd mirror comes parallel to M_1. Find the value of θ.

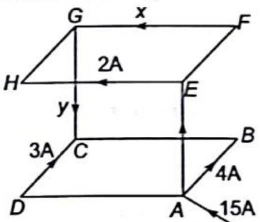


9. A body is thrown upwards with an initial velocity u and simultaneously another body is dropped from a height h along the same line. Prove that they will meet each other after a time of $t = \frac{h}{u}$ seconds.

10. Find current i in the following circuit.



11. In the given circuit, find the values of x and y.



12. A body is projected with an initial velocity of 30 ms^{-1} at an angle of $30°$ with the horizontal from the ground. Find the maximum height reached by the body. Also find the horizontal range, and its velocity after 1 s.

Section II

B. Fill in the Blanks

13. The value of universal gravitational constant (G) is ....

14. Dimensional formula for refractive index is ....

15. If a body covers one-third of a distance with an uniform speed of 20 ms^{-1}, next one-third of the distance with 30 ms^{-1} and the last one-third of the distance with 60 ms^{-1}, then the average speed of the body for the total journey is ....

Section III

C. Only One Option Correct

16. Focal length of a convex lens is 16 cm. It is dipped into water. The refractive indices of the material of lens and water are 1.5 and 1.33, respectively. The new focal length will be
(a) 64 cm (b) 18 cm (c) 24.24 cm (d) 16 cm

17. Which of the following four statements is false ?
(a) A body can have zero velocity and still be accelerated.
(b) A body can have a constant velocity and still have a varying speed.
(c) A body can have a constant speed and still have a varying velocity.
(d) The direction of the velocity of a body can change when its acceleration is constant.

18. The mass of three wires of copper are in the ratio 1 : 3 : 5 and lengths are in the ratio 5 : 3 : 1. Then the ratio of their electrical resistances are
(a) 1:3:5 (b) 5:3:1 (c) 1:15:25 (d) 125:15:1

19. The speed of sound in a medium depends upon
(a) the elastic property, but not on the inertia property
(b) the inertia property, but not on the elastic property
(c) the elastic property, as well as the property of inertia
(d) neither the elastic property nor the property of inertia

20. A convex lens having both surface of radius of curvatures as 20 cm is made of a material of refractive index 1.5. Now find out which of the following statements are true?
(a) Power of the convex lens is + 5D.
(b) For an object distance of 10 cm, a virtual erect and twice magnified image is formed.
(c) For an object distance of 15 cm, a virtual erect and twice magnified image is formed.
(d) For an object distance of 30 cm, a real inverted and twice magnified image is formed.

Answers

1. $\frac{32}{3}$ N **2.** 30°, $\frac{C}{2}$, 150° **3.** $20\sqrt{2}$ m/s, $3\sqrt{2}$ s **4.** Rs 1540 **5.** 100 Ω
7. 800 N **8.** 50° **10.** 0.2 A **11.** $x = 6A$, $y = 8A$
12. $H = 11.25$ m, $R = 45\sqrt{3}$ m, $10\sqrt{7}$ m/s **13.** 6.67×10^{-11} Nm^2/kg^2 **14.** Dimensionless **15.** 30 m/s
16. (a) **17.** (b) **18.** (d) **19.** (c) **20.** (a)

Workout 2

Section I

A. Subjective Questions

1. Particles A and B are free to move along parallel paths. B is moving at a constant velocity of 20 ms^{-1} and A starts moving at the moment B passes it. If A accelerates at 2 ms^{-2} for 5 s and then travels at a constant velocity, then find the distance between A and B 10s after B passes A.
2. A ball travels 4 m in the first 2 s and 4.4 m in the next 4 s. What will be the velocity of ball at the end of seventh second from start ?
3. A body starting from rest travels $\frac{7}{16}$ of the total distance in the last second. If the acceleration is 2 ms^{-2}, then find the total distance travelled.
4. A boat covers certain distance between two spots on a river taking 4 h going downstream and 6 h going upstream. What will be the time taken by the boat to cover the same distance in still water ?
5. A particle starting from rest from O accelerates uniformly along a straight line and crosses two points A and B which are 5 m apart in 2 s. If velocity at B is 2 ms^{-1} more than the velocity at A, find the distance OA.

Section II

B. Only One Option Correct

6. An object of height 5 cm is placed 2 m in front of a concave mirror of radius of curvature 40 cm. The size of image is
   (a) 0.25 cm (b) 0.45 cm (c) 1.10 cm (d) 2.20 cm
7. A man runs towards a mirror at a speed 15 ms^{-1}. The speed of the image relative to the man is
   (a) 15 ms^{-1} (b) 30 ms^{-1} (c) 35 ms^{-1} (d) 20 ms^{-1}
   (e) 25 ms^{-1}
8. A virtual image larger than the object can be obtained by
   (a) concave mirror (b) convex mirror (c) plane mirror (d) None
9. When a plane mirror is rotated through an angle θ, the reflected ray turns through the angle 2θ, then the size of the image
   (a) is doubled (b) is halved
   (c) remains the same (d) becomes infinite

10. A car starts from rest and moves with constant acceleration. The ratio of distance covered by the car in nth second to that covered in n seconds is

(a) $\frac{2n-1}{n^2}$ (b) $\frac{n^2}{2n-1}$ (c) $\frac{2n-1}{n}$ (d) $1:n$

11. A stone falls from rest. The total distance covered by it in the last second of its motion is equal to the distance covered in the first three seconds of its motion. How long does the stone remain in the air ? (Take $g = 10\ ms^{-2}$.)

(a) 4 s (b) 5 s (c) 6 s (d) 7 s

12. A body moves with velocity v, $2v$ and $3v$ in the first, second and third, "one-third" distance of the path travelled. Its average velocity is

(a) $\left(\frac{6}{11}\right)v$ (b) $\left(\frac{12}{11}\right)v$ (c) $\left(\frac{18}{11}\right)v$ (d) $\left(\frac{36}{11}\right)v$

13. A train starts from rest and acquires a speed v with uniform acceleration α. Then it comes to stop with uniform retardation β. What will be the average velocity of the train ?

(a) $\frac{\alpha\beta}{\alpha+\beta}$ (b) $\frac{\alpha+\beta}{\alpha\beta}$ (c) $\frac{v}{2}$ (d) v

14. A person is throwing balls into the air one after the other. He throws the second ball when the first ball is at the highest point. If he is throwing two balls every second, how high do they rise ?

(a) 5 m (b) 3.75 m (c) 2.50 m (d) 1.25 m

15. Which of the following can be zero when the particle is in motion for some time ?

(a) Displacement (b) Distance covered (c) Speed (d) None of these

16. A car moving with a speed of 40 kmh^{-1} can be stopped by applying brakes after at least 2 m. If the same car is moving with a speed of 80 kmh^{-1}, what is the minimum stopping distance ?

(a) 8 m (b) 2 m (c) 4 m (d) 6 m

17. A car travels the first half distance between two places with a speed of 30 kmh^{-1} and the remaining half with a speed of 50 kmh^{-1}. The average speed of the car is

(a) 37.5 kmh^{-1} (b) 10 kmh^{-1} (c) 42 kmh^{-1} (d) 40 kmh^{-1}

Answers

1. 125 m	**2.** 0.2 m/s	**3.** 13.5 m	**4.** 24 hr	**5.** $\frac{9}{8}$ m
6. (b)	**7.** (b)	**8.** (a)	**9.** (c)	**10.** (a)
11. (b)	**12.** (c)	**13.** (c)	**14.** (d)	**15.** (a)
16. (a)	**17.** (a)			

Workout 3

Section I

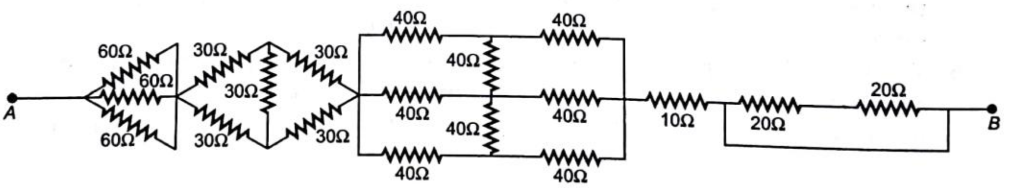
A. Subjective Questions

1. (a) Two identical spheres A and B of charges $4Q$ and $2Q$, respectively, are connected by a wire. Find the ratio of the two forces between the spheres A and B before and after connection with the wire.

(b) 10^{19} electrons are removed from a sphere of radius 1 m. Find its potential.

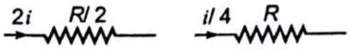
2. Two bulbs of wattages 60 W and 100 W, respectively, each rated at 230 V, are connected in series with a supply of 440 V. Which bulb will fuse ?

3. Find the equivalent resistance between points A and B in the given figure.
If the current flowing through 60 Ω resistor is $3A$ then find, the current through 10 Ω resistor. Also find the potential difference across AB.



4. An electric device draws a current of 2 A for 1.25 min. If the resistance of the element present in it is 100 Ω, calculate the electric energy drawn by the device in kilo joule.

5. A student in a hostel used a 60 W lamp for 5 h a day and a 100 W electric iron for half an hour a day on the average. What will be the bill of electricity for a month (30 days) if the cost of one unit of electricity is 50 paise ?

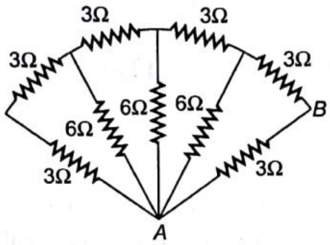
6. 

In the above circuits, if the power in the first circuit is K watt, the power in the second circuit would be ....

Section II

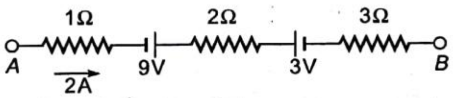
B. Only One Option Correct

7. The resistance between points A and B is



(a) $9\,\Omega$ (b) $2\,\Omega$
(c) $12\,\Omega$ (d) $8\,\Omega$

8. The potential difference between points A and B in the given branch of a circuit is



(a) 6 V (b) 12 V
(c) 9 V (d) 0 V

9. Three wires having resistances $1\,\Omega$, $2\,\Omega$ and $6\,\Omega$ are joined in parallel across a battery. If a current 0.1 A flows through $6\,\Omega$ resistor, the total current drawn by the combination is

(a) 0.6 A (b) 0.3 A
(c) 0.1 A (d) 1 A

10. The material used in heater coils is
(a) Nichrome (b) Copper
(c) Silver (d) Manganese

11. Five balls numbered 1, 2, 3, 4, 5 are suspended using separated threads. The balls (1, 2), (2, 4) and (4, 1) show electrostatic attraction, while balls (2, 3) and (4, 5) show repulsion. Therefore, the ball 1 must be
(a) negatively charged (b) positively charged
(c) neutral (d) made of metal

12. The work done (in joule) in carrying a charge of 100 C between two points having a potential difference of 10 V is
(a) 0.1 (b) 10
(c) 100 (d) 1000

13. If an electron experiences a force equal to its weight when placed in an electric field, the intensity of the electric field is (mass of electron $= 9 \times 10^{-31}$ kg, $g = 10\ \text{ms}^{-2}$)
(a) $5.62 \times 10^{-11}\ \text{NC}^{-1}$ (b) $5.62 \times 10^{11}\ \text{NC}^{-1}$
(c) $5.62 \times 10^{-9}\ \text{NC}^{-1}$ (d) $5.62 \times 10^{9}\ \text{NC}^{-1}$

Section III

C. Assertion and Reason Type

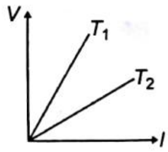
In each of the following questions a statement of Assertion (A) is given followed by a corresponding statement of Reason (R) just below it. Of the statements mark the correct answer as.

(a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.

14. **(A)** No two electric lines of force can intersect each other.
(R) Tangent at any point of electric line force gives the direction of electric field.

15. **(A)** Electric potential of earth is zero.
(R) The electric field of earth is zero.

16. **(A)** When a wire is stretched to double its length, its resistivity becomes twice.
(R) $R = \frac{Sl}{A}$



17. **(A)** V-I graphs for a conductor at two different temperatures T_1 and T_2 are as shown in the figure, here $T_1 > T_2$.
(R) Resistance of a conductor decreases with increase of temperature.

Answers

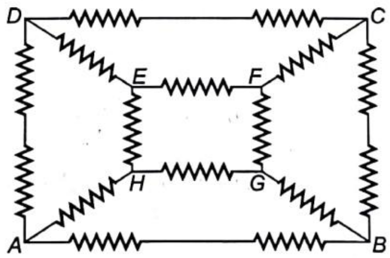
1. (a) $\frac{8}{9}$ (b) 14.4×10^9 volt		**2.** 60 W	**3.** $\frac{260}{3}\ \Omega$, 9 A	**4.** 30 kJ
5. ₹ 11.5	**6.** $\frac{K}{32}$	**7.** (b)	**8.** (a)	**9.** (d)
10. (a)	**11.** (d)	**12.** (d)	**13.** (a)	**14.** (a)
15. (c)	**16.** (d)	**17.** (c)		

Workout 4

Section I

A. Subjective Questions

1. In the given figure all the resistors are equal. If the resistors between points E, H and F, G are removed, the effective resistance between A and B is R. Now the removed resistors are reconnected and the resistors between points E, F and G, H are removed. Find the effective resistance between points A and B now.



2. In a car race, car A takes a time of t second less than car B at the finish and passes the finishing point with a velocity v ms^{-1} more than the car B. Assuming that the car starts from rest and travels with constant accelerations a_1 and a_2 respectively, show that $v = (\sqrt{a_1 a_2})\, t$.

3. A plane mirror is fixed on the wall of a room and a man stands in front of the mirror at the middle of the room. Find the minimum height of the mirror through which the man can see the full image of the wall behind him.

4. A convex lens of focal length produces twice the magnified image of an object on a screen placed 180 cm from the object. Find the power of the convex lens.

5. An object weighs 20 kgf on earth surface. Find the weight of the object on a planet whose mass and radius are twice that of the earth planet.

6. A block placed on a horizontal surface is being pushed by a force F making an angle θ with the vertical. If the coefficient of friction is μ, how much force is needed to get the block just started. Discuss the situation when $\tan\theta < \mu$.

Section II

B. Only One Option Correct

7. The angular velocity of the minutes' hand and the hour's hand in a clock are in the ratio of

(a) 1:12
(b) 6:1
(c) 12:1
(d) 1:6

8. A particle of mass m is executing a uniform circular motion along a path of radius r. If the magnitude of its momentum is p, then the radial force acting on the particle will be

(a) pmr
(b) $\frac{rm}{p}$
(c) $\frac{mp^2}{r}$
(d) $\frac{p^2}{mr}$

9. A stone is projected with a velocity 9.8 ms^{-1} horizontally from a tower of height 100 m. Its velocity after 1 s will be .... ms^{-1}.

(a) 9.8
(b) 4.9
(c) $9.8\sqrt{2}$
(d) $4.9\sqrt{2}$

10. A man sitting in a train in motion is facing the engine. He tosses a coin up and the coin falls in front of him. The train is

(a) moving forward with uniform speed
(b) moving backward with uniform speed
(c) moving forward with acceleration
(d) moving forward with deceleration

Section III

C. Assertion and Reasoning Type Questions

In each of the following questions a statement of Assertion **(A)** is given followed by a corresponding statement of Reason **(R)** Just below it. Of the statements mark the correct answer.

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) Assertion is true but Reason is false.
(d) Assertion is false but Reason is true.

11. **(A)** On a banked curved track, the vertical component of normal reaction provides the necessary centripetal force.
(R) Centripetal force is always required for the motion in a curved path.

12. **(A)** A person sitting in an artificial satellite revolving around the earth feels weightlessness.
(R) There is no gravitational force on the satellite.

13. **(A)** The algebraic sum of currents meeting at junction in a closed circuit is zero.
(R) The Kirchhoff's law does not obey the law of conservation of charge.

14. **(A)** The acceleration of a body down a rough inclined plane is less than the acceleration due to gravity.
(R) The body is able to slide on an inclined plane only when its acceleration is greater than the acceleration due to gravity.

Section IV

D. More Than One Options Correct

15. A convex lens having focal length 30 cm is taken. Which of the following statements is/are correct ?
(a) The object distance for a real, inverted and four times magnified image is 37.5 cm.
(b) The object distance for a real, inverted and four times magnified image is 22.5 cm.
(c) The object distance for a virtual, erect and four times magnified image is 22.5 cm.
(d) The object distance for a virtual, erect and four times magnified image is 37.5 cm.

16. The refractive index of glass is $\frac{3}{2}$ and that of water is $\frac{4}{3}$. Which of the following statements is/are false ?
(a) The refractive index of water with respect to glass is 1.125.
(b) The ratio of the time taken by light to travel 20 m in glass to the time taken by light to travel 60 m in water is 3 : 8.
(c) The ratio of the time taken by light to travel 20 m in glass to the time taken by light to travel 60 m in water is 8 : 3.
(d) The ratio of the speed of light in water to that in glass is 1.125.

Answers

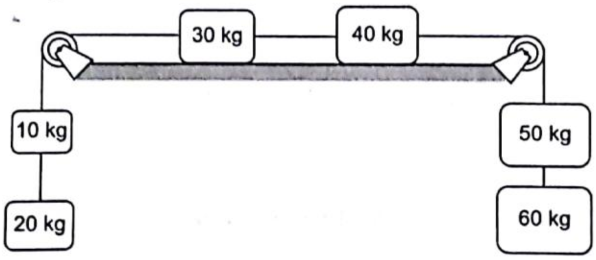
1. $\frac{55R}{39}$	**3.** $\frac{\text{Height of wall}}{3}$	**4.** $\frac{5}{6}$ D	**5.** 10 kgf
6. $\frac{\mu mg}{\sin\theta + \mu\cos\theta}$	**7.** (c)	**8.** (c)	**9.** (c)
10. (b)	**11.** (d)	**12.** (c)	**13.** (c)
14. (d)	**15.** (a, c)	**16.** (a, b)	

Workout 5

Section I

A. Subjective Questions

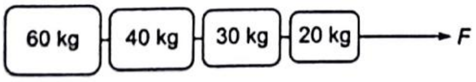
1. In the given figure, the coefficient of friction between the bodies and the surface is 0.1. Find acceleration of 60 kg block.



2. Two men can go around a curved track in 6 min and 12 min, respectively. If they start from the same place, then find the time interval for successive crossings if they travel in (a) opposite sense (b) same sense.

3. In case of uniform circular motion prove that linear acceleration exists in radial direction and its value is $\frac{4\pi^2 r}{T^2}$.

4. A heavier body and a lighter body both moving with same kinetic energy are brought to rest by applying the same retarding force. Then prove that both will travel same distance before coming to rest but the lighter body will stop first.

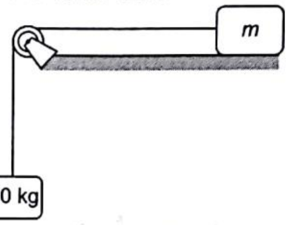
5. 

If the force F is 300 N, then find the force on 60 kg body.

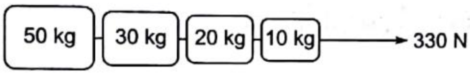
6. A force of 200 N acts on a body of mass 50 kg at rest for 10 s after which the force ceases to act and the body continues to move. Later on, an another force of 100 N acts on the body and brings it to rest. If the total distance travelled by the body is 700 m, then find the total time taken for the journey. Also find the time-interval for which the body moved uniformly.

7. A ball of mass 500 g is dropped from a certain height onto the ground. It was observed that it loses 36% of its total energy in every collision and rises upto a height of 39.4 m after the 1st collision. Find the height from which ball is released ?

8. In the given figure, if the acceleration of the system is 3 ms^{-2} and coefficient of friction between m and surface be 0.1, then find the value of m.



9. Keeping the angle of banking of a road constant, the maximum speed of the vehicles is to be increased by 20%. Find the new radius of curvature of the road if it was initially 24 m.

10. 

If the coefficient of friction between the bodies and the surface be 0.25, then find out the net force on 20 kg body.

11. A 10 kg ball moving with a velocity of 4 ms^{-1} approaches another ball of mass 5 kg moving with a velocity of 1 ms^{-1} in the same direction as the first ball.

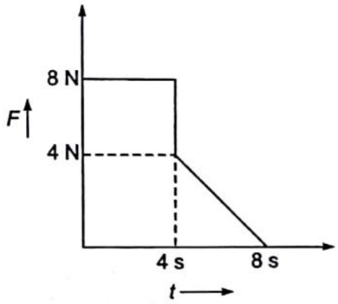
(a) If the collision is perfectly elastic, then find the velocities after collision.

(b) If the collision is perfectly inelastic, then find the loss in kinetic energy after the collision.

12. A father (60 kg) and his daughter (20 kg) are both at rest on a frictionless ice pond. The father lifts a 1 kg ball and throws it to his daughter with a horizontal speed 5 ms^{-1}, and the daughter catches it. Find the speeds of both father and daughter.

13. (a) A force of 200 N is acting on a body for 8 s and then ceases to act. The body is moving 200 m in the next 5 s. Find the mass of the body.

(b) In the given force-time graph if the body starts from rest, then find the velocity of the body after 8 s. Given that the mass of the body is 200 g.



14. A fly wheel rotates at the rate of 20 rpm. Find the time required to stop if it is slowing down at the rate of 2 $rads^{-2}$. Also find the number of revolutions it will make before coming to rest.

15. A boy of mass 50 kg is walking on a stationary boat of mass 200 kg with a speed of 6 ms^{-1}. Find his distance with respect to the ground in 10 s.

Answers

1. 3.5 m/s^2 **2.** (a) 4 min, (b) 12 min **5.** 120 N

6. 32.5 s, 2.5 s **7.** 61.56 m **8.** 35 kg **9.** 34.56 m

10. 10 N **11.** (a) 2 m/s, 5 m/s (b) 15 J **12.** $v_f = \frac{1}{12}$ m/s, $v_d = \frac{5}{21}$ m/s

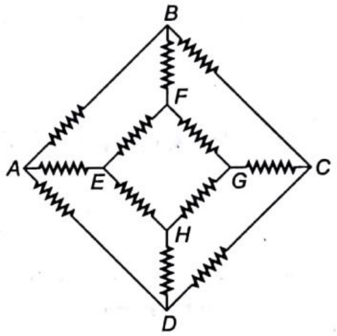
13. (a) 40 kg, (b) 200 m/s **14.** $\frac{\pi}{3}$ s, $\frac{\pi}{18}$ revolutions **15.** 75 m

Workout 6

Section I

A. Subjective Questions

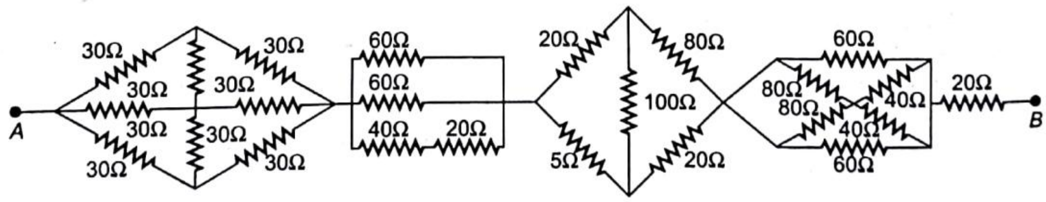
1.



If all resistances are equal to 120 Ω, then find the equivalent resistance between points C and G. Also find the equivalent resistance between points A and C.

2. Three copper wires having masses in the ratio of 3 : 4 : 5 are taken with their lengths in the ratio of 5 : 4 : 3 and connected in series across a battery of emf 106 V. Find the voltages across each wires.

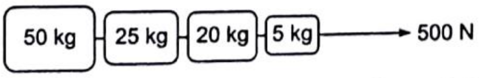
3.



If a current of 4 A is passing through 5 Ω resistor in the given circuit, then find the potential difference between points A and B.

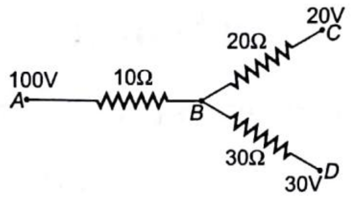
4. (a) A ball of certain mass moving with a velocity of 20 ms^{-1} collides with a stationary ball of 4 times its mass. If the balls stick together after the collision, then findout the speed of the stuck mass after the collision.

(b)



Find the force acting between 25 kg and 20 kg body.

5.



Find the current passing through 20 Ω and 10 Ω resistor. Also find the potential at B.

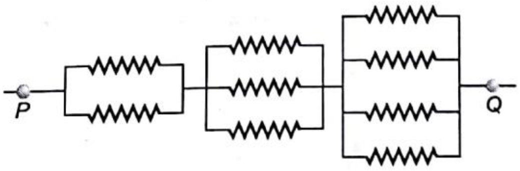
6. A stone of mass 5 kg falls from the top of a tower 80 m high and buries itself one metre deep in sand. Find the average resistance offered by the sand.

7. A house is fitted with 8 lamps, each of 40 W, and two fans, each consuming a current of 0.25 A. The energy is supplied at the rate of 220 V. If the lamps are lighted for 3 hrs a day and the fans work for 6 hrs a day, find out the electricity bill for 30 days. The cost of energy consumed is at the rate of 40 paise per kWh.

Section II

B. Only One Option Correct

8. A number of 24 Ω resistors are connected as shown below. Then the effective resistance between points P and Q is



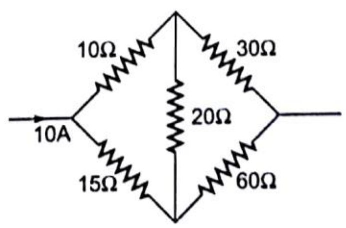
(a) 21.6 Ω
(b) $\frac{24}{3}$ Ω
(c) 26 Ω
(d) 36 Ω

9. A wire of resistance 10 Ω is elongated by 10%. The resistance of the elongated wire is

(a) 11 Ω
(b) 11.1 Ω
(c) 12.1 Ω
(d) 13.1 Ω

10. The unit of magnetic moment in SI system is ....

11. In the given figure, the current passing through 20 Ω resistor is ....



Section III

C. Assertion and Reason Type

In each of the following questions a statement of Assertion (**A**) is given followed by a corresponding statement of Reason (**R**), just below it. Of the statements mark the correct answer as

(a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.

12. **(A)** A hollow sphere is filled with water and used as a simple pendulum. If water leaks out gradually, then the time period first increases and then decreases till it reaches original value.
(R) When water is leaking out the height of centre of gravity decreases and when the total water has leaked out it moves back to its original position.

13. **(A)** Basic difference between an electric line and magnetic line of force is that former is discontinuous while the later is continuous or endless.
(R) No electric lines of forces exist inside a charged body but magnetic lines do exist inside a magnet.

14. **(A)** Current is a scalar quantity.
(R) Electric current arises due to continuous flow of the charged particles or ions.

15. **(A)** At null point, a compass needle shows the North and South directions.
(R) At null point, $B_{total} = 0$.

Answers

1. 70 Ω, 90 Ω	**2.** 62.5 V, 30 V, 13.5 V		**3.** 500 V	**4.** (a) 4 m/s, (b) 375 N
5. $\frac{25}{11}$ A, $\frac{13}{11}$ A, $\frac{720}{11}$ V	**6.** 4 kN	**7.** ₹ 19.44	**8.** (c)	**9.** (c)
10. A-m^2	**11.** Zero	**12.** (a)	**13.** (b)	**14.** (b)
15. (d)				